1. Consider a system of two springs and two masses that are connected together by a damper as shown:

The damper produces a force that is proportional to the relative velocity between the two ends. That is, if \( x_1 \) represents the position of the mass on the left, and \( x_2 \) represents the position of the mass on the right, then the damper will exert an equal and opposite force with magnitude \( b(\dot{x}_2 - \dot{x}_1) \) on each mass. Furthermore, assume that the damping is weak, so that \( \gamma = \frac{b}{m} < \omega_0 = \sqrt{\frac{k}{m}} \).

(a) Determine the net force that acts on each mass and write the set of coupled differential equations for \( x_1 \) and \( x_2 \), which you can assume are defined with respect to their equilibrium positions.

(b) Assume that a solution is of the form \( x_i = A_i e^{\alpha t} \), substitute it into the set of differential equations and write the resulting system of algebraic equations in matrix form, \( Ax = 0 \).

(c) Write an expression for the characteristic polynomial of which \( \alpha \) must be a root in order to satisfy the matrix equation in part (b).

(d) Describe why you would expect \( \alpha = \pm i\omega_0 \) to be two of the roots of the characteristic polynomial? What kind of motion would this correspond to?

(e) Use polynomial division to factor \( (\alpha^2 + \omega_0^2) \) out of the characteristic polynomial and use the quadratic formula to find expressions for the other two roots.

(f) Calculate the eigenvectors corresponding to the roots of the polynomial found in (d) and (e) and describe the resulting motion.
2. Consider a system of two identical masses hanging from two identical springs as shown:

Solve for the motion of each mass about their equilibrium positions when both masses are initially at rest but the bottom one is pulled down by a force $F$ and then released at $t = 0$.

Completing this problem will require several steps. Follow the examples discussed in class and in the text and be sure to explain clearly each step in your analysis.