1.3 Equipartition Theorem

→ The average energy of any quadratic degree of freedom is $\frac{1}{2}k_B T$

Examples KE $\frac{1}{2}mv_x^2$, $\frac{1}{2}mv_y^2$, $\frac{1}{2}mv_z^2$, $\overset{\text{rotation}}{\frac{1}{2}I\omega_x^2}$, $\frac{1}{2}I\omega_y^2 \ldots$

$\qquad\qquad \frac{1}{2}k_BT \quad \frac{1}{2}k_BT \quad \frac{1}{2}k_BT \quad \frac{1}{2}k_BT \quad \frac{1}{2}k_BT$

Harmonic oscillator $\frac{1}{2}k_s x^2$, $\frac{1}{2}k_s y^2 \ldots$

$\qquad\qquad \frac{1}{2}k_BT \quad \frac{1}{2}k_BT \quad \ldots$

For a system of N molecules and f quadratic degrees of freedom

$$U_{therm} = N \cdot f \cdot \frac{1}{2}k_BT$$

** <u>Important</u>: This is an approximation. It only works to the extent that quantum effects are unimportant.
(First quantum effect explained was a violation of equipartition.)

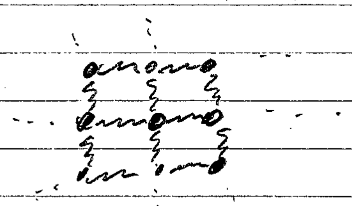
Quantum effect can freeze out motion. Example, if $k_BT \ll \hbar\omega$ for harmonic oscillator, then $U \simeq \frac{1}{2}\hbar\omega$ independent of T.
Another example, particle in a box will only have $\overline{KE} = \frac{1}{2}k_BT$ if $(E_2 - E_1) \ll k_BT$

Example: monatomic gas He, Ne, Ar, ... $f = 3$

Example: N_2 3 translation 2 rotation (rotation about axis frozen out)
2 vibration

The 2 vibration degree of freedom are frozen out until high T
" " rotation " " are frozen out only at very low T

In a solid: $f \to 0$ as $T \to 0$
$f \simeq 6$ for "medium" T (not frozen out, not anharmonic)



Prob 1.24 Calc U for 1g of lead at room temp (no frozen out modes)

Atomic mass of lead = 207 => $1g = \frac{6.02 \times 10^{23}}{207} = 2.9 \times 10^{21}$ atoms

$U = 2.9 \times 10^{21} \cdot 6 \cdot \frac{1}{2} 1.38 \times 10^{-23} \frac{J}{K} 300K = 36 J$

Suppose you have 1g of lead at room temp.
Some how you add 1 J so that $U = 37 J$
what is new T?

$$37 J = 2.9 \times 10^{21} \; 6 \cdot \frac{1}{2} \; 1.38 \times 10^{-23} \frac{J}{K} T$$

$T = \underline{308 K}$

Prob 1.25 List all degrees of freedom for H_2O molecule

3 - translation

3 - rotation

3 - vibration