Problem set 7, Due Mar 24

March 24, 2011

• Find relation between helicity and magnetic energy in a constant-α force-free fields. Hint: derive an expression similar to curlB = αB for the vector potential A.

For constant-α force-free fields

\[ \text{curl } B = \text{curl curl } A = \alpha B = \alpha \text{curl } A \]
\[ \text{curl } A = \alpha A \]
\[ A \cdot B = \frac{1}{\alpha} \text{curl } A \cdot B = \frac{1}{\alpha} B^2 \]  

(1)

• Estimate magnetic Reynolds (Lundquist) number for the Sun. Solar mass is \( M_\odot = 2 \times 10^{33} \) g, Solar radius is \( R_\odot = 7 \times 10^{10} \) cm, magnetic field \( B = 1 \) G.

Magnetic diffusivity \( \kappa \)

\[ \partial_t B = \nabla \times v \times B + \kappa \Delta B \]  

(2)

Resistivity \( \eta \)

\[ E = \eta j \]  

(3)

Conductivity \( \sigma \)

\[ \sigma = \frac{1}{\eta} \]  

(4)

Magnetic diffusivity \( \kappa \)

\[ \kappa = \frac{c^2}{4\pi \sigma} = \frac{c\eta}{4\pi} \]  

(5)

Magnetic Reynolds = Lundquist number

\[ Lu = \frac{V_A L}{\kappa} \]  

(6)

\[ \eta = \frac{m_e \nu_{\text{coll}}}{e^2 n} = \frac{e^2 \sqrt{m_e}}{T^{3/2}} \ln \Lambda \]
\[
T = \frac{GMm_p}{R} \\
V_A = \frac{B}{\sqrt{4\pi\rho}} \\
\rho = M/((4\pi/3)R^3) \tag{7}
\]

\[
Lu \approx \left(\frac{Gm_p}{\rho}\right)^{3/2} \frac{M\cdot R}{\sqrt{\pi\varepsilon}} \approx 10^{10} \tag{8}
\]

- A plasma with the conductivity \( \eta \) is embedded in the magnetic field of the kind \( B = B_0 \tanh(x/L) \) at \( t = 0 \). Find the magnetic field evolution if there is no plasma flows, \( \mathbf{v} \equiv 0 \).

\[
\partial_t B = \kappa \partial_x^2 B \\
B(x, t) = \int dx_0 G(x - x_0, t) B(x_0) \\
G(x - x_0, t) = \frac{1}{\sqrt{4\pi\kappa t}} e^{-\frac{(x-x_0)^2}{4\kappa t}} \tag{9}
\]

Figure 1: Evolution of the magnetic field.