1. (French, 5-9) The CO$_2$ molecule can be likened to a system made up of a central mass $m_2$ connected by equal springs of spring constant $k$ to two masses $m_1$ and $m_2$ (with $m_1 = m_2$) as shown:

(a) Set up and solve the equations for the two normal modes in which the masses oscillate along the line joining their centers. The equation of motion for $m_3$ is

$$m_3 \frac{d^2 x_3}{dt^2} = -k(x_3 - x_2)$$

and there are similar equations for $m_1$ and $m_2$.

(b) Putting $m_1 = m_3 = 16$ units, and $m_2 = 12$ units, what would be the ratio of the frequencies of the two modes, assuming this classical description were applicable?
2. (*French, 5-10*) Two equal masses are connected as shown with two identical massless springs of spring constant $k$. Considering only motion in the vertical direction, show that the angular frequencies of the two normal modes are given by $\omega^2 = (3 \pm \sqrt{5})k/2m$ and hence that the ratio of the normal mode frequencies is $(\sqrt{5} + 1)/(\sqrt{5} - 1)$. Find the ratio of amplitudes of the two masses in each separate mode.

*Note:* You don’t need to consider the gravitational forces acting on the masses because these are independent of the displacements.
3. Consider the following three-loop circuit in which all three capacitors have the same capacitance, $C$, and the two inductors have the same inductance, $L$.

(a) Write the set of three coupled, differential equations that describes the currents that flow in each loop.
(b) Show that solutions of the form $i(t) = I e^{i\omega t}$ will satisfy the set of differential equations, provided $\omega$ satisfies the following matrix equation, in which $\omega_0 = \sqrt{1/LC}$:

$$
\begin{pmatrix}
\omega_0^2 - \omega^2 & \omega^2 & 0 \\
\omega^2 & \omega_0^2 - 2\omega^2 & \omega^2 \\
0 & \omega^2 & \omega_0^2 - \omega^2
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
I_3
\end{pmatrix} = 0
$$

(c) Calculate the frequencies of the three normal modes of oscillation, expressing them in terms of the natural oscillation frequency $\omega_0 = \sqrt{1/LC}$.