Midterm Phys661  
(Oct.3 12:00- Oct.4 3:00pm)

• Problem 1
Consider an one dimensional quantum system. There is no potential. i.e. The single particle Hamiltonian is just $H = \frac{p^2}{2m}$. However, we assume that the wavefunction satisfies periodic boundary condition, $\Psi(x) = \Psi(x + L)$.
(a) Write down the single particle eigenstates and eigenfunctions.
(b) Consider a small perturbation $H_1 = V_0 \cos(qx)$. Find the first order and second order energy corrections as well as the first order correction of wavefunctions.

• Problem 2
Following the problem 1, now we consider that the single particle is spin 1/2 fermion and there are two identical particles in the system,
(a) without any perturbation, what is the ground state energy and wavefunction of this two particle system?
(b) Consider a perturbation which is an interaction between the spins of the two particles, $H_I = V_0 \delta(x_1 - x_2) \sigma_1^z \sigma_2^z$, where $\sigma_1, \sigma_2$ are Pauli matrix representing the spins. Calculate the energy correction of the ground state up to second order.

• Problem 3
An electron and a nuclon are localized in the space so that we only consider their spin degrees of freedom. In the presence of an external magnetic field, the system is described by the following Hamiltonian

$$H_0 = \mu_e B \sigma_e^z + \mu_n B \sigma_n^z$$

where $\sigma_e$ and $\sigma_n$ are Pauli matrix representing the spins of the electron and the nuclon respectively. $\mu_e$ is much larger than $\mu_n$.
(a) There is a hyperfine coupling between the electron and nuclear spins that is given by

$$H_1 = J \sigma_e^z \sigma_n^z.$$  

What is the ground state?
(b) At time $t = 0$, an additional small magnetic field is applied to the electron spin, that is described by $H_2 = b \cos(\omega t) \sigma_e^x$. Calculate the probability of the nuclear spin pointing to $+z$ direction at time $t$ up to the second order of $b$.

• Problem 4
Consider a particle in the following potential,

$$V(r) = V_0 r^n.$$ 

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where $r = \sqrt{x^2 + y^2 + z^2}$, $-2 < m < 0$ and $V_0 > 0$. Using the variational method to calculate the ground state of the system by taking

(a) the trial wavefunction $\psi \sim e^{-\lambda r / 2}$

(b) the trial wavefunction $\psi \sim e^{-\lambda r^2 / 2}$.

(c) Which one is the better ground state for this problem?