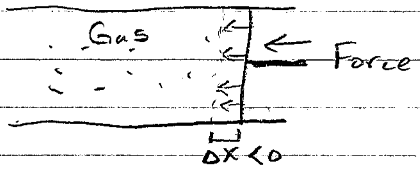
1.5 Compression Work



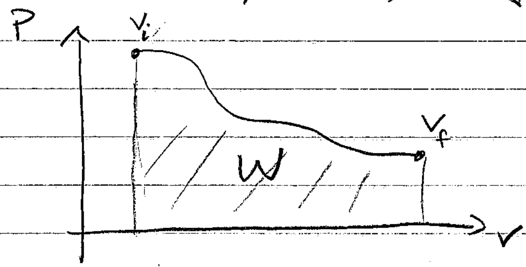
move to left

Remember work done on object $\vec{F} \cdot d\vec{r}$

If there is no friction and you push at constant speed
$|\vec{F}| = P \cdot A$

$$W = \vec{F} \cdot d\vec{r} = -PA\Delta x = -P\Delta V$$

If ΔV is negative, positive work is done on gas
If " positive, negative " " " "

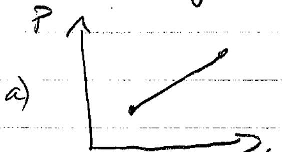


$$W = -\int_{V_i}^{V_f} P(V)\,dV$$

why does P depend on V?

Prob 1.31 1 liter of He at 1 atm goes to 3 liters so $P(V) \propto V$

a)



$$P(V) = \frac{1.01 \times 10^5 \text{ Pa}}{10^{-3} \text{ m}^3} V$$

b) $W = -\int_{V_i}^{V_f} P(V)\,dV = -1.01 \times 10^8 \frac{\text{Pa}}{\text{m}^3} \left(\frac{V_f^2 - V_i^2}{2}\right)$
$= -4.04 \times 10^2 \text{ J} = -404 \text{ J}$

c) $\Delta U = \frac{3}{2} N k_B (T_f - T_i) = \frac{3}{2}(P_f V_f - P_i V_i) = 12.12 \times 10^2 \text{ J} = 1212 \text{ J}$

d) $Q = \Delta U - W = 1616 \text{ J}$

e) Put a flame under canister . . .

The compression of an ideal gas provides examples with closed expressions.

<u>Isothermal compression</u> – $T =$ constant $\Rightarrow$ $P \propto 1/V$

$$W = -\int_{V_i}^{V_f} P\,dV = -\int_{V_i}^{V_f} \frac{N k_B T}{V}\,dV = \boxed{N k_B T \ln\left(\frac{V_i}{V_f}\right)}$$

Note $\Delta U = \Delta\left(\frac{1}{2} N k_B T\right) = 0\,!$ $\Rightarrow$ $Q = \Delta U - W = N k_B T \ln\left(\frac{V_f}{V_i}\right)$

For ideal gas, isothermal compression gives $Q = -W$. That is heat input is minus the work done on gas.

<u>Adiabatic compression</u> $Q = 0$ (no heat in/out)

$$\Rightarrow \Delta U = W$$

Use $U = \frac{f}{2} N k_B T$ $\Rightarrow$ $dU = \frac{f}{2} N k_B\, dT = -P\,dV$

For ideal gas $P = \frac{N}{V} k_B T$ $\Rightarrow$ $\frac{f}{2} \cancel{N}\cancel{k_B}\, dT = -\frac{\cancel{N}}{V} \cancel{k_B} T\, dV$

$$\Rightarrow \frac{f}{2}\frac{dT}{T} = -\frac{dV}{V} \quad \Rightarrow \quad \frac{f}{2} \ln\left(\frac{T_f}{T_i}\right) = -\ln\left(\frac{V_f}{V_i}\right) = \ln\left(\frac{V_i}{V_f}\right)$$

$$\Rightarrow \left(\frac{T_f}{T_i}\right)^{f/2} = \frac{V_i}{V_f} \quad \Rightarrow \quad V_f T_f^{f/2} = V_i T_i^{f/2}$$

$$V T^{f/2} = \text{constant}.$$

Prob 1.35 use $PV = N k_B T$ $\Rightarrow$ $V^{(1+f/2)} P^{f/2} = \text{constant}$

raise both sides to $\frac{2}{f}$ power $V^{\frac{2+f}{f}} P = \text{constant}$

$$V^{\gamma} P = \text{constant}$$

$$\gamma = \frac{2+f}{f}$$

Prob 1.37 In diesel engine, air compressed to $1/20$ of volume quickly.

$\Rightarrow$ $Q = 0$ $\quad T_f = T_i\,(V_i/V_f)^{2/f} = 20^{2/f}\, T_i$

$f = 5$ for N_2 $\quad T_i = 300\,K$ $\quad T_f = 1000\,K$

Prob 1.40 Show $\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}$ when gas expands adiabatically

$V T^{f/2} = \text{constant}$ use $V = \frac{N k_B T}{P}$ $\Rightarrow$ $T^{(f/2+1)} P^{-1} = \text{const.}$

$\Rightarrow T = \text{const}\ P^{\frac{1}{f/2+1}} = \text{const}\ P^{\frac{2}{f+2}}$

$$\frac{dT}{dP} = \frac{2}{f+2}\ \text{const}\ P^{\frac{2}{f+2}-1} = \frac{2}{f+2}\frac{T}{P}$$

From Prob 1.16 $\frac{dP}{dz} = -\frac{mg}{k_B}\frac{P}{T}$

$$\Rightarrow \frac{dT}{dz} = \frac{dP}{dz}\frac{dT}{dP} = -\frac{mg}{k_B}\frac{P}{T}\frac{2}{f+2}\frac{T}{P} = -\frac{mg}{k_B}\frac{2}{f+2}$$

use $f = 5$ $m = 4.65 \times 10^{-26}$ kg $\Rightarrow$ $\frac{dT}{dz} \simeq -9.4 \times 10^{-3}\ \frac{K}{m}$

$\simeq -9.4\ \frac{K}{km}$

$= -9.4\ {}^\circ C/km$

If $\frac{dT}{dz}$ decreases faster than this, then warm, low density air will rise and cold high density air will sink giving convection