Physics 422 - Spring 2015 - Assignment #1, Due February 2nd

1. Consider the polynomial
   \[ Ax^2 + Bx + C = 0. \]
   (a) What are the roots of the polynomial?
   (b) What conditions must \( A, B, \) and \( C \) satisfy for the roots to be real?

2. Show that the complex valued function
   \[ z(t) = ae^{i\alpha}e^{i\omega t} + be^{i\beta}e^{i\omega t} \]
   can be written in the form
   \[ z(t) = re^{i(\omega t + \varphi)} \]
   and find expressions for \( r \) and \( \varphi \) in terms of the real numbers \( a, b, \alpha \) and \( \beta \).

3. Consider two springs with spring constants \( k_1 \) and \( k_2 \).
   (a) Show that the effective spring constant, \( k_p \), that would result if the springs were connected in parallel, is given by
   \[ k_p = k_1 + k_2. \]
   (b) Show that the effective spring constant, \( k_s \), that would result if the springs were connected in series, is given by
   \[ k_s = \left( \frac{1}{k_1} + \frac{1}{k_2} \right)^{-1}. \]

4. Consider an object made out of elastic material of length \( L \) that has a uniform elastic modulus, \( Y \), and a cross sectional area given by the function \( A(\ell) \), where \( 0 \leq \ell \leq L \). Find an expression for the spring constant of the object.
5. A mass, \( m \), is attached to one end of a spring with spring constant \( k \) and equilibrium length \( \ell \). The other end of the spring moves with constant velocity so that its position \( X(t) \) at time \( t \) is given by

\[
X(t) = \ell + vt. 
\]

Find an expression for the position of the mass as a function of time, \( x(t) \), if at time \( t = 0 \) the mass is initially located at \( x(0) = 0 \) and is initially at rest, \( \dot{x}(0) = 0 \).