Analytical Mechanics

SEVENTH EDITION

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First mode
$N = 1$
$a_k = A \sin \left( \frac{k\pi}{4} \right)$

Second mode
$N = 2$
$a_k = A \sin \left( \frac{2k\pi}{4} \right)$

Third mode
$N = 3$
$a_k = A \sin \left( \frac{3k\pi}{4} \right)$
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