Phys 241 Final
April 29, 1996

1. A convex lens has a focal length of f = 40 cm. An object of 3 cm height is placed 30 cm in front of the lens. What is the magnification of the image? Is it a real or virtual image?

$p = 30$ cm

$$\frac{1}{i}+\frac{1}{p}=\frac{1}{f} \Rightarrow i=\frac{1}{\frac{1}{f}-\frac{1}{p}}=\frac{1}{\frac{1}{40}-\frac{1}{30}}=-120\text{ cm}$$

$$m=-\frac{i}{p}=-\left(\frac{-120}{30}\right)=\boxed{+4}$$

$$i<0 \Rightarrow \boxed{\text{virtual}}$$

	m	image type
(a)	−20	real
(b)	+4	virtual
(c)	−16	real
(d)	+16	virtual
(e)	+40	virtual

2. Consider two charges $q = 3 \times 10^{-3}$ C a distance $a = 10^{-4}$ m apart. How much work is done by the field to turn the electric dipole from $\theta = 150°$, to $\theta = 30°$ when a uniform $|\vec{E}| = 10^{7}$ V/m?

$\vec{p}$ = electric dipole moment $|\vec{p}| = |q|a$

$$U(\theta) = -\vec{p}\cdot\vec{E} = -|\vec{p}||\vec{E}|\cos\theta$$

$$-W = \Delta U = U_f - U_i = U(30°) - U(150°) = (-|qa||\vec{E}|\cos 30°) - (-|qa||\vec{E}|\cos 150°)$$

W = work done by forces

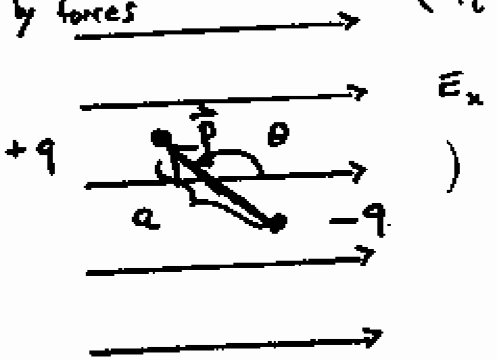
(a) −5.2 J

(b) +5.2 J

(c) −9.6 J

(d) +9.6 J

(e) +3.4 J



$$W = |qa||\vec{E}|(\cos 30° - \cos 150°)$$

$$= \underbrace{(3\times10^{-3}\,C)}_{q}\underbrace{(10^{-4}\,m)}_{a}\underbrace{\left(10^{7}\,\frac{V}{m}\right)}_{|\vec{E}|}(\cos 30° - \cos 150°) = \boxed{5.196\text{ J}}$$

3. A diffraction grating produces a line in 2nd order at $\theta = 35°$. If the grating has $\frac{3000 \text{ lines}}{\text{cm}}$. What is the wavelength of the light?

$$d = \text{grating spacing} = \frac{\text{width of grating}}{\#\text{ rulings}} = \frac{w}{N}$$

(a) 1210 nm

(b) 956 nm

(c) 3132 nm

(d) 810 nm

(e) 319 nm

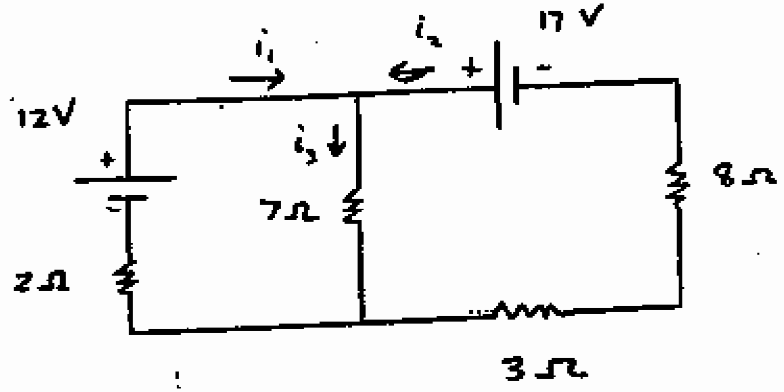
$d \sin\theta = m\lambda$ m=2 (line is second order)

$$\lambda = \frac{d\sin\theta}{m} = \frac{d\sin\theta}{2} = \frac{\frac{w}{N}\sin\theta}{2} = \frac{\sin\theta}{2\left(\frac{N}{w}\right)}$$

$$= \frac{\sin 35°}{2\left(\frac{3000\text{ lines}}{\text{cm}}\right)} = 9.559\times10^{-5}\text{ cm}$$

$$= \boxed{955.9\text{ nm}}$$

4. Consider the two loop circuit. What is the magnitude of the current through the 8Ω resistor?



(a) 7.6 A

(b) 4.3 A

(c) 1.6 A

(d) 0.61 A

(e) 0.32 A

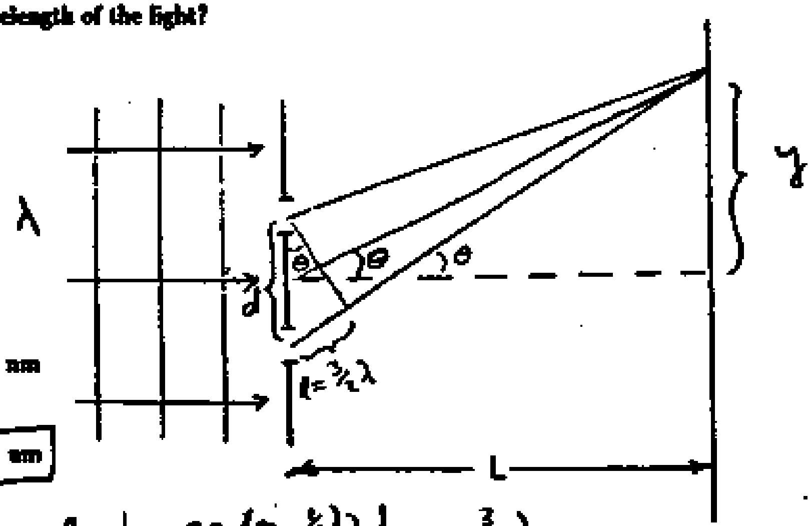
$$-(i_1+i_2)(7\Omega) - i_1(2\Omega) + 12 = 0$$

$$-(i_1+i_2)(7\Omega) - i_2(8+3\ \Omega) + 17 = 0$$

$$\begin{bmatrix}-9 & -7\\ -7 & -18\end{bmatrix}\begin{bmatrix}i_1\\ i_2\end{bmatrix} = \begin{bmatrix}-12\\ -17\end{bmatrix}$$

$$i_2 = \frac{\begin{vmatrix}-9 & -12\\ -7 & -17\end{vmatrix}}{\begin{vmatrix}-9 & -7\\ -7 & -18\end{vmatrix}} = \frac{69}{113} = \boxed{.61}$$

5. Consider a double slit interference experiment where the distance between the slits is $d = 7 \times 10^{-4}$ m and the distance to the screen is 30 meters. The second interference minimum is $y = 20$ cm from the normal line between the slits and the screen. What is the wavelength of the light?



(a) 1230 nm

$\boxed{\text{(b) 3110 nm}}$

(c) 1460 nm $\quad \ell = d\sin\theta = (m - \tfrac{1}{2})\lambda \Big|_{m=2} = \frac{3}{2}\lambda$

$\tan\theta \approx \frac{y}{L} \Rightarrow \theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{20\text{ cm}}{3000\text{ cm}}\right) = .382^0$

(d) 760 nm $\quad \lambda = \frac{d\sin\theta}{3/2} = \frac{(7\times10^{-4}m)\sin(.382^\circ)}{3/2} = \boxed{3110\text{ nm}}$

(e) 920 nm

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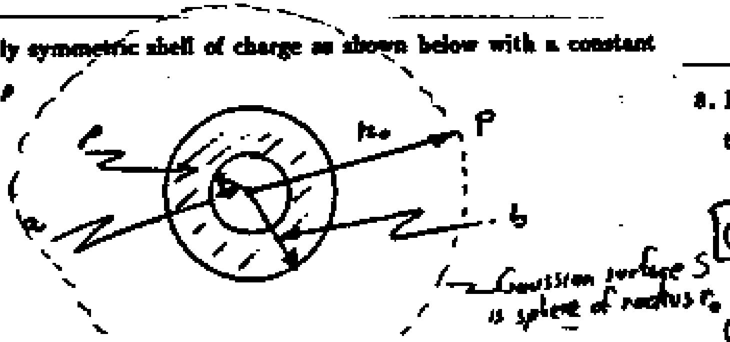
6. Consider the spherically symmetric shell of charge as shown below with a constant volume charge density ρ



Let $\rho = 10^{-8}$ C/m³, the inner radius $a = 0.3$ m, the outer radius $b = 0.6$ m and the distance to P is $r_0 = 8$ m. What is the magnitude of the electric field at point P?

by Gauss' Law $\quad \oint_S \vec{E}\cdot d\vec{A} = \frac{\text{charge in } S}{\epsilon_0}$

(a) 22.9 V/m $\quad \boxed{\text{(d) 1.1 V/m}}$ $\quad E \cdot 4\pi r_0^2 = \frac{\rho\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)}{\epsilon_0}$

(b) 1.7 V/m $\quad$ (e) .5 V/m $\quad E = \frac{\rho \cdot \frac{4}{3}\pi(b^3 - a^3)}{4\pi\epsilon_0 r_0^2} = \frac{\rho(b^3-a^3)}{3\epsilon_0 r_0^2}$

(c) 10 V/m $\quad E = \frac{(10^{-8})((.6\,m)^3 - (.3\,m)^3)}{3(8.85\times10^{-12})(8\,m)^2} = 1.11\ V/m$

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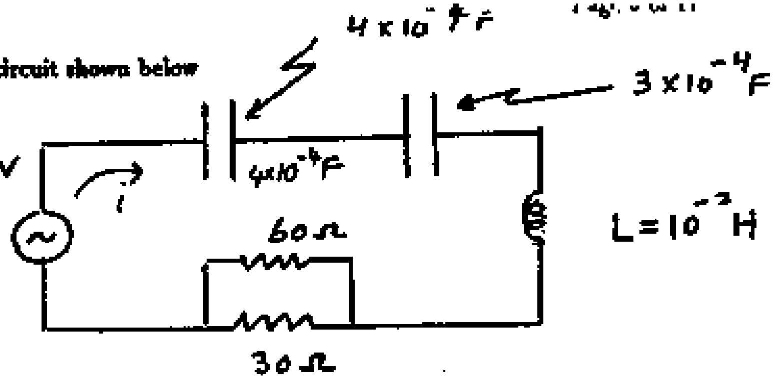
7. Consider the AC circuit shown below

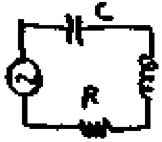
$\mathcal{E}_{rms} = 220V$

$\omega = 2\pi f$



What is the resonant frequency? What is the power P dissipated at resonance?

Equivalent to series LRC



	f_0	P
(a)	7.3 × 10³ Hz	0.92 KW
(b)	1.21 × 10² Hz	2.42 KW
(c)	4.6 × 10² Hz	4.73 KW
(d)	1.21 × 10² Hz	1.73 KW
(e)	4.6 × 10² Hz	2.42 KW

$\frac{1}{C} = \frac{1}{4\times10^{-4}} + \frac{1}{3\times10^{-4}}$

$\Rightarrow C = 1.714\times10^{-4}F$

$R = \frac{1}{\frac{1}{60}+\frac{1}{30}} = 20\ \Omega$

resonant freq $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(10^{-3}H)(1.714\times10^{-4}F)}}$

$= 763.76\ rad/s$

$f_0 = \frac{\omega_0}{2\pi} = \frac{763.76}{2\pi} = 121.55\ Hz.$

impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonance, $X_L - X_C = 0 \Rightarrow Z = R$

$i_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{\mathcal{E}_{rms}}{R}$

$\bar{P} = i_{rms}^2 R = \frac{\mathcal{E}_{rms}^2}{R} = \frac{(220\ V)^2}{20\,\Omega} = 2.42\ kW$

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8. Electromagnetic radiation has a wavelength range from 10^{-3} to 500 meters. What is the corresponding frequency range?

$\boxed{\text{(a) } 3\times10^{11} \leftrightarrow 6\times10^{5} \text{ Hz}}$

(b) 3 × 10⁸ ↔ 2 × 10³ Hz

(c) 2 × 10¹² ↔ 6 × 10⁵ Hz

(d) 6 × 10¹⁰ ↔ 2 × 10³ Hz

(e) 3 × 10¹⁰ ↔ 2 × 10³ Hz

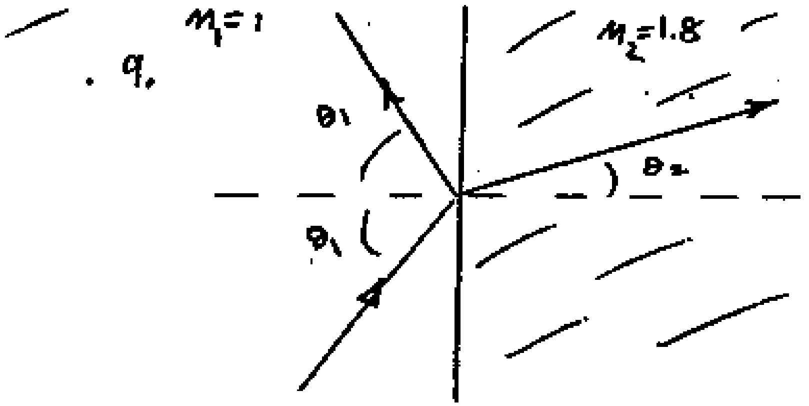
$10^{-3}\ m < \lambda < 500\ m$

$10^{-3}\ m < \frac{c}{f} < 500\ m$

$\frac{1}{10^{-3}\ m} > \frac{f}{c} > \frac{1}{500\ m}$

$\frac{3\times10^8\ m/s}{10^{-3}\ m} > f > \frac{3\times10^8\ m/s}{500\ m}$

$3\times10^{11}\ Hz > f > 6\times10^{5}\ Hz$



A light ray λ = 800 nm is incident on a half plane (n = 1.8) at θ_1 = 67°. What is the refracted angle θ_2? What is the wavelength of the refracted ray?

	θ_2	λ_2
(a)	31°	444 nm
(b)	73°	1440 nm
(c)	58°	1440 nm
(d)	43°	444 nm
(e)	37°	520 nm

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{(1)\sin 67^\circ}{1.8}\right)$

$= \boxed{30.75^\circ}$

refracted ray is in medium with index of refraction n_2

⇒ wavelength in this medium is

$\frac{\lambda}{n_2} = \frac{800\ nm}{1.8} = \boxed{444\ nm}$

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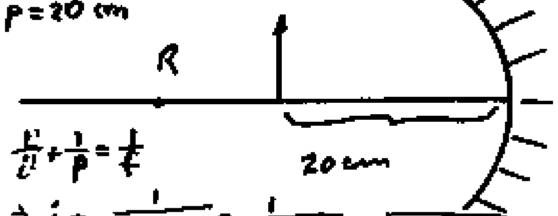
10. A concave spherical mirror has a radius of curvature of 35 cm. An object 7 cm high is placed 20 cm from the mirror. What is the position and the magnification of the image?

concave ⇒ $f > 0,\ r > 0$. ∴ $f = \frac{r}{2} = \frac{35\ cm}{2}$

$p = 20$ cm

	i	m
(a)	9.3cm	+4
(b)	76.5cm	−3
(c)	140 cm	−7
(d)	126cm	−6.3
(e)	12.7cm	2.7



$\frac{1}{i} + \frac{1}{p} = \frac{1}{f}$

$\Rightarrow i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{35/2} - \frac{1}{20}} = \boxed{140\ cm}$

$m = -\frac{i}{p} = -\frac{140\ cm}{20\ cm} = \boxed{-7}$

11. A light ray travels inside a Zircon light pipe. The index of refraction of the cubic Zirconia is 2.2. What is the critical angle for total internal reflection?

(a) 27°

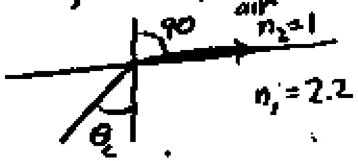
(b) 32°

(c) 14°

(d) 61°

(e) 19°

Zircon light pipe assumed to be in air

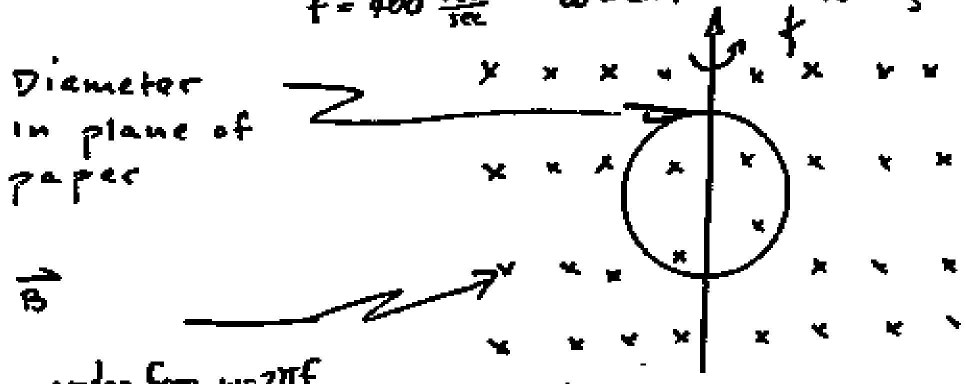


$n_1 \sin\theta_c = n_2 \sin 90^\circ = n_2$

$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) = \boxed{27.03^\circ}$

12. A uniform $\vec{B}$ field of 1.7 Tesla is directed into the plane of the paper. A 130 turn coil is rotated with 400 revolutions per second about a diameter as shown below.

$\underbrace{400 \text{ revolutions per second}}_{f = 400\ \frac{rev}{sec}}$ $\omega = 2\pi f = 2.513\times10^3\ \frac{rad}{s}$



angular freq $\omega = 2\pi f$

The radius of the loop is 0.6 m. What is the maximum voltage developed by the coil?

(a) 1.6×10^{5} V

(b) 6.3×10^{5} V

(c) 9.4×10^{5} V

(d) 4.2×10^{5} V

(e) 5.3×10^{5} V

Let $d\vec{A} = \hat{n}\, dA$, where $\hat{n}$ is normal to the plane of the loop (note that $\hat{n}$ changes with time)

flux $\Phi = \int_{loop} \vec{B}\cdot d\vec{A} = \int_{loop} B\, dA \cos\omega t$

$= B\cos\omega t \underbrace{\int_{loop} dA}_{\pi r^2} = B\pi r^2 \cos\omega t$

emf $\mathcal{E} = -N\frac{d\Phi}{dt} = -NB\pi r^2(-\omega \sin\omega t)$

emf $\mathcal{E} = NB\pi r^2\omega \sin\omega t$ maximum voltage when $\sin\omega t = 1$

$= 6,3\times10^{-5}$ V

maximum emf $= \underline{NB\pi r^2\omega} = (130)(1.7\ T)\pi(.6)^2(2.5\times10^3\ \frac{ra}{}$

13. What is the voltage across capacitor C_1?

(a) 12.0 V

Replace C_1, C_2 by equivalent capacitance C_{12}

$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{18\,\mu F} + \frac{1}{10\,\mu F}$ $C_1 = 18\,\mu F$ $C_2 = 10\,\mu F$

(b) 9.3 V

$\Rightarrow C_{12} = 6.43\,\mu F$

voltage across C_{12} is 16 V

$\boxed{\text{(c) 5.7 V}}$

charge on $C_{12} = q = C_{12} V = (6.43\,\mu F)(16\,V)$

(d) 11.2 V

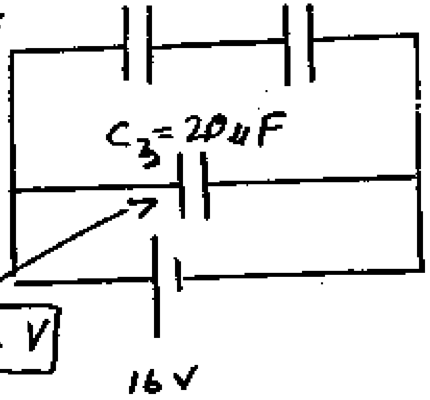
$= 1.029 \times 10^{-4}\,\mu C$

charge on C_1 = charge on $C_{12} = q$

(e) 8.3 V

voltage across $C_1 = \frac{q}{C_1} = \frac{1.029 \times 10^{-4}\,C}{18 \times 10^{-6}\,F} = \boxed{5.714\ V}$

$C_3 = 20\,\mu F$



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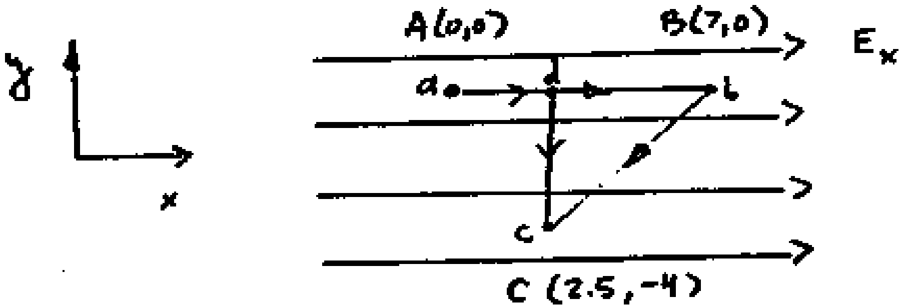
14. A charged particle $q = 4 \times 10^{-4}\ C$ is moved from a to b to c in a uniform electric field as shown below. :



When $|\vec{E}| = 3 \times 10^7\ V/m$ and the distances are in centimeters, how much work is done by the field going from a to c directly?

(a) 300 J

Electric force is conservative force ⇒ work done in going from a to b to c same as work done going from a to d to c.

(b) 76 J

$$W = \int_a^d \vec{F} \cdot d\vec{r} + \int_d^c \vec{F} \cdot d\vec{r} = \int_a^d q\vec{E} \cdot d\vec{r} + \int_d^c \cancelto{0}{q\vec{E} \cdot d\vec{r}}$$

$\boxed{\text{(c) 300 J}}$

$\vec{E} \cdot d\vec{r} = 0$ on path from d to c

$$= qE_x \int_a^d dx$$

(d) 210 J

$$= (4 \times 10^{-4}\,C)(3 \times 10^7\,\tfrac{V}{m})(2.5\,cm - 0\,cm)$$

(e) 430 J

$$= \boxed{300\ J}$$

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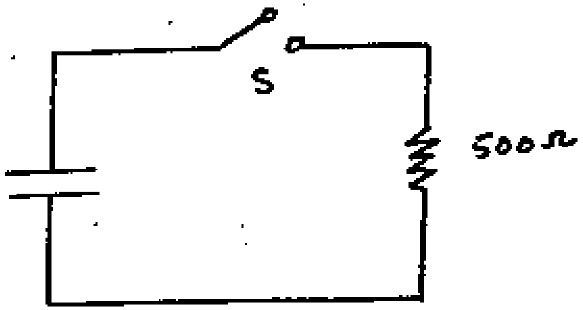
15. A capacitor system is charged to a voltage $V + 3000$ V and then discharged through a resistor as shown below. What is the time constant of the discharge?

$C = 6.1\,\mu F$



(a) 4.61×10^{-3} S

(b) 2.05×10^{-3} S

$\tau = RC = (500\,\Omega)(6.1 \times 10^{-6}\,F)$

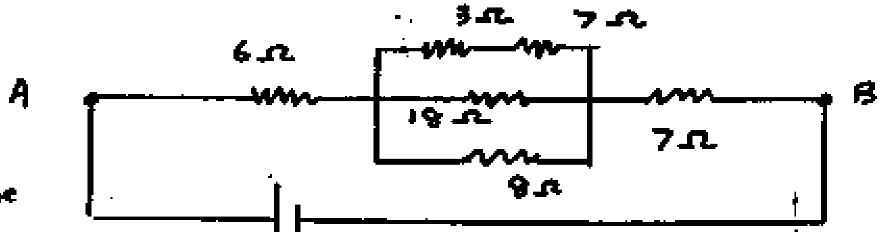
$= \boxed{3.05 \times 10^{-3}\ s}$

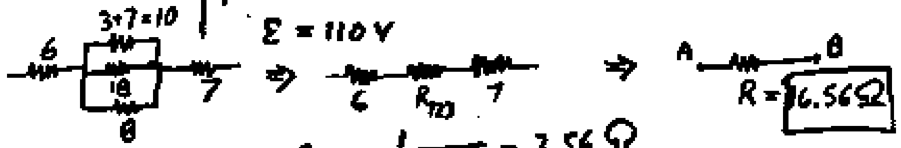
(c) 7.23×10^{-3} S

$\boxed{\text{(d) } 3.05 \times 10^{-3}\ S}$

(e) 5.27×10^{-3} S

16: Consider the resistor network shown below. What is the equivalent resistance R_{AB}? What is the power dissipated in the network?





$\mathcal{E} = 110\ V$

$R = \boxed{16.56\,\Omega}$

$R_{123} = \frac{1}{\frac{1}{10} + \frac{1}{18} + \frac{1}{8}} = 3.56\,\Omega$

	R	Power
(c)	19 Ω	870 W
(a)	4.1 Ω	1422 W
$\boxed{\text{(d)}}$	$\boxed{16.6\,\Omega}$	$\boxed{731\ W}$
(b)	7.2 Ω	1140 W
(e)	11 Ω	985 W

$R = 16.56\,\Omega$

$P = \frac{\mathcal{E}^2}{R} = \frac{(110\,V)^2}{16.56\,\Omega}$

$= \boxed{730.48\ W}$