Lab 7: Rotational Motion (M7)

Objectives

- Study the relationships among torque, angular acceleration and moment of inertia for a rigid body.
- Use the conservation of angular momentum to understand motion on a rotating platform.

Theory

Earlier, you explored Newton's Second Law of Motion in its linear form ($F = ma$). We are using the following, important concepts in rotational motion:

torque τ - the ability of a force to rotate a mass about an axis, $\tau = Fr)$,

moment of inertia *I* - how mass is distributed about the axis of rotation,

for a rotating disk $I = \frac{1}{2}MR^2$, for a rotating ring $I = \frac{1}{2}M(R_{inner}^2 + R_{outer}^2)$, and

for a rotating point mass $I = MR^2$;

the unit for the moment of inertia is kg*m^2

angular velocity $\omega = \Delta\theta/\Delta t$, related to linear (tangential) velocity through the equation

$$v = \omega r, \tag{1}$$

angular acceleration α (related to linear acceleration through the equation $a = \alpha r)$.

Newton's Second Law may be extended to include rotational motion: ***torque is equal to the moment of inertia times the angular acceleration.***

$$\tau = I\alpha \tag{2}$$

Additionally,

$$ma = mg - T \Rightarrow T = mg - ma = m(g-a) \tag{3}$$

is obtained from the linear version of Newton's Second Law balancing the tension force T and the weight. Eliminating tension force T and rearranging terms yields the equations:

$$\tau = Fr = Tr = m(g\text{-}a)r = I\alpha = I(a/r) \tag{4}$$

$$\alpha = a/r \Rightarrow \tau = m(g\text{-}a)r = I(a/r) = I\alpha \tag{5}$$

$$\tau = I\alpha \quad \text{or} \quad m(g\text{-}a)r = I\alpha \tag{6}$$

Now we can find the moment of inertia. After you have made measurements, you will be able to calculate the driving torque: $\tau = m(g\text{-}a)r$ for each mass. Once we know the value of the torque and the angular acceleration, we can calculate the moment of inertia simply dividing the torque by angular acceleration $\tau = I\alpha$ so $I = \tau/\alpha$.

A rotating platform has an angular velocity (ω) measured in radians per second. The relation between the angular velocity and linear (tangential) velocity is similar to that between angular and linear accelerations: $v = \omega r$.

Angular momentum (**L**), a conserved quantity, is defined by the vector equation:

$$L = I\omega \ , \qquad L \text{ is measured in kg*m}^2\text{/s.} \tag{7}$$

Kinetic energy (KE) of the rotating object is given by the following equation:

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{v}{r}\right)^2, \qquad KE \text{ is measured in joules (J).} \tag{8}$$

<u>Setup for Activities 1 and 2:</u>

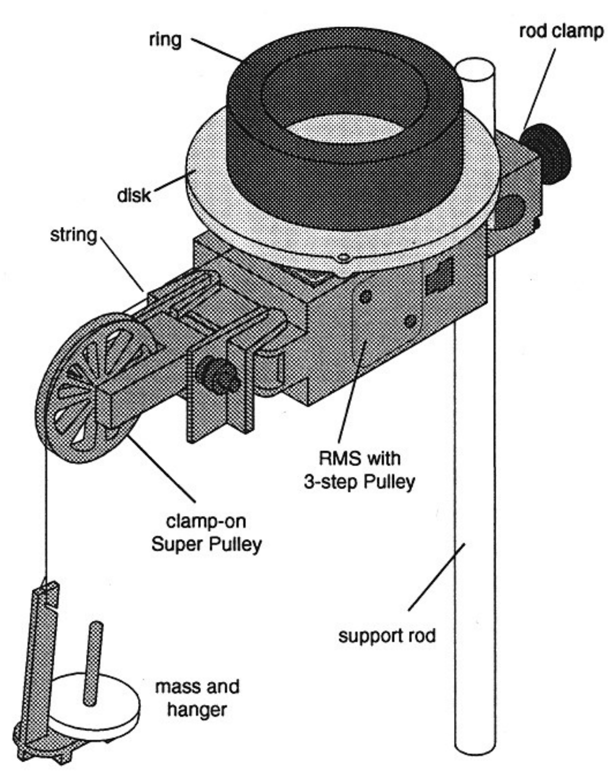


Fig. 1. Rotational inertia setup of disk and ring

The Aluminum disk can be attached to the Rotary Motion Sensor (RMS) by screwing the disk to a 3-step pulley. A black ring is placed on top of the Aluminum disk and is held in place by two notches that allow the ring not to slip off the disk. A string, which is redirected by the clamp-on Super Pulley, carries a mass m. The string is wound around the 3-step pulley directly under the Aluminum disk.

NOTE: The clamp-on Super Pulley must be adjusted at an angle so the thread runs in a line tangent to the point where it leaves the 3-step pulley and straight down the middle of the groove on the clamp-on Super Pulley.

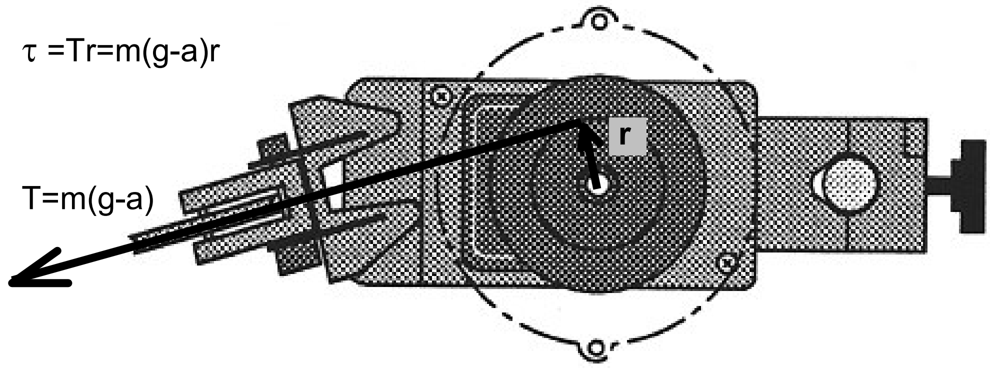
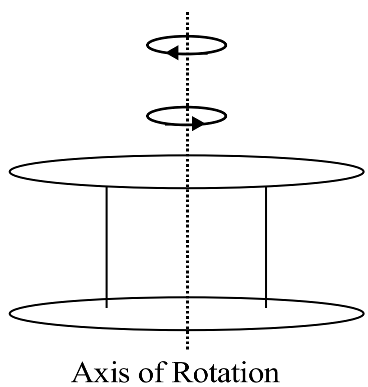


Fig. 2. Adjustment for clamp-on Super Pulley[1]

Setup for Activities 3 and 4:

The platforms will be used to illustrate the conservation of angular momentum.

Each platform is large enough for a person to **sit** on it. Always sit down slowly and carefully on the top of these platforms because they are free to rotate. Try to evenly distribute your mass along the axis of rotation.



The tops of the platforms are free to rotate in either direction about the axis of rotation.

Procedure:

Activity 1: Moment of Inertia

Download files for experiment M7 from Brightspace page for Physics 220.

In this activity, you will calculate the combined moment of inertia of the small Aluminum disk and the black ring. You will use the theoretical formulas for the moment of inertia of a

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[1] Rotary Motion Sensor – Instruction Manual by PASCO Scientific, 1996, pp. 3, 16.

disk and a ring. The rest of the apparatus, i.e., the 3-step pulley, is less than 1% of the overall moment of inertia and should not play a major role in calculations (you may ignore it).

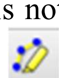
Remove the black ring and unscrew the Aluminum disk from the Rotary Motion Sensor (RMS). Measure the mass the black ring and the Aluminum disk and record them on the data sheets. Use the digital caliper to find the radius of the Aluminum disk and the inner and outer radius of the black ring. When finding the radius, it will be easier to find the diameter and divide by two. Ask your TA if you need help with the caliper. Using these values, **calculate** the theoretical value of the total moment of inertia of the Aluminum disk and the black ring combined.

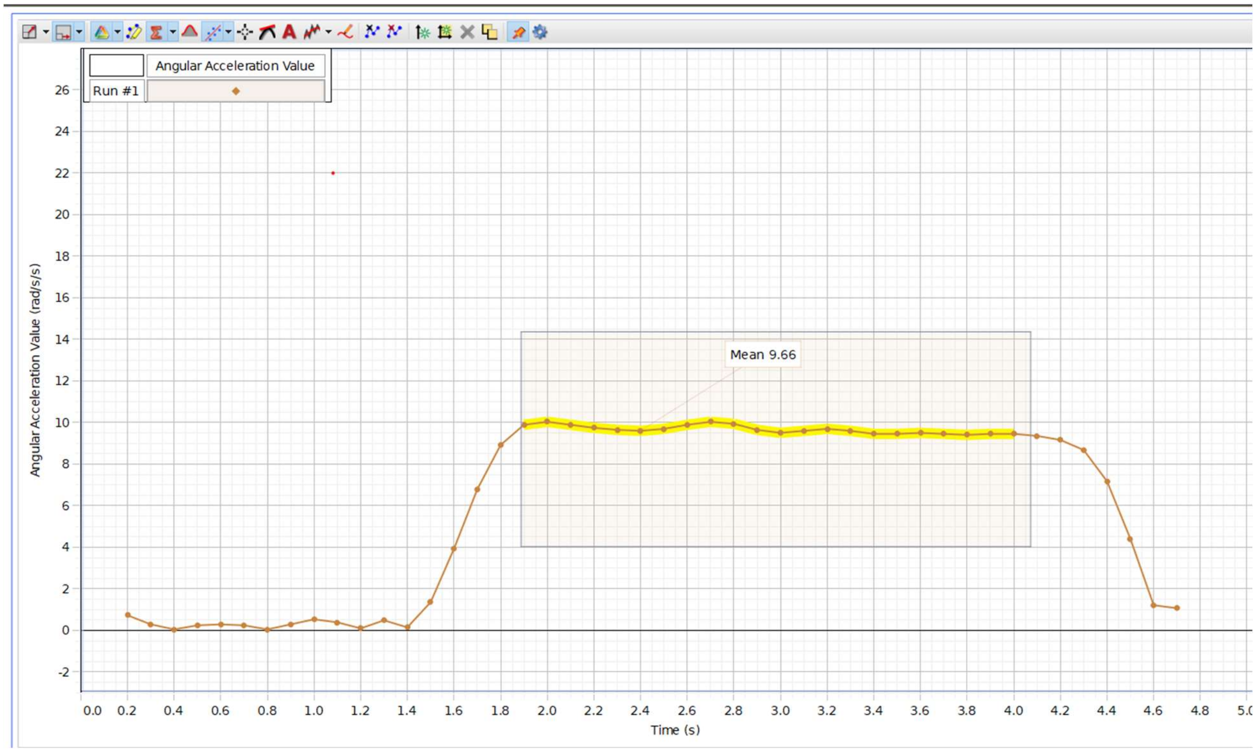
Activity 2: Moment of Inertia from Angular Acceleration

Double-click on the “M7 Activity 2” icon to run it.

In this activity, you will measure angular acceleration due to a descent of a known mass. Then, you will calculate the moment of inertia of the Aluminum disk and the black ring combined. The time of descent is proportional to the moment of inertial of the disk + ring and inversely proportional to the mass of the hanging object.

Before putting the Aluminum disk and the black ring system back on top of the Rotary Motion Sensor, measure and record the radius of the **middle disk** on the 3-step pulley. This middle disk on the 3-step pulley is where you will wind the string around. You should use the caliper to measure the disk's radius. Do not confuse the radius with disk's diameter.

For each trial, **wind enough string around the middle disk** of the 3-step pulley. Start recording data. As soon as a couple of data points appear on the screen, release the weight. Allow the weight to fall from the extension rod to the table. When the weight reaches the table, slowly stop the Rotary Motion Sensor. Focus on the constant angular acceleration section of the graph; this should be the flat part of the graph. Use the “Smart Tool”  button to read the average value of the angular acceleration. Ignore the sign of the angular acceleration, i.e., use the absolute value. The sign of the angular acceleration simply indicates the direction of the rotation and is not going to change the moment of inertia. You may also use the “Data Highlighter” tool  to select data points that have approximately constant value (ignoring points at the beginning and points after the weight hit the “landing” foam. This method allows you to find more accurate average angular acceleration value in the selected range. Here is an example image with “Data Highlighter” tool.



Use masses of 45, 65 and 85 grams. The brass discs are stamped with the mass value and the plastic hanger mass is equal to 5 grams.

Measure the average acceleration |α| and then calculate the torque τ that is applied by the string with hanging mass to the disk and the ring spinning together with the same angular velocity. The torque $\tau = m(g\text{-}a)r = I|\alpha_{AVE}|$ should be proportional to the average angular acceleration α_{AVE} with moment of inertia I as the proportionality constant. Calculate the **moment of inertia *I*** for each value of the hanging mass. Next, calculate the average value of the moment of inertia for the disk and the ring combined in Activity 2.

Find the percent difference between the value of the moment of inertia (for the disk and the ring combined) measured in Activity 2 versus the combined moment of inertia from Activity 1.

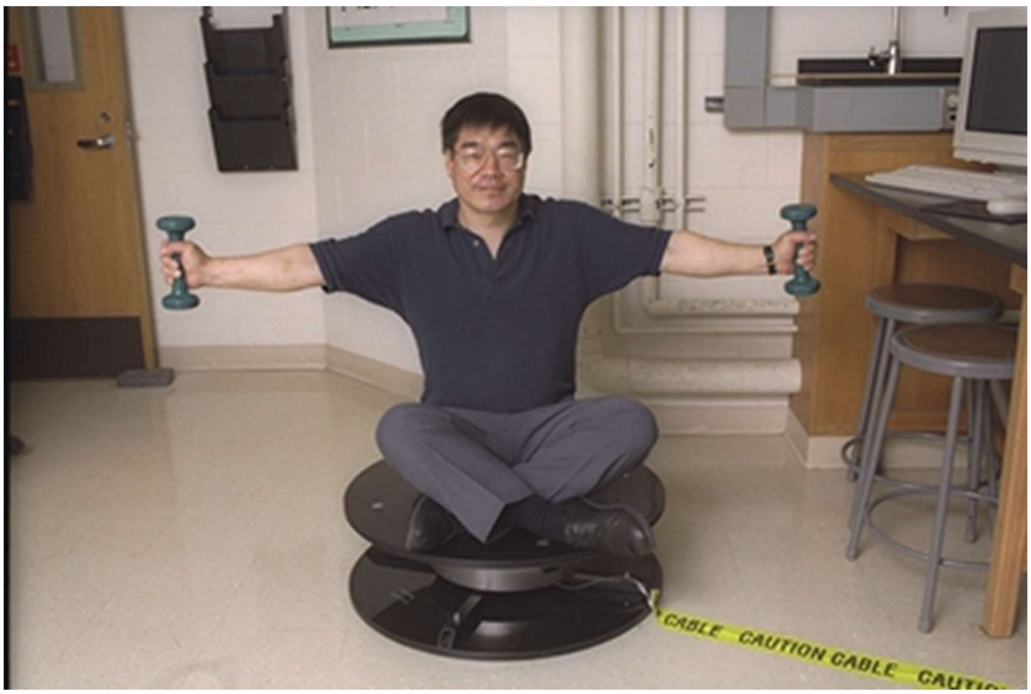
Activity 3: Changing Moment of Inertia

For the final two activities, you need to team up with another group and select a platform.

Warning: In *Activities* 3 and 4 you will need to be very careful. You will be spinning on a platform. If at any time you feel disoriented or sick while spinning, immediately stop and get off the platform.

The platform tops are free to rotate. Hold the platform steady while one person from your group sits down carefully on top of it. Do **not** allow anyone to stand on the platform. The person should **sit near the center** of the platform because the moment of inertia is affected by the distribution of the mass about the axis of rotation. Make sure that the person's legs and feet are completely on the platform (do not allow them to dangle over the side).

Once the person is situated on the platform, hand the person the two equal weights. Put one weight in each hand. Have the person extend his/her arms so that the weights are away from the person's body. Because the space around the platform is limited, you need to make sure that the weights will not hit any of the chairs or table legs.



Place a person at the center of the platform.

Do not spin the platform too fast. Remember, what may look slow to you, may be too fast for the person on the platform. Always listen to your partner. If at any time the person wishes to get off the platform, immediately begin to gently slow and then stop the platform.

Once you have made sure that the person sitting on the platform and holding the weights will not bump into anything, provide a torque and allow the platform to spin. After the platform has spun around several times, tell the person to quickly move the weights in towards his/her body. Observe the resulting changes in the person's angular velocity.

Quickly move the weights in towards the center of mass.

When the platform has spun around several more times, have the person slowly extend the weights back to the original position. Again, note any resulting changes in the person's angular velocity. When finished, slow and then stop the platform. Take the weights from the person and then hold the platform steady while the person gets off the platform.

Allow someone else to get onto the platform and repeat the activity. Repeat this procedure for each person in your group. When you are all finished, answer the questions on your data sheet.

Activity 4: Conservation of Angular Momentum

Warning: In this activity you will be using a spinning bicycle wheel. When walking across the room, handing the wheel to the person on the platform or manipulating the wheel while on the platform, always maintain a firm grasp on the axis of rotation. When handing the wheel to the person sitting on the platform make sure that he/she has control of the wheel before you let go. Because the wheel is spinning, it has a large angular momentum and energy, so please be careful.

Hold the top of the platform steady while one person from your group carefully sits down near the center of the platform. Do **not** allow anyone to stand on the platform.

Along the axis of rotation of the bicycle wheel, there are two rods (two handles). Even when these rods are held firmly, the wheel is free to rotate. Take the bicycle wheel over to the spinner. Firmly grip the bicycle wheel along the axis of rotation with both hands, one on the rod below the wheel and one on the rod above. The spinner is operated by a foot pedal. While pushing down on the pedal with your foot, hold the wheel against the spinning metal. The wheel should begin to rotate. Hold the wheel away from your body because any contact with your body or your clothing will cause the wheel to slow down. Hold the wheel against the spinner until it begins to rotate quickly (maximum angular speed is about 300 rpm).

Carefully bring the wheel back to your platform being sure that it does not come into contact with your body or clothing. Hand the bicycle wheel to the person sitting on the platform with the **axis of the wheel horizontal**. Make sure that the person has control of the wheel before you let go.

While firmly gripping the rods (handles), the person on the platform should turn the wheel 90˚, so that its **axis of rotation is vertical,** then flip it 180° (upside down). Hold each position for a few seconds so that the group may observe what happens to the motion of the person on the platform. Repeat the wheel positions several times until the wheel slows or stops spinning.

When the person on the platform is finished, slow and then stop the platform. Take the wheel from the person and hold the platform steady while the person gets off the platform.

Let another person from your group get onto the platform and repeat the activity. Repeat this procedure for each person in your group. When you are all finished, answer the questions from the data sheets.

Complete the lab report and return it to the lab TA.

Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. Your completed Data Sheets. (6.5 points)
2. Return the completed lab report to your lab TA.