Homework 2 Phys661  
(due day: Sep. 19)

• Problem 1  
Derive the following third order energy correction for a non-degenerate state,

\[ E_3^n = \sum_{m \neq n} \sum_{m' \neq n} \frac{V_{nm} V_{nm'} V_{m'n}}{(E_0^n - E_0^m)(E_0^n - E_0^{m'})} - V_{nn} \sum_{m \neq n} \frac{|V_{nm}|^2}{(E_0^n - E_0^m)^2} \]  

(1)

• Problem 2  
For the one dimensional harmonic oscillator,

\[ H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 x^2 \]  

(2)

assume a small perturbation \( v = \lambda^2 m\omega^2 x^2 \) acting on the system. Calculate the energy correction for the state \( |n> \) up to the third order of \( \lambda \) and show your result is consistent with the exact solution.

• Problem 3  
A relativistic particle with a low speed is approximated by the following Hamiltonian,

\[ \hat{H} = H_0 = \frac{\hat{p}^2}{2m} + V(\hat{r}) - \frac{\hat{p}^4}{8m^3c^2} \]  

(3)

where the last term is a correction to relativistic effect.

(a). If we take \( V(\hat{r}) \) as the potential for the electron in hydrogen atom, i.e. \( V(\hat{r}) = -e^2/r \). Based on symmetry, discuss how the above relativistic effect changes the degeneracy of the states in the system.

(b). Calculate the first order energy correction due to the relativistic term for each state

• Problem 4  
A slightly anisotropic three-dimensional harmonic oscillator has \( \omega_x = \omega_y = \omega \) and \( \delta\omega = \omega_z - \omega > 0 \). A charged particle moves in the field of this oscillator and is at the same time exposed to a uniform magnetic field in the x-direction.

a. Write the Hamiltonian

b. Consider the magnetic field is small. Evaluate the energy correction of the ground state and first excited state of the system.

• Problem 5  
A free spin 1/2 fermions move in a one dimension quantum well, \( 0 \leq x \leq 1 \)
$L$, which the wavefunctions are known to satisfy the periodic boundary condition, $\psi(0) = \psi(L)$.

(a) Write the eigenstate wavefunctions and the energy eigenvalues of the system. Show states are doubly degenerated.

(b) If the system is perturbed by a potential $V = -V_0 e^{-(x-L/2)^2/a^2}$, $a << L$, calculate the first order energy correction and wavefunction correction for the degenerate states.