Homework 6 Phys661  
(due day: Nov. 19)

• Problem 1
Consider Klein-Gordon equation,
\[
[(\partial_t^2 - \vec{\nabla}^2) + m^2] \psi(r, t) = 0. \tag{1}
\]
We introduce the following two component wavefunction \( \phi(r, t) \) which satisfies
\[
i(\partial_t \phi(r, t)) = \hat{H} \phi(r, t), \tag{2}
\]
where
\[
\hat{H} = (\sigma_z + i\sigma_y) \vec{P}^2 + m\sigma_z, \tag{3}
\]
(a) Prove that there is one to one correspondence between \( \psi(r, t) \) and \( \phi(r, t) \). Therefore, Klein-Gordon equation is equivalent to the above new equation.
(b) Define the density and current operators and derive the continuity equation in the above new representation.

• Problem 2
Consider the matrix \( \gamma^a, a = 1, 2, 3, 4, 5 \) defined in Dirac equations. Let \( \gamma^{ab} = \frac{i}{2}[\gamma^a, \gamma^b] \). Derive the commutators: \( [\gamma^{ab}, \gamma^{cd}] \) and \( [\gamma^{ab}, \gamma^c] \).

• Problem 3
Consider the Dirac particle with mass \( m \) in an infinite quantum well, \( V(x, y, z) = 0 \), \( 0 < x < L; \infty, x > L, x < 0 \). Obtain the bound state solutions.

• Problem 4
Consider relativistic theory in \( 2 + 1 \) dimension, i.e. the space is a two dimensional space. So momentum operator is \( \vec{P} = (p_x, p_y) \), and \( E^2 = \vec{P}^2 + m^2 \).
(a) Construct your ‘Dirac’ equation in \( 2 + 1 \) dimension.
(b) Obtain your plane wave solution of the equation.
(c) what will be ‘Dirac’ equation in \( 1 + 1 \) dimension.