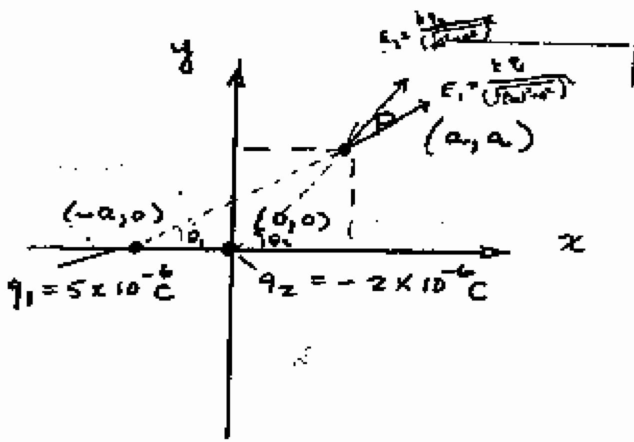
Physics 241
Exam I
Feb 25, 1998

Multiple Choice

1. Consider two point charges located at (x_i, y_i) as shown below, where $a = 3m$



What is the value of the y component of the electric field, E_y at the point P?

(a) 1110 V/m

(b) 700 V/m

(c) −260 V/m

(d) −450 V/m

(e) none of the above

$E_y = E_1 \sin\theta_1 + E_2 \sin\theta_2$

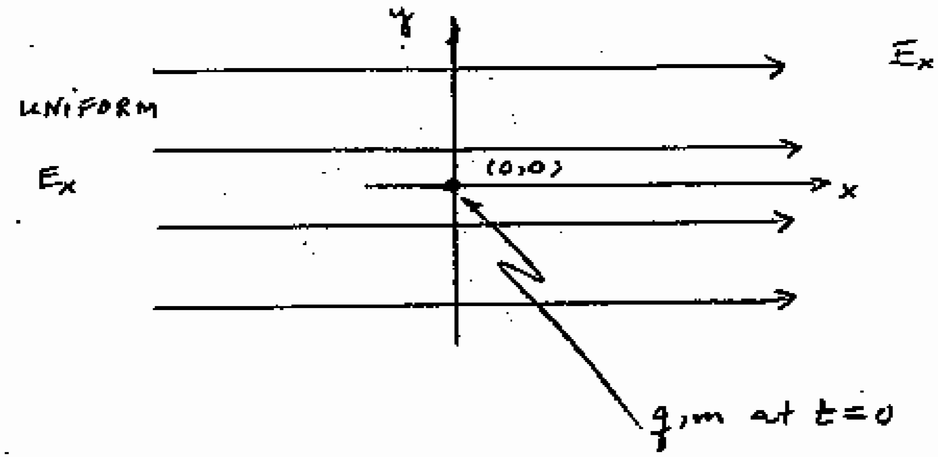
$= \left(\frac{kq_1}{5a^2}\right)\frac{a}{\sqrt{a^2+(2a)^2}} + \frac{kq_2}{2a^2}\frac{a}{\sqrt{a^2+a^2}}$

$= \frac{kq_1 a}{(5a^2)^{3/2}} + \frac{kq_2 a}{(2a^2)^{3/2}} = \frac{k}{a^2}\left(\frac{q_1}{5^{3/2}} + \frac{q_2}{2^{3/2}}\right)$

$= \frac{9\times10^9 \frac{Nm^2}{C^2}}{(3m)^2}\left(\frac{5\times10^{-6}C}{5^{3/2}} + \frac{-2\times10^{-6}C}{2^{3/2}}\right)$

$= -259.6$ V/m

2. Consider a uniform electric field where $E_x = 100$ V/m. A negative charged particle with $q = -8 \times 10^{-19}$C and mass $m = 9 \times 10^{-26}$ Kg is released at rest from the origin $x = 0, y = 0$ at time $t = 0$. Where is the charge located after $t = 10^{-4}$ seconds?



(a) $x =$ 47.6 m

(b) $x =$ −47.0 m

(c) $x =$ 8.90 m

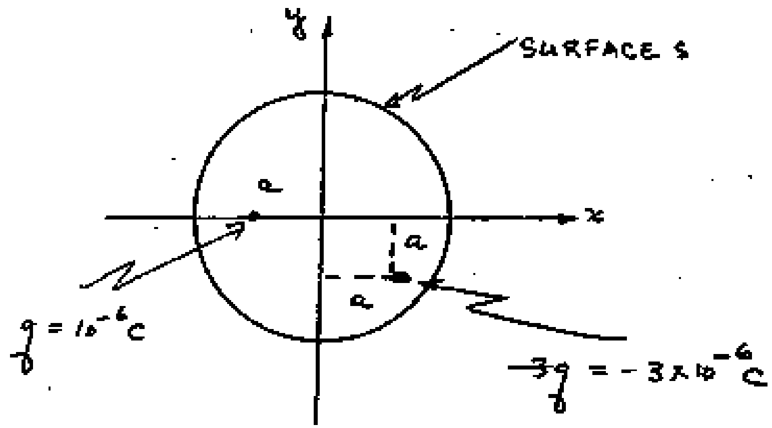
(d) $x =$ −4.40 m

(e) none of the above

$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}t^2 = \frac{1}{2}\frac{qE}{m}t^2$

$= \frac{1}{2}\frac{(-8\times10^{-19}C)(100\ V/m)(10^{-4}s)^2}{9\times10^{-26}\ kg} = -4.44$ m

3. An electric charge distribution consists of a charge $+q$ located at $(x=-a, y=0)$ and a negative charge $-3q$ located at $(x=a, y=-a)$ as shown below. The value of q is $q=10^{-6}$C. What is the total electric flux emitted by these charges?



(a) 1.13×10^{6} N/C m^2

(b) -2.25×10^{5} N/C m^2

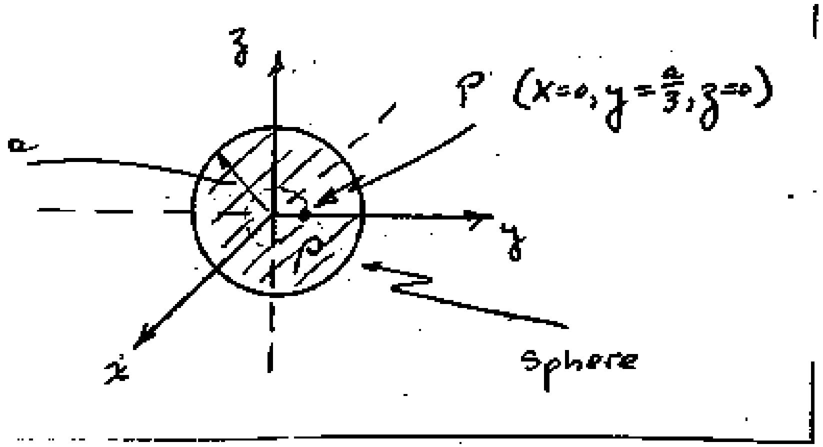
(c) -3.34×10^{6} N/C m^2

(d) 4.45×10^{5} N/C m^2

(e) none of the above

$\Phi = \frac{-3q+q}{\epsilon_0} = \frac{-2q}{\epsilon_0} = -\frac{2\times10^{-6}\,C}{8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}} = -2.25\times10^{5}\,\frac{N\,m^2}{C}$

4. Consider a spherical charge distribution with a uniform volume charge density ρ.



The sphere has a radius a. What is the magnitude of the electric field at the point P which is located at $(x=0, y=\frac{a}{3}, z=0)$?

(a) $2/3\ \rho a/\epsilon_0$

(b) $1/9\ \rho a/\epsilon_0$

(c) $1/6\ \rho a/\epsilon_0$

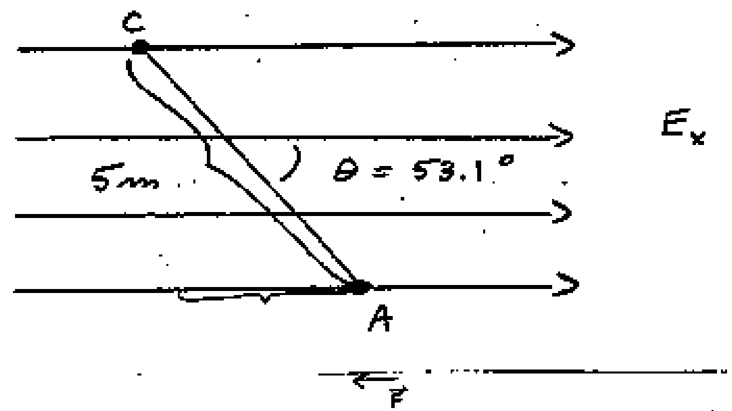
(d) $1/2\ \rho a/\epsilon_0$

(e) none of the above

$\oint_S \vec{E}\cdot d\vec{A} = \frac{\text{charge in } S}{\epsilon_0} = \frac{\rho\cdot\frac{4}{3}\pi r^3}{\epsilon_0}$

$E(r)\cdot4\pi r^2 \Rightarrow E = \frac{\rho r}{3\epsilon_0} = \frac{\rho a}{9\epsilon_0}$

5.



A uniform electric field $E_x = 1500$ V/m is shown above. How much work is done by the electric field when a charge $q = -3 \times 10^{-3}$C is moved from A to C?

(a) + 3.00 J

(b) + 13.5 J

(c) − 4.55 J

(d) − 14.7 J

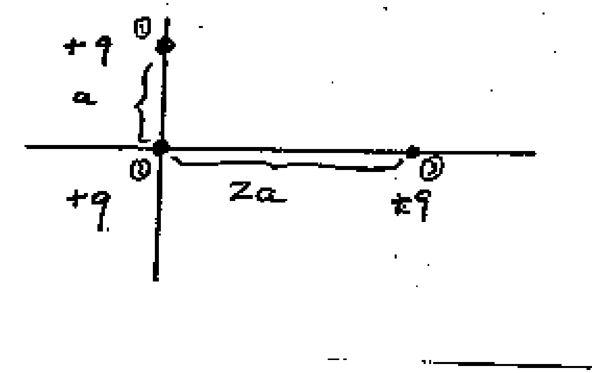
(e) none of the above

$W = \int \vec{F} \cdot d\vec{s} = F\ (5 \cos 53.1°)$

$= qE_x\ (5 \cos 53.1°)$

$= (3 \times 10^{-3}\ C)(1500\ V/m)\ 5 \cos 53.1° = 13.50\ J$

6. Three charges are placed as shown



What external work was required to move these charges from $r = \infty$ where their mutual potential energy is zero to the above configuration? (note $k = 1/4\pi\epsilon_0$)

(a) $+1.95\ k\,q^2/a$

(b) $+1.55\ k\,q^2/a$

(c) $-1.55\ k\,q^2/a$

(d) $-2.30\ k\,q^2/a$

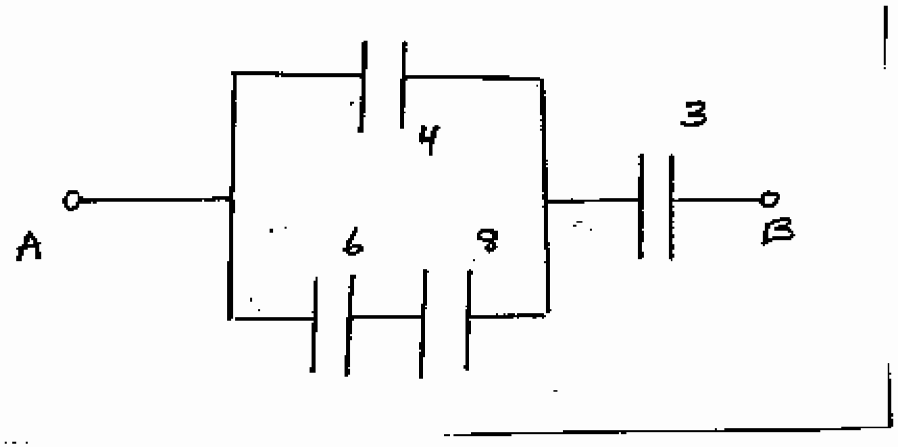
(e) none of the above

$U = U_{12} + U_{13} + U_{23}$

$= \frac{kq^2}{a} + \frac{kq^2}{\sqrt{a^2 + (2a)^2}} + \frac{kq^2}{\sqrt{(2a)^2}}$

$= kq^2\left(\frac{1}{a} + \frac{1}{\sqrt{5}a} + \frac{1}{2a}\right) = \frac{kq^2}{a}\left(\frac{3}{2} + \frac{1}{\sqrt{5}}\right) = 1.95\ \frac{kq^2}{a}$

7. Consider the capacitance network where all capacitances are in μF. (10^{-6}F)



What is the equivalent capacitance between A and B?

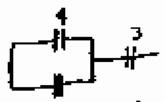
(a) 2.57 μF

(b) 3.20 μF

(c) 1.60 μF

(d) 2.14 μF

(e) none of the above

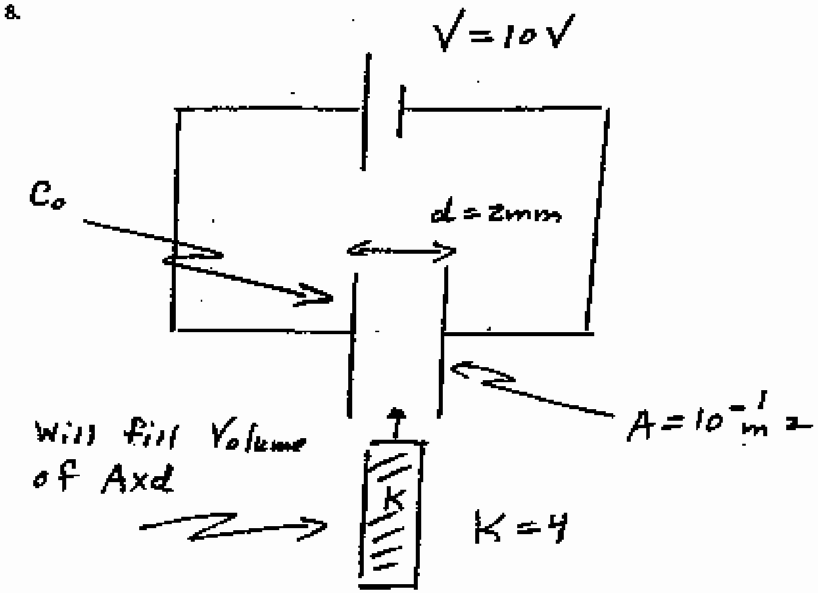


$\frac{1}{\frac{1}{6}+\frac{1}{8}} = \frac{1}{\frac{4+3}{24}} = \frac{24}{7}$

$4+\frac{24}{7} = \frac{52}{7}$ 3

$\frac{1}{\frac{1}{\frac{52}{7}}+\frac{1}{3}} = \frac{1}{\frac{7}{52}+\frac{1}{3}} =$ 2.14 μF

8.



A dielectric slab with dielectric constant $K = 4$ is inserted into a parallel plate capacitor with $A = 10^{-1}m^2$ and $d = 2$mm. How much charge flows onto the plates when the dielectric is inserted into the air dielectric capacitor?

(a) 1.85 × 10^{-8}C

(b) 2.85 × 10^{-8}C

(c) 1.33 × 10^{-8}C

(d) 1.77 × 10^{-8}C

(e) none of the above

Let C_o = capacitance before inserting dielectric $= \frac{\epsilon_0 A}{d}$

initial charge $q_0 = C_o V$

Capacitance after inserting dielectric is $C = KC_o$

final charge $q = CV = KC_o V$

$q - q_0 = KC_oV - C_oV = (K-1)C_oV = (K-1)\frac{\epsilon_0 A}{d}V$

$= (4-1)\frac{(8.85\times10^{-12}\ \frac{N\,m^2}{C^2})(.1\ m^2)(10\ V)}{2\times10^{-3}\ m} = 1.327\times10^{-8}$ C

9. 45 grams of Cu with mass density of 9 grams/cm^3 is extruded (drawn) into a long Cu wire with a circular cross section whose diameter is 1 mm. What is the resistance of the wire if the resistivity of Cu is 1.78 × 10^{-6} ohm-cm ?

radius $r = \frac{\text{diameter}}{2} = \frac{1\text{ mm}}{2} = .5\text{ mm} = .05\text{ cm}$

area $A = \pi r^2$

volume $V = \frac{45\text{ g}}{9\text{ g/cm}^3} = 5\text{ cm}^3$

length $L = \frac{V}{A} = \frac{V}{\pi r^2}$

$R = \frac{\rho L}{A} = \frac{\rho V}{A \cdot A} = \frac{\rho V}{(\pi r^2)^2} = \frac{(1.78\times10^{-6}\ \Omega\text{-cm})(5\text{ cm}^3)}{(\pi (.05\text{ cm})^2)^2}$

$= .144\ \Omega$

(a) 1.44 × 10^{-1} ohm

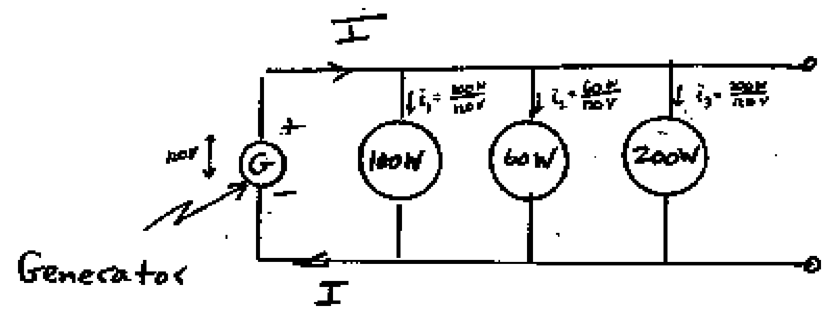
(b) 1.76 × 10^{+1} ohm

(c) 8.12 × 10^{-1} ohm

(d) 9.65 × 10^{-2} ohm

(e) none of the above

10. Consider three different wattage light bulbs hooked up to a constant 120 V potential difference provided by a generator G.



$I = i_1 + i_2 + i_3 = \frac{100}{120} + \frac{60}{120} + \frac{200}{120} = \frac{360}{120} = 3\ A$

What is the current through the generator?

(a) 4.6 A

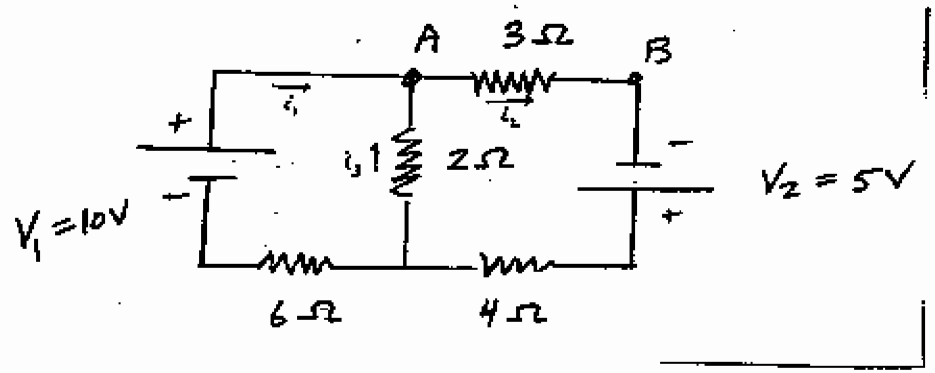
(b) 1.3 A

(c) 2.5 A

(d) 3.0 A

(e) none of the above

11. Consider the two loop circuit.



What current flows from A to B?

(a) +0.88 A

(b) −0.29 A

(c) −0.76 A

(d) +2.17 A

(e) none of the above

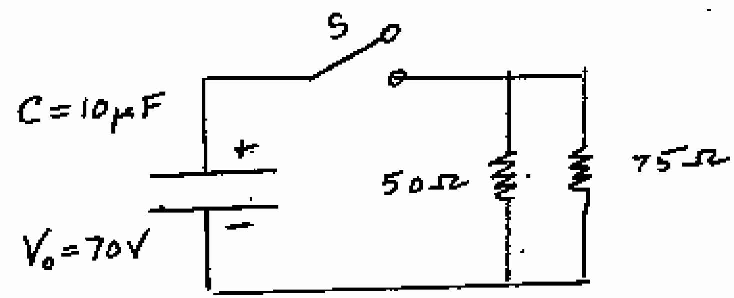
$$\begin{cases} 10 + 2i_3 - 6i_1 = 0 \\ 5 - 4i_2 - 2i_3 - 3i_2 = 0 \\ i_1 + i_3 = i_2 \Rightarrow i_3 = i_2 - i_1 \end{cases}$$

$$\begin{cases} 10 + 2(i_2 - i_1) - 6i_1 = 0 \\ 5 - 7i_2 - 2(i_2 - i_1) = 0 \end{cases}$$

$$10 = 8i_1 - 2i_2 \qquad 5 = -2i_1 + 9i_2 \qquad \begin{bmatrix} 8 & -2 \\ -2 & 9 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$i_2 = \frac{\begin{vmatrix} 8 & 10 \\ -2 & 5 \end{vmatrix}}{\begin{vmatrix} 8 & -2 \\ -2 & 9 \end{vmatrix}} = \frac{40+20}{72-4} = \frac{60}{68} = \boxed{.882\ A}$$

12. A capacitor $C = 10\mu F$ is charged to a potential difference of 70 V, then the switch S is closed at $t = 0$?



What is the charge on the capacitor at $t = 1.5 \times 10^{-4}$s?

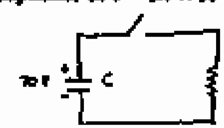
(a) 4.24×10^{-4}C

(b) 7.37×10^{-4}C

(c) 7.17×10^{-5}C

(d) 5.72×10^{-4}C

(e) none of the above



$$R_{eq} = \frac{1}{\frac{1}{50} + \frac{1}{75}} = \frac{1}{\frac{5}{150}} = \frac{150}{5} = 30\,\Omega$$

time constant $\tau = R_{eq}C$

initial charge $q_0 = CV_0$

for a discharging capacitor

$$q = q_0 e^{-t/\tau} = CV_0 e^{-\frac{t}{R_{eq}C}}$$

$$= (10\times10^{-6}F)(70\ V)\, e^{-\frac{(1.5\times10^{-4}s)}{(30\Omega)(10\times10^{-6}F)}}$$

$$= 4.24\times10^{-4}\ C$$