ANNOUNCEMENT

*Exam 1*: Friday December 14, 2012, 8 AM – 10 AM
*Location*: Elliot Hall of Music
*Covers all readings, lectures, homework from Chapters 29 through 33.
*The exam will be multiple choice.

Be sure to bring your student ID card and a handwritten one-page (two sided) crib sheet plus the crib sheets that you prepared for exams 1 and 2.

NOTE THAT FEW EQUATIONS WILL BE GIVEN – YOU ARE REMINDED THAT IT IS YOUR RESPONSIBILITY TO CREATE WHATEVER TWO-SIDED CRIB SHEET YOU WANT TO BRING TO THIS EXAM.

The equation sheet that will be given with the exam is posted on the course homepage. Click on the link on the left labeled “EquationSheet”
Clicker Question

A beam of parallel light rays from a laser is incident on a solid transparent sphere of index of refraction $n$. If a point image is produced at the back of the sphere, what is the index of refraction of the sphere?

(A) 0
(B) 0.5
(C) 1.0
(D) 1.5
(E) 2.0
Clicker Question

This is refraction at a single surface.

\[
\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}
\]

• Parallel incident rays: \( s \to \infty \) so that \( n_1/s \to 0 \)
• In air, \( n_1 = 1 \)
• Image distance, \( s' = 2r \)

\[
\frac{n_2}{2r} = \frac{n_2 - 1}{r} \quad \Rightarrow \quad n_2 = 2n_2 - 2 \quad \Rightarrow \quad n_2 = 2
\]
Interference of Light

- Interference phenomena are a consequence of the wave-like nature of light
- Electric field: \( E_x(z, t) = E_0 \sin(kz - \omega t) \)
- Principle of superposition: \( \vec{E}_{total} = \vec{E}_1 + \vec{E}_2 \)
- Useful trigonometric identity:

\[
\sin \alpha + \sin \beta = 2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)
\]
Interference of Light

• Consider two waves with the same wavelength and frequency but different phase:

\[ E_1(z, t) = E_0 \sin(kz - \omega t) \]
\[ E_2(z, t) = E_0 \sin(kz - \omega t + \delta) \]

• Their superposition has the same wavelength and frequency, but an *amplitude* that depends on the phase, \( \delta \):

\[ E_{total}(z, t) = (2E_0 \cos(\delta/2)) \sin(kz - \omega t) \]

  – If \( \delta = \pi, 3\pi, 5\pi, \ldots \) then \( \cos(\delta/2) = 0 \) (destructive)
  – If \( \delta = 0, 2\pi, 4\pi, \ldots \) then \( \cos(\delta/2) = 1 \) (constructive)
$\delta = 0$: Constructive interference

$\delta = \pi$: Constructive interference

Some other value of $\delta$: Constructive interference
Interference of Light

• To observe interference effects with light, the two interfering waves must be “coherent”:
  – They must have the same $\omega$ and $\lambda$
  – They must maintain the same phase difference over relatively large distances

• “Coherence length” is the distance over which the phase, $\delta$, remains constant

• Necessary conditions to observe interference
Producing Coherent Light

• The easiest way to produce coherent light is to produce two waves using the same source.
• Incandescent lights produce a broad distribution of $\lambda$ (incoherent).

![Spectral lines]

• Gas discharge lamps produce spectral lines with the same $\lambda$ and $\omega$ but random phase (also incoherent).
• **Lasers are a good source of coherent light.**
Interference of Light

• Lasers are a good source of coherent light.
• To observe interference effects you need two waves with different phases.
• Three ways to produce a phase difference:
  1. Travelling through media with different indices of refraction
  2. Traveling along paths of different length
  3. By reflecting from a boundary between two media.
Different Media

• Frequency remains constant but the speed of light is lower in media with $n > 1$:
  
  – Speed of light, $v = c/n$
  – Wavelength, $\lambda = c/\pi f$

  \[
  \text{Phase is } \varphi_1 = \frac{2\pi L}{\lambda_1} = 2\pi \frac{ln_1}{\lambda}
  \]

  \[
  \varphi_2 = \frac{2\pi L}{\lambda_2} = 2\pi \frac{ln_2}{\lambda}
  \]

  – Phase difference is $\delta = 2\pi \frac{L}{\lambda} (n_1 - n_2)$
Different Path Lengths

- Huygens’ principle:
  - Light propagates by continuously re-emitting spherical waves
  - The waves emitted from each slit interfere with each other

Thomas Young experiment (1801)
Interference: Different Path Lengths

Interference maxima condition:
\[ d \sin \theta_m = m\lambda, \quad m = 0, 1, 2, ... \]
\( m = \) order number

Interference mimima condition:
\[ d \sin \theta_m = \left(m - \frac{1}{2}\right)\lambda, \quad m = 1, 2, 3, ... \]
\( m = \) order number
Different Path Lengths

- Small angle approximation: $\sin \theta \approx \tan \theta \approx \theta$.

$\tan \theta = \frac{y}{L}$

Maximum intensity occurs at

$$y = m\frac{\lambda L}{d}$$
Double-Slit Diffraction

- Spacing between fringes: \( \Delta y = \lambda \frac{L}{d} \).
Reflection

• Light is partially reflected from an interface between two materials with different indices of refraction (lecture 23):

\[ I_r = I_0 \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 \]

• The reflected light has a phase difference of \( \delta = \pi \ (180^\circ) \) when it enters a material with a greater index of refraction.

• When light is reflected from an interface into a medium with a lesser index of refraction, there is no change in phase.
Reflection with phase inversion:

Reflection without phase inversion:
Reflection

- mirror
- light beam projector
- screen
- soap film
Reflection from a Boundary

*The phase difference between two waves can change if one or both are reflected.

<table>
<thead>
<tr>
<th>Reflection</th>
<th>Reflection Phase Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off lower index</td>
<td>zero</td>
</tr>
<tr>
<td>Off higher index</td>
<td>180°</td>
</tr>
</tbody>
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QUESTION: Which ray experiences a phase shift due to reflection?

a) Ray 1  b) Ray 2  c) none  d) both
Interference: Soap Bubble

The phase difference between the two waves is caused by:

1. Phase shift on reflection for ray 1 of 180°. Ray 2 does not phase shift on reflection.

2. Path length difference between ray 1 and ray 2 is $2t$. (Ray 2 crosses the film twice.)

For in-phase waves (maxima - bright fringes):

\[ 2t = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \]

For out of-phase waves (minima - dark fringes):

\[ 2t = m \frac{\lambda}{n_2} \]

The total difference between ray 1 and ray 2 is the extra difference traveled plus the shift due to reflection.
Question

*The phase difference between two waves can change if one or both are reflected.

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Newton’s Rings:

Fringes from a wedge-shape film of air indicating how optically flat are the glass plates.
Coating a Glass Lens to Suppress Reflections:

180° phase change at both \( a \) and \( b \) since reflection is off a more optically dense medium.

How thick should the coating be for destructive interference?

\[
2t = \frac{\lambda'}{2}
\]

\[
t = \frac{\lambda'}{4} = \frac{\lambda}{4n_2}
\]

What frequency to use? Visible light: 400-700 nm.
Coating a Glass Lens to Suppress Reflections:

For $\lambda=550$ nm and least thickness ($m=1$)

$$t = \frac{\lambda}{4n}$$

$$= \frac{550 \text{ nm}}{4 \times 1.38} = 99.6 \text{ nm}$$

Note that the thickness needs to be different for different wavelengths.
Clicker Question

• Bright light of wavelength 585 nm is incident perpendicularly on a soap film \((n = 1.33)\) of thickness 1.21 microns, suspended in air. Is the light reflected by the two surfaces of the film closer to interfering fully destructively or fully constructively?

(A) fully constructively

(B) fully destructively

(please don’t enter E...)