Table of Orthogonal Functions Arising from Sturm - Liouville Systems

Name and Physical Application	Rodrique's Formula	Generating Funcion	Differential Equation	S -L Form of D. E.	Orthonormality
Legendre Polynomials 1) Multiple Expansion 2) ∇^2 in sph. coor.	$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$	$\sum_{n=0}^{\infty} P_n(x)t^n=\frac{1}{\sqrt{1-2xt+t^2}}$ $(0<t<1)$	$(x^2-1)P_n''+2xP_n'-n(n+1)P_n = 0$	$\frac{d}{dx}\left((1-x^2)P_n'\right)+n(n+1)P_n=0$	$\int_{-1}^{1} P_n P_m \, dx = \delta_{nm}\frac{2}{2n+1}$
Hermite Polynomials Quantum Oscillator	$H_n(x)=(-1)^n e^{x^2}\frac{d^n e^{-x^2}}{dx^n}$	$\sum_{n=0}^{\infty}\frac{H_n(x)}{n!}t^n=e^{-t^2+2tx}$ $(t>0)$	$H_n''-2xH'+2nH_n=0$	$\frac{d}{dx}(e^{-x^2}H_n')+2ne^{-x^2}H_n=0$	$\int_{-\infty}^{\infty}H_n H_m e^{-x^2}dx=\delta_{nm}\sqrt{\pi}\,2^n n!$
Laguerre Polynomials H - atom	$L_n(x)=e^x\frac{d^n}{dx^n}(x^n e^{-x})$	$\sum_{n=0}^{\infty}\frac{L_n(x)}{n!}t^n=\frac{e^{\frac{-xt}{1-t}}}{1-t}$ $(0<t<1)$	$xL_n''+(1-x)L_n'+nL_n=0$	$\frac{d}{dx}(xe^{-x}L_n')+ne^{-x}L_n=0$	$\int_0^{\infty}L_n L_m e^{-x}dx=\delta_{nm}(n!)^2$
	Series Presentation				
Bessel's Function (of integral order) ∇^2 in cylindrical coordinates	$J_n(x)=\sum_{m=0}^{\infty}\frac{(-1)^m\left(\frac{x}{2}\right)^{2m+n}}{(n+m)!\,m!}$	$\sum_{n=-\infty}^{\infty}J_n(x)t^n=e^{\frac{1}{2}x\left(t-\frac{1}{t}\right)}$ $(t>0)$	$x^2J_n''+xJ_n'+(x^2-n^2)J_n=0$	$\frac{d}{dx}(xJ_n')+\left(x-\frac{n^2}{x}\right)J_n=0$	
Trigonometric Functions Classical Oscillator	$f_n=\sin nx=\sum_{m=0}^{\infty}(-1)^m\frac{(nx)^{2m+1}}{(2m+1)!}$ $g_n=\cos nx=\sum_{m=0}^{\infty}(-1)^m\frac{(nx)^{2m}}{(2m)!}$		$f_n''+n^2f_n=0$ $g_n''+n^2g_n=0$	$\frac{d}{dx}(f_n')+n^2f_n=0$ $\frac{d}{dx}(g_n')+n^2g_n=0$	$\int_{-\pi}^{\pi}f_n f_m\,dx=\delta_{nm}\pi$ $\int_{-\pi}^{\pi}g_n g_m\,dx=\delta_{nm}\pi$ $\int_{-\pi}^{\pi}f_n g_m\,dx=0$