6.6 Partition functions for Composite Systems

Suppose you have 2 particles that are distinguishable

$$Z_{TOT} = \sum_{s_1} \sum_{s_2} e^{-\beta(E(s_1)+E(s_2))} = \left(\sum_{s_1} e^{-\beta E(s_1)}\right)\left(\sum_{s_2} e^{-\beta E(s_2)}\right) = Z_1 Z_2$$

If you have N distinguishable particles

$$Z_{TOT} = Z_1 Z_2 Z_3 \ldots Z_N$$

What about indistinguishable particles?
If particle 1 is in state 813 and particle 2 is in state 1127, this is same as 1 in state 1127 and 2 in 813.
If the temperature is high, the number of states where $S_1 = S_2$ is minimal. In this case,

$$Z_{TOT} = \frac{1}{2} Z_1 \cdot Z_2 = \frac{1}{2} Z_1^2 \quad (\text{because } Z_1 = Z_2)$$

If there are 3 particles, $Z_{TOT} = \frac{1}{1 \cdot 2 \cdot 3} Z_1 Z_2 Z_3 = \frac{1}{3!} Z_1^3$

In General $Z_{TOT} = \frac{1}{N!} Z_1^N$

Z_1 = partition function for single particle
Z_{TOT} = partition function for N-particles