Homework 6 Phys661
(due day: April 24)

• Problem 1
  Consider the Dirac particle with mass $m$ in an infinite quantum well,
  $V(x, y, z) = 0, 0 < x < L; \infty, x > L, x < 0$. Obtain the bound state
  solutions.

• Problem 2:
  Let’s consider a two dimensional massless Dirac equation,
  \[ H = \sigma_x P_x + \sigma_y P_y \] (1)
  (a) Find eigenstates and eigenvalues of above Hamiltonian
  (b) Show that $\frac{\sigma_z}{2} + L_z$ is conserved, where $L_z$ is z-direction orbital angular
      momentum.
  (c) Apply an external uniform magnetic field in z direction to the system. Ignoring the Zeeman energy and assuming the particle carry charge $e = 1$, write the Hamiltonian.
  (d) Define $a^\pm = \frac{\sigma_x \pm i \sigma_y}{2}$, and $b^\pm = \frac{p_x \pm ip_y - i(x\pm iy)}{2}$. Calculate the commutation relation, $[a^+, a^-]$ and $[b^+, b^-]$.
  (f) Using the operators in (d) to obtain the Landau levels for the Hamiltonian in (c).
  (g) What is the ground state?

• Problem 3: Consider the Dirac equation
  \[ i \frac{\partial \Psi}{\partial t} = [\vec{\alpha} \cdot \vec{P} + \beta m] \Psi \] (2)
  The current operator is given by $\vec{j} = \Psi^\dagger \vec{\alpha} \Psi$. Derive the approximated form of $\vec{j}$ in non-relativistic approximation.