PHYS515 (Spring 2019), Test #2, V1 Name: ______________________

Show work to get credit. There are problems where the answer is a specific number.

See text above Eq. (3.49)

(1) (5 pts) 2 grams of ice at 0° C is melted to 2 grams of water at 0° C. This occurs at 1 atm of pressure. a) What is the change of entropy of this system? b) Did the number of states of this system increase or decrease? c) Determine the ratio (final number of states)/(initial number of states).

From 3.57 $dS = \frac{1}{T}dU + \frac{P}{T}dV - \cancel{\frac{\mu}{T}dN}^{\to 0}$

From 3.47 $dU = Q + W = Q - PdV$ $\Rightarrow dS = \frac{Q}{T}$

From top pg 33 $Q = 333\frac{J}{g}\,2g = 666J$ $T = 0°C = 273K$

a) $dS = \frac{666J}{273K} = 2.44\frac{J}{K}$

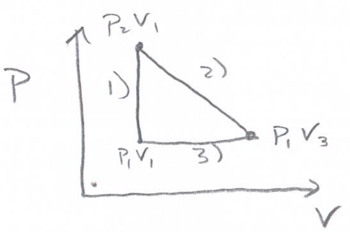
b) From Eq 2.45 Increase because $\Omega = e^{S/k_B}$ and S increase

c) $\frac{\Omega_{fin}}{\Omega_{init}} = e^{dS/k_B} = e^{\frac{2.44 J/K}{1.38\times10^{-23} J/K}} = e^{1.77\times10^{23}}$

Prob 4.1

(2) (5 pts) You make a heat engine which has He gas in a piston. It goes through 3 steps. 1) Starting at P_1, V_1 the pressure is increased to P_2 keeping the volume fixed. 2) Then the volume is expanded until the pressure is back to P_1 but the volume is now V_3. For this step the pressure is linearly dependent on V. 3) Then the volume is decreased back to V_1 keeping the pressure constant. a) Draw this cycle on a PV diagram; label each step and the various P_i, V_i points. b) Make a table that has the amount of heat added to the gas and the work done *on* the gas for each step. c) Compute the total work performed in one cycle and the total heat added to the gas in one cycle.

He has 3 quad. D.O.F.



$W = -\int_{V_i}^{V_f} PdV$

$\Delta U = \frac{3}{2}Nk_B\Delta T = \frac{3}{2}(P_fV_f - P_iV_i)$

$Q = \Delta U - W$

	W	Q
1)	0	$\frac{3}{2}(P_2-P_1)V_1$
2)	$-\frac{(P_2+P_1)}{2}(V_3-V_1)$	$\frac{3}{2}(P_1V_3-P_2V_1) + \frac{(P_2+P_1)}{2}(V_3-V_1)$
3)	$(V_3-V_1)P_1$	$\frac{3}{2}(P_1V_1-P_1V_3) - (V_3-V_1)P_1$

on gas

$W_{TOT} = (V_3-V_1)\left[P_1 - \frac{P_2+P_1}{2}\right] = -\frac{1}{2}(V_3-V_1)(P_2-P_1)$

$W_{performed} = -W_{TOT} = \frac{1}{2}(V_3-V_1)(P_2-P_1)$

cancel

$Q = \frac{3}{2}(P_2-P_1)V_1 + \frac{3}{2}(P_1V_3-P_2V_1) + \frac{3}{2}(P_1V_1-P_1V_3) - W_{TOT}$

$Q = -W_{TOT} = \frac{1}{2}(V_3-V_1)(P_2-P_1)$

a) Prob 5.5
b) Prob 5.2

(3) (5 pts) Consider the reaction of C (graphite) plus $2H_2$ going to CH_4 that takes place at standard temperature and pressure. a) Specify how much energy *you* have to add or remove to make this reaction go for each mole of CH_4. b) For this reaction, verify that the data for the Gibbs free energy, enthalpy, and entropy are consistent.

Need to check ΔG for the left and right side $C + 2H_2 \to CH_4$

$E_{you} = \Delta G_{CH_4} - \Delta G_C - 2\Delta G_{H_2} = -50.72\text{ kJ} - 0 - 0 = -50.72\text{ kJ}$

You get out $\underline{50.72\text{ kJ}}$

b) From Eq (5.9)

$\Delta G = \Delta H - T\Delta S$

$-50.72\times10^3\text{ J} \stackrel{?}{=} -74.81\times10^3\text{ J} - 298\text{K}\left(.186.26\frac{\text{J}}{\text{K}} - 5.74\frac{\text{J}}{\text{K}} - 2\times130.68\right)$

$= -74.81\text{ kJ} + 24.09\text{ kJ}$

$\checkmark\!= -50.72\text{ kJ}$ matches

Prob 6.4 (4) (5 pts) *For this problem, I want all answers to 5 significant digits.* Snedley Frumble has discovered a quantum system where the energy levels are $E_n = \varepsilon n^3$ with $n = 0, 1, 2, \ldots$ and there is one state with $n = 0$, two states with $n = 1$, three states with $n = 2$, etc. For the case $\varepsilon/(k_BT) = \ln(5/4)$, a) compute the probability to have energy 0 and b) compute the probability to have energy ε.

$Z = 1\cdot1 + 2\cdot e^{-\beta\varepsilon} + 3\left(e^{-\beta\varepsilon}\right)^8 + \ldots$

$= 1 + 2\frac{4}{5} + 3\left(\frac{4}{5}\right)^8 + 4\left(\frac{4}{5}\right)^{27} + 5\left(\frac{4}{5}\right)^{64} = 3.11299$

a) $P_{E=0} = 1/Z = 0.32123$

b) $P_{E=\varepsilon} = \frac{2\cdot4/5}{Z} = 0.51398$

Prob 5.28

(5) (10 pts) Aluminum silicate, Al_2SiO_5, has three different crystalline forms: kyanite, andalusite, and sillimanite. a) Which is the stable form at $T = 298$ K and 1 bar? Explain why you chose that form. b) As you raise the pressure keeping $T = 298$ K, are there phase changes? If there are phase changes, determine all of the pressures where they occur. c) Instead of raising the pressure, you raise the temperature keeping $P = 1$ bar. Are there phase changes? If there are phase changes, determine all of the temperatures where they occur.

a) Find the smallest ΔG — Kyanite

b) Use $dG = V dP$

The phase changes occur when

$$\Delta G_a^\circ + V_a \Delta P = \Delta G_b^\circ + V_b \Delta P$$

$$\Delta P = \frac{\Delta G_a^\circ - \Delta G_b^\circ}{V_b - V_a}$$

From the table $V_{andal.} > V_{kyan}$ and $V_{silli.} > V_{kyan}$

This means $\Delta G_{kyan} + V_{kyan}\Delta P$ is always less than that for the other forms $\Rightarrow$ No Phase Transition

c) Use $dG = -S\,dT$

The phase changes occur when $\Delta G_a^\circ - S_a \Delta T = \Delta G_b^\circ - S_b \Delta T$

$$\Delta T = \frac{\Delta G_a^\circ - \Delta G_b^\circ}{S_a - S_b}$$

andal. $\Delta T = \dfrac{1.22\times10^3\,J}{9.41\,J/K} = 130K \Rightarrow T = 428K$

silli $\Delta T = \dfrac{2.89\times10^3\,J}{12.3\,J/K} = 235K \Rightarrow T = 533K$

$\leftarrow$ lower means kyanite $\rightarrow$ andalusite at 428K

Now check whether andalusite $\rightarrow$ sillimanite

$\Delta T = \dfrac{1.67\times10^3\,J}{2.89\,J/K} = 578K \Rightarrow T = 876K$

andalusite $\rightarrow$ sillimanite at 876K

Sec. 3.3

(6) (10 pts) A system consists of $N \gg 1$ quantum objects each of which has ~~two~~ two states with energies: $E_1 = 0$ and $E_2 = \varepsilon$. Define N_1 to be the number of objects in state 1 and N_2 to be the number of objects in state 2. This system is isolated. It is known to have internal energy $U = N_2\varepsilon$. a) Determine the entropy for this system in terms of U and N. b) Determine the temperature and chemical potential in terms of U and N.

Use $S = k_B \ln \Omega$

For 2 state system $\Omega = \dfrac{N!}{N_2!\,(N-N_2)!}$ Eq. (3.27)

$$S = k_B \ln \frac{N!}{N_2!\,(N-N_2)!}$$

$$\simeq k_B \ln \frac{\left(\frac{N}{e}\right)^N}{\left(\frac{N_2}{e}\right)^{N_2}\left(\frac{N-N_2}{e}\right)^{N-N_2}}$$ Stirling approx

$$= k_B \{ N \ln N - N_2 \ln N_2 - (N-N_2)\ln(N-N_2)$$ Eq. (3.28)

$$= k_B \{ N \ln N - \frac{U}{\varepsilon}\ln\left(\frac{U}{\varepsilon}\right) - \left(N - \frac{U}{\varepsilon}\right)\ln\left(N - \frac{U}{\varepsilon}\right)]$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} = -\frac{k_B}{\varepsilon}\ln\frac{U}{\varepsilon} - \frac{k_B}{\varepsilon} + \frac{k_B}{\varepsilon}\ln\left(N-\frac{U}{\varepsilon}\right) + \frac{k_B}{\varepsilon}$$ Eq. (3.5)

$$= \frac{k_B}{\varepsilon}\ln\left(\frac{N\varepsilon - U}{U}\right)$$

$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V} = -k_B T\left(\ln N + 1 - \ln\left(N-\frac{U}{\varepsilon}\right) - 1\right)$$ Eq. (3.59)

$$= k_B T \ln\left(\frac{N\varepsilon - U}{N\varepsilon}\right)$$

$$= \frac{\varepsilon \ln\left(\frac{N\varepsilon - U}{N\varepsilon}\right)}{\ln\left(\frac{N\varepsilon - U}{U}\right)}$$