KEY

Solutions - Codrington

PHYSICS 241

EXAM II **April 03, 1997**

Name:________________________ **ID#:**________________________

USEFUL CONSTANTS:

1. $\epsilon_o = 8.854 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$
2. $\kappa = 8.9875 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$
3. $\mu_o = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$
4. Magnitude of charge on an electron $q_e = 1.602 \times 10^{-19}$ C
5. Magnitude of charge on a proton $q_p = 1.602 \times 10^{-19}$ C
6. Mass of an electron $m_e = 9.11 \times 10^{-31}$ kg
7. Mass of a proton $m_e = 1.67 \times 10^{-27}$ kg

14. (6 points) A circular loop of wire of radius R is placed in a uniform magnetic field **B** and is then spun at a constant angular velocity ω about an axis through its diameter. If the axis of rotation is perpendicular to **B**, the magnetic flux through the loop varies with time given by the relation:

(A)( ) $\pi R^2 B \cos(\omega t)$
(B)( ) $\pi R^2 B^2 \cos(\omega t)$
(C)( ) $\pi R^2 B \cos(\omega t/2)$
(D)( ) $\pi R^2 B^2 \cos(\omega t/2)$
(E)( ) 0

$\Phi = \int \vec{B} \cdot d\vec{A} = \int B\, dA \cos \omega t$

$= B \cos(\omega t) \int dA$

$= B \pi R^2 \cos(\omega t)$

1. (6 points) A singly charged ion of mass m_1 is accelerated from rest by a potential difference V_o. It is then deflected by a uniform magnetic field (perpendicular to the ion's velocity) into a semicircle of radius R_1. Now a triply-charged ion of mass m_2 is accelerated through the same potential difference and is deflected by the same magnetic field into a semicircle of radius $R_2 = 2R_1$. The ratio of the ions masses m_2/m_1 is:

(A)( ) 8
(B)( ) 12
(C)( ) 18
(D)( ) 24
(E)( ) 48

$\frac{1}{2}m_1v_1^2 = qV_o \Rightarrow v_1 = \sqrt{\frac{2qV_o}{m_1}}$

$\frac{1}{2}m_2v_2^2 = 3qV_o \quad v_2 = \sqrt{\frac{6qV_o}{m_2}}$

$R_1 = \frac{m_1v_1}{qB} = \frac{\sqrt{2qV_om_1}}{qB}$

$R_2 = \frac{m_2v_2}{3qB} = \frac{\sqrt{6qV_om_2}}{3qB} = 2R_1 = \frac{2\sqrt{2qV_om_1}}{qB}$

$\frac{\sqrt{6}}{3}\left(\frac{m_2}{m_1}\right)^{1/2} = 2\sqrt{2} \Rightarrow \frac{m_2}{m_1} = \left(\frac{6\sqrt{2}}{\sqrt{6}}\right)^2 = (\sqrt{12})^2 = 12$

2. (6 points) Two parallel conductors, separated by a distance $a = 30$ cm, carry currents in the same directions. If $I_1 = 2.0$ A and $I_2 = 7.5$ A. The force per unit length exerted on each conductor by the other is:

$i_1 = 2$ $i_2 = 7.5$

(A)( ) 4.4×10^{-4} N/m attractive
(B)( ) 4.4×10^{-4} N/m repulsive
(C)( ) 1.0×10^{-5} N/m attractive
(D)( ) 1.0×10^{-5} N/m repulsive
(E)( ) 7.2×10^{-6} N/m repulsive

$B_1 = \frac{\mu_0 i_1}{2\pi a}$

$|\vec F| = |i_2 \vec L \times \vec B_1| = i_2 L B_1 = \frac{\mu_0 i_1 i_2 L}{2\pi a}$

$\therefore \frac{F}{L} = \frac{\mu_0 i_1 i_2}{2\pi a} = \frac{(4\pi\times10^{-7})(2\ A)(7.5\ A)}{2\pi\,(.3\ m)}$

$= 1.0\times10^{-5}$ N attractive

3. (6 points) A small airplane with a wing span of 15 m is flying due north at a speed of 100 m/s over a region where the vertical component of the Earth's magnetic field is 1.2 μT. The potential difference developed between the wing tips is:

(A)( ) 1.44 mV
(B)( ) 1.80 mV
(C)( ) 0.21 V
(D)( ) 0.84 V
(E)( ) 1.60 V

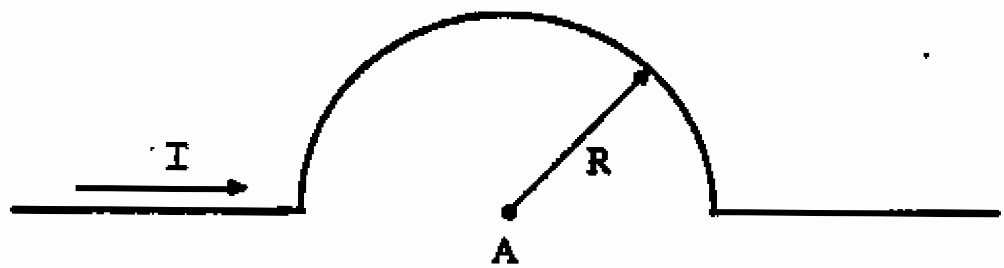
$\vec F = q(\vec E + \vec v \times \vec B) = 0 \Rightarrow -\vec E = \vec v \times \vec B$

$\Delta V = -\int \vec E \cdot d\vec l = +\int (\vec v \times \vec B)\cdot d\vec l = (\vec v \times \vec B)\cdot \vec L$

$= vBL = (100\ m/s)(1.2\times10^{-6}\ T)(15\ m)$

$= 1.80\times10^{-3}$ V

4. (6 points) The segment of wire in the figure below carries a current of $I = 20.0$ A in the direction shown by the arrow. The radius of the arc is $R = 5.0$ cm. The magnetic field at point A is:



$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0 i\, ds}{4\pi R^2} \quad \text{into page}$$

$$B = \int \frac{\mu_0 i\, ds}{4\pi R^2} = \frac{\mu_0 i}{4\pi R^2}\underbrace{\int ds}_{R\phi} = \frac{\mu_0 i \phi}{4\pi R}$$

$$\phi = \pi \Rightarrow B = \frac{\mu_0 i \pi}{4\pi R} = \frac{(4\pi\times10^{-7})(20\text{ A})\pi}{4\pi(.05\text{ m})}$$

into page

(A)( ) $\pi \times 10^{-6}$ T out of the page
(B)( ) $\pi \times 10^{-6}$ T into the page
(C)( ) $2\pi \times 10^{-7}$ T in the plane of the page
(D)( ) $4\pi \times 10^{-5}$ T out of the page
(E)( ) $4\pi \times 10^{-5}$ T into the page

5. (8 points) A circular loop of wire, of radius $R = 50$ cm, with two turns lies in a plane perpendicular to a uniform magnetic field of magnitude $B = 0.40$ T. If in 0.10 s the wire is reshaped into a square with four turns, but remains in the same plane, what is the magnitude of the average induced emf (in Volts) in the wire during this time?

area of circle $= A_1 = \pi R^2$
Length of wire $= (2 \text{ turns})(2\pi R) = 4\pi R$ $\quad N_1 = 2$ turns
side of square $= a = \dfrac{\text{length of wire}}{(4\text{ turns})(4\text{ sides})} = \dfrac{4\pi R}{16} = \dfrac{\pi R}{4}$
area of square $= A_2 = a^2 = \dfrac{\pi^2 R^2}{16}$
$N_2 = 4$ turns

$$\text{Emf} = \frac{N_1\Phi_1 - N_2\Phi_2}{\Delta t} = \frac{N_1 B A_1 - N_2 B A_2}{\Delta t} = \frac{N_1 B \pi R^2 - N_2 B \frac{\pi^2 R^2}{16}}{\Delta t}$$

$$= \frac{BR^2}{\Delta t}\left(N_1\pi - \frac{N_2\pi^2}{16}\right) = \underbrace{\frac{(.4\text{ T})(.5\text{ m})^2}{(.10\text{ s})}}_{1}\left(2\pi - \frac{4\pi^2}{16}\right)$$

$$= 2\pi\left(1 - \frac{\pi}{8}\right)$$

(A)( ) $2\pi[1 - \pi/16]$
(B)( ) $2\pi[1 - \pi/4]$
(C)( ) $2\pi[1 - \pi/8]$
(D)( ) $4\pi[1 - \pi/4]$
(E)( ) $4\pi[1 - \pi/8]$

6. (8 points) A long solenoid, of radius R, with n number of turns per meter carries a current which varies with time as $I = I_o \sin(\omega t)$. Inside the solenoid and coaxial with it is a loop that has a radius $r < R$ and consists of a total of N turns of fine wire. The emf induced in the loop is given by:

> Problem: μ_o missing.

(A)( ) $-\pi R^2 nN\omega I_o \cos(\omega t)$
(B)( ) $-\pi r^2 nN\omega I_o \cos(\omega t)$
(C)( ) $\pi r^2 n^2 \omega I_o \cos(\omega t)$
(D)( ) $\pi R^2 n^2 \omega I_o \cos(\omega t)$
(E)( ) $\pi r^2 nN\omega I_o \sin(\omega t)$

$B = \mu_o n I = \mu_o n I_o \sin(\omega t)$

$\Phi = BA = \mu_o n I_o \sin(\omega t)\, \pi r^2$

$\varepsilon = -N\frac{d\Phi}{dt} = -N\mu_o n I_o \omega \cos(\omega t)\, \pi r^2$

7. (6 points) A 6-V battery is used to charge a 50-μF capacitor. The capacitor is then discharged through a 0.34-mH inductor. The maximum current in the circuit is:

(A)( ) 2.3 A
(B)( ) 2.8 A
(C)( ) 4.2 A
(D)( ) 5.1 A
(E)( ) 7.5 A

$q_{max} = CV = (50\times10^{-6})(6\ V) = 300\times10^{-6}\ C$

$U_E = \frac{1}{2}\frac{q_{max}^2}{C}$

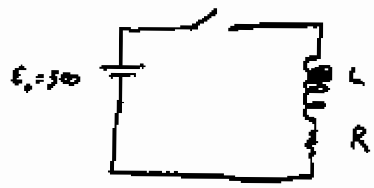
$U_B = \frac{1}{2}L i_{max}^2$

$U_E = U_B \Rightarrow \frac{1}{2}\frac{q_{max}^2}{C} = \frac{1}{2}L i_{max}^2$

$\Rightarrow i_{max} = \frac{q_{max}}{\sqrt{LC}} = \frac{300\times10^{-6}\ C}{\sqrt{(.34\times10^{-3}\ H)(50\times10^{-6}\ F)}} = 2.30\ A$

8. (8 points) At $t = 0$, a source of emf with 500 V, is applied to a coil that has an inductance of 0.80 H and a resistance of 30 Ω. The energy stored in the magnetic field when the current reaches half its maximum value is:

(A)( ) 14.7 J
(B)( ) 27.8 J
(C)( ) 38.2 J
(D)( ) 45.1 J
(E)( ) 64.8 J



Max value $i_{max} = \frac{\varepsilon_o}{R} = \frac{500}{30} = \frac{50}{3}\ A$ (Replace L by wire)

$U_B = \frac{1}{2}L\left(\frac{1}{2}i_{max}\right)^2 = \frac{1}{2}(.8\ H)\left(\frac{1}{2}\cdot\frac{50}{3}\right)^2 = 27.77\ J$

9. (8 points) A sinusoidal voltage $V(t) = (40\ \text{V})\sin(100t)$ is applied to a series LCR circuit with $L = 160$ mH, $C = 99\ \mu$F, and $R = 68\ \Omega$. The phase angle ϕ between the current and the voltage is:

$\omega_d = 100$

(A)( ) $-39.8°$
(B)( ) $39.8°$
(C)( ) $-53.1°$
(D)( ) $53.1°$
(E)( ) $0°$

$\Rightarrow$ ...blem
...nswer is $-51.3°$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - \frac{1}{\omega_d C}}{R}\right)$$

$$= \tan^{-1}\left(\frac{100(160\times10^{-3}\ H) - \frac{1}{100(99\times10^{-6}\ F)}}{68\ \Omega}\right)$$

$$= -51.3°$$

10. (6 points) An RLC circuit is used in a radio to tune into an FM station broadcasting at 99.7 MHz. The resistance in the circuit is 12 Ω and the inductance is 1.40 μH. What value of capacitance should be used?

(A)( ) 1.15 pF
(B)( ) 1.82 pF
(C)( ) 1.26 μF
(D)( ) 2.29 μF
(E)( ) 5.32 μF

$$\omega' = \left(\omega_0^2 - (R/2L)^2\right)^{1/2} = \left(\frac{1}{LC} - (R/2L)^2\right)^{1/2}$$

$$\omega^2 + (R/2L)^2 = \frac{1}{LC}$$

$$\Rightarrow C = \frac{1}{L\left(\omega^2 + (R/2L)^2\right)} = \frac{1}{L\left((2\pi f)^2 + (R/2L)^2\right)}$$

$$= \frac{1}{(1.40\times10^{-6}\ H)(2\pi\cdot 99.7\times10^{6}\ Hz)^2 + \frac{(12\ \Omega)^2}{2(1.40\times10^{-6}\ H)}}$$

$$= \boxed{1.82\times10^{-12}\ F}$$

11. (8 points) A long straight conducting wire of diameter 4.0 mm carries a current of 20 A. Considering that the current is uniformly distributed over the cylindrical cross-section of the wire, calculate the magnetic field at a point 1.0 mm from the axis of the wire.

(A)( ) 0.40 mT
(B)( ) 1.0 mT
(C)( ) 0.50 T
(D)( ) 2.00 T
(E)( ) None of the above

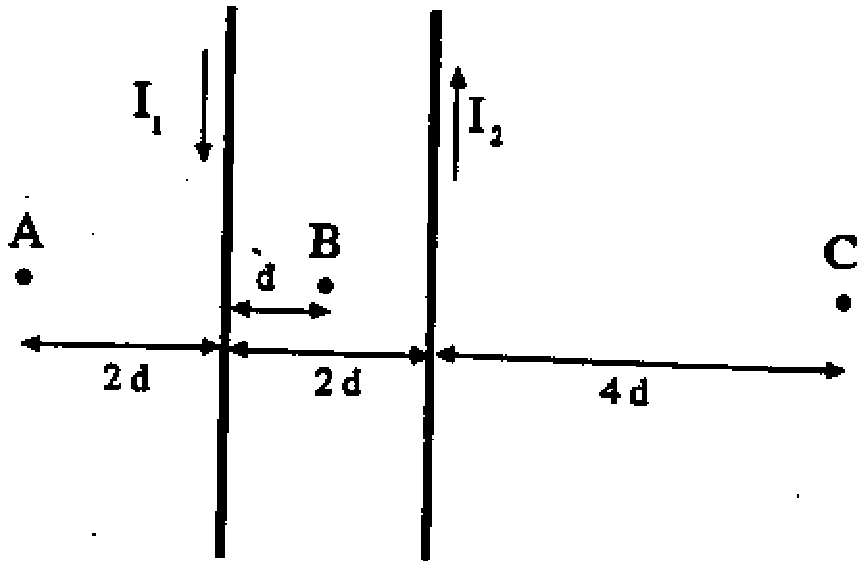
$$J = \frac{i}{\pi R^2}$$

$$i_{enc} = J\cdot\pi r^2 = i\frac{r^2}{R^2}$$

$$B\cdot 2\pi r = \oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} = \mu_0 i\frac{r^2}{R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r = \frac{(4\pi\times10^{-7})(20)(1.0\times10^{-3})}{2\pi(2.0\times10^{-3}\ m)^2} = 1.0\times10^{-3}\ T$$

12. (8 points) Two straight, very long, parallel conductors carry currents I_1 and I_2 in the directions as shown in the figure below. If the magnetic field at point C is to be zero, the ratio of the currents I_1/I_2 must be:



(A)( ) 1.5
(B)( ) 2.0
(C)( ) 2.5
(D)( ) 3.0
(E)( ) At C magnetic field can not be zero.

$B(c) = \frac{\mu_0 I_1}{2\pi(6d)} - \frac{\mu_0 I_2}{2\pi(4d)} = 0$

$\Rightarrow \frac{\mu_0 I_2}{2\pi d}\underbrace{\left(\frac{I_1}{I_2 6} - \frac{1}{4}\right)}_{=0} = 0$

$\Rightarrow \frac{I_1}{I_2} = \frac{6}{4} = 1.5$

13. (8 points) An 800-eV electron traveling along the $+x$ axis enters a region of uniform magnetic field of magnitude 0.02 T. If the direction of the magnetic field is along $-z$, determine the magnitude and direction of the electric field necessary to keep the electron moving along its original direction.

(A)( ) $3.36 \times 10^5 \hat{j}$ V/m
(B)( ) $-3.36 \times 10^5 \hat{j}$ V/m
(C)( ) $-3.36 \times 10^5 \hat{i}$ V/m
(D)( ) $3.36 \times 10^5 \hat{i}$ V/m
(E)( ) $7.72 \times 10^5 \hat{k}$ V/m

$\frac{1}{2} m_e v^2 = 800 \text{ eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} = 1.28 \times 10^{-16}$ J

$\Rightarrow v = \left(\frac{2(1.28 \times 10^{-16} \text{ J})}{9.1 \times 10^{-31} \text{ kg}}\right)^{1/2} = 1.676 \times 10^7$ m/s

$E = vB = (1.676 \times 10^7 \text{ m/s})(.02 \text{ T}) = 3.35 \times 10^5 \frac{V}{m}$