Long-Lived Memory for Mesoscopic Quantum Bits

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We describe a technique to create long-lived quantum memory for quantum bits in mesoscopic systems. Specifically we show that electronic spin coherence can be reversibly mapped onto the collective state of the surrounding nuclei. The coherent transfer can be efficient and fast and it can be used, when combined with standard resonance techniques, to reversibly store coherent superpositions on the time scale of seconds. This method can also allow for "engineering" entangled states of nuclear ensembles and efficiently manipulating the stored states. We investigate the feasibility of this method through a detailed analysis of the coherence properties of the system.

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A broad effort is now under way to develop new techniques for coherently controlling quantum degrees of freedom in mesoscopic systems [1]. These efforts are stimulated in part by the proposals to use these systems as quantum bits in the context of quantum information science. The fast decoherence associated with solid-state environments proves to be the main obstacle for experimental realization of such control.

Spin degrees of freedom of electrons confined in semiconductor quantum dots are attractive qubit candidates [2,3] with relatively long decoherence times. Techniques for the coherent manipulation and measurement of electron spins are now being developed. For the latter, coupling of spin and charge degrees of freedom is probably necessary. Experimental measurements of the spin relaxation times indicate sub-MHz rates [4], although it is not yet clear what will determine the ultimate coherence lifetimes.

This Letter describes a technique for greatly extending the lifetimes of electron-spin qubits in confined structures by coherently mapping an arbitrary spin superposition state into the spins of proximal, polarized nuclei. This is achieved by effective control of the spin-exchange part of hyperfine contact interaction. After the transfer is completed, the resulting superpositions could be stored for a very long time — up to seconds — and mapped back into the electron-spin degrees of freedom on demand. We further show that the stored states can be manipulated using an extension of standard resonance techniques.

Although it is widely known that nuclear spins can possess exceptionally long coherence times due to their weak environmental coupling, single nuclear spins are very difficult to manipulate and measure in practice [5]. In the present approach these problems are circumvented by using collective nuclear degrees of freedom. We demonstrate that such collective states are extremely robust with respect to realistic imperfections, such as partial initial polarization and spin diffusion, and decoherence. As a result the present technique combines the strengths of electron-spin (or charge) manipulation with the excellent long-term memory provided by nuclei.

When uncontrolled, the coupling of electronic spin degrees of freedom to nuclei may be considered an environmental decoherence process. Interesting features of this process arise from its non-Markovian nature [6–8]. The present Letter shows that properly controlled coupling of electrons to nuclei can be used to greatly extend the effective coherence time of electron spins. This study parallels recent work involving the use of atomic ensembles as quantum information carriers [9].

To illustrate the technique, consider a single electron localized in quantum dot. The effective Hamiltonian for the electron and N spin-I_0 nuclei in a magnetic field B_0 along the z axis is

$$\hat{H} = g^* \mu_B B_0 \hat{S}_z + g_n \mu_n B_0 \sum_j \hat{I}_z^j + \hat{V}_{\mathrm{HF}}. \qquad (1)$$

The first two terms of Eq. (1) correspond to the Zeeman energy of the electronic and nuclear spins; the third term is the hyperfine contact interaction between the s-state conduction electrons and the nuclei in the dot, $\hat{V}_{\mathrm{HF}} = \sum_j a_j \hat{\vec{I}}^j \cdot \hat{\vec{S}}$. The coefficients $a_j = A v_0 |\psi(\vec{r}_j)|^2$ correspond to the one-electron hyperfine interaction with the nuclear spin at site $\vec{r}_j$, where A is the hyperfine contact interaction constant, v_0 is the volume of a unit cell, and $\psi(\vec{r})$ is the envelope function of the localized electron. Spin-orbit effects are strongly suppressed in confined structures [10] and here only lead to electron-spin decoherence. The hyperfine term can be written $\hat{V}_{\mathrm{HF}} = \hat{V}_D + \hat{V}_\Omega$ where $\hat{V}_D = \sum_j a_j \hat{I}_z^j \hat{S}_z$ and $\hat{V}_\Omega = \sum_j a_j/2(\hat{I}_-^j \hat{S}_+ + \hat{I}_+^j \hat{S}_-)$. $\hat{V}_D$ produces an effective magnetic field for the electron $\hat{B}_{\mathrm{eff}} = B_0 - 1/g^* \mu_B \sum_j a_j \hat{I}_z^j$, which results in the well-known Overhauser shift. However, when $g^* \mu_B \hat{B}_{\mathrm{eff}} \ll \hat{V}_\Omega$, spin-exchange dominates.

We start with a perfectly polarized nuclear ensemble $|\mathbf{0}\rangle_n = |I_0, ..., I_0\rangle_n$, as shown in Fig. 1(a). Because of conservation of total spin only a spin-down initial electron state can undergo nontrivial evolution. When the dynamics are governed by $\hat{V}_\Omega$ ($\hat{B}_{\mathrm{eff}} \to 0$), there is a coherent exchange of excitations between electronic and nuclear degrees of freedom. For the inital state $|\downarrow\rangle_e \otimes |\mathbf{0}\rangle_n$,

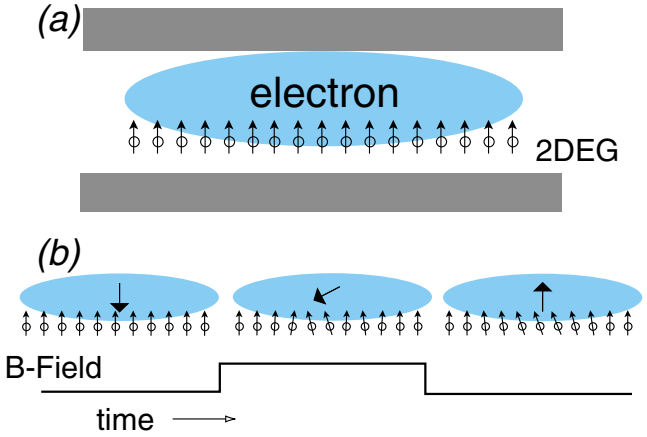

FIG. 1 (color online). (a) A schematic of an electron trapped in a quantum dot with polarized nuclei. (b) Storage, where a pulse in the magnetic field starts amd brings the spins into resonance for a π pulse. Readout is the same process in reverse.

spin-exchange couples this state to the collective nuclear excitation with one flipped spin $|\uparrow\rangle_e \otimes |\mathbf{1}\rangle_n$ with

$$|\mathbf{1}\rangle_n \equiv \left(\sum_j |a_j|^2\right)^{-1/2} \sum_j a_j |I_0, \ldots, (I_0 - 1)_j, \ldots, I_0\rangle_n.$$

Hence the evolution is given by the two-level dynamics

$$\begin{bmatrix} |\downarrow\rangle|\mathbf{0}\rangle \\ |\uparrow\rangle|\mathbf{1}\rangle \end{bmatrix}(t) = \begin{bmatrix} \cos(\Omega t/2) & -i\sin(\Omega t/2) \\ i\sin(\Omega t/2) & \cos(\Omega t/2) \end{bmatrix} \begin{bmatrix} |\downarrow\rangle|\mathbf{0}\rangle \\ |\uparrow\rangle|\mathbf{1}\rangle \end{bmatrix}(0).$$

The ensemble displays Rabi-oscillations with

$$\Omega = \sqrt{2I_0 \sum_j |a_j|^2}.$$

At the same time, the spin-up electronic state $|\uparrow\rangle_e \otimes |\mathbf{0}\rangle_n$ is an eigenstate of both $\hat{V}_D$ and V_Ω. Hence, by pulsing the applied field to go from $g^*\mu_B \hat{\boldsymbol{B}}_{\text{eff}} \gg \hat{V}_\Omega$ to $\hat{\boldsymbol{B}}_{\text{eff}} \sim 0$ for a time $t = \pi/\Omega$ as diagrammed in Fig. 1(b), an arbitrary superposition of the electron-spin will evolve as

$$(\alpha|\uparrow\rangle_e + \beta|\downarrow\rangle_e) \otimes |\mathbf{0}\rangle_n \rightarrow |\uparrow\rangle_e \otimes (\alpha|\mathbf{0}\rangle_n + i\beta|\mathbf{1}\rangle_n), \quad (2)$$

demonstrating that an electronic spin state can be coherently mapped into nuclei.

The Rabi-flopping process can be controlled by removing the electron from the dot or by changing the effective magnetic field, B_0. Away from the resonance condition ($|B_{\text{eff}}| \gg \Omega/g^*\mu_B$), the system is far detuned and no evolution will occur. For the perfectly polarized state the effective detuning is $\delta = (g^*\mu_B - g_n\mu_n)B_0 + I_0 A + (I_0 - 1)A/N$. The Rabi frequency depends upon the distribution of the a_j coefficients, $\Omega = \sqrt{2I_0}A/\sqrt{N}[1 + (\Delta a^2)/\bar{a}^2]^{1/2}$ with bars denoting averages over the set $\{a_j\}$. For a GaAs dot with 10^4 nuclei and $\Delta a^2 \sim \bar{a}^2$, $A \approx 90\mu$ eV and $I_0 = 3/2$, and the speed of transfer is determined by $\Omega/2\pi \simeq 0.6$ GHz, which exceeds the expected spin coherence time (including contributions due to spin-orbit coupling) by 3 orders of magnitude. The resonance condition is fulfilled for $|B_{\text{eff}}| \ll 50$ mT. Retrieval of the stored qubit can be implemented by reversing this process: we either inject a spin-polarized electron into the dot or change the effective magnetic field, bringing the levels into resonance, and Rabi oscillations pick up at the same point as before. The rise time of such a pulse must be much less than the Rabi time for high fidelity transfer. This requirement can be greatly alleviated by using adiabatic passage techniques (see, e.g., Ref. [9])

Before proceeding with a detailed description of the coherence properties and imperfections we note that the above results can be easily generalized to the preparation of complex collective nuclear states. For example, injection of a series of spin-down electrons into spin-up polarized nuclei will lead to a progressive increase of the nuclear spin. In the basis of collective angular momentum, $\hat{\boldsymbol{I}} = \sum_i \hat{\boldsymbol{I}}^i$, we define the total angular momentum nuclear states $|\mathbf{m}\rangle_n = |I = NI_0, I_z = I - m\rangle_n$. Each electron can effect the transfer $|\downarrow\rangle_e \otimes |\mathbf{m}\rangle_n \rightarrow |\uparrow\rangle_e \otimes |\mathbf{m} + \mathbf{1}\rangle_n$. When injected electrons are prepared in different superposition states this process can be used to effectively "engineer" collective states of nuclear ensembles. In fact using a proper sequence of electrons an arbitrary state of the type $|\Psi\rangle_n = \sum_{m=0}^{I} c_m |\mathbf{m}\rangle_n$ can be prepared [11]. We note, in particular, that the highly entangled states of the kind $(|\mathbf{0}\rangle_n + |\mathbf{m}\rangle_n)/\sqrt{2}$, with large $\mathbf{m}$ could be used for high-resolution NMR spectroscopy in analogy with related atomic physics studies [12,13]. Such states can also be prepared by manipulating one-electron in the dot with fast electron-spin resonance (ESR) pulses.

Injection of polarized electrons, combined with ESR pulses, can also be used to perform manipulation of the stored nuclear state. For example, the qubit stored in nuclear-spin could be mapped into the injected electron, manipulated by ESR, and then mapped back. Alternatively off-resonant ($|\delta| \gg \Omega$) coupling of storage states to electron-spin can be used to shift the resonance frequencies of transitions $|\mathbf{m}\rangle_n \rightarrow |\mathbf{m} + \mathbf{1}\rangle_n$ relative to each other. For example in the case of two lowest states $m = 1, 2$ the differential shift is on the order of $\Omega^4/|\delta|^3$. Whenever this shift is large compared to decoherence rate and the spectral width of excitation pulse, the lowest two states of collective manifold $|\mathbf{0}\rangle_n$, $|\mathbf{1}\rangle_n$ can be considered as an effective two-level system and can be manipulated through NMR pulses and other means. These ideas could be extended to the spin-exchange coupled qubits proposal [2].

We now turn to the consideration of various decoherence mechanisms and imperfections that limit the performance of the storage technique. To evaluate the effects of spatial inhomogeneity we note that the collective state $|\uparrow\rangle_e \otimes |1\rangle_n$ is not an eigenstate of $\hat{V}_D$ unless the a_j's are identical. The effect is determined by the distribution of eigenenergies under $\hat{V}_D$ of the states

$|\uparrow\rangle_e \otimes |I_0, \ldots, I_0 - 1_j, \ldots, I_0\rangle_n$, given by

$$E_j = (g^*\mu_B/2 - g_N\mu_N I_0)B_0 + I_0 A/2 - a_j/2.$$

Since $\overline{\Delta E^2} \sim \overline{\Delta a^2}$, the inhomogeneous linewidth is $\sim \bar{a} = A/N$. Hence inhomogeneous broadening is smaller than the relevant time scale for Rabi flopping by a factor of $\sqrt{N}$, and is negligible during transfer operation. After the mapping, its effect can be mitigated by either removing the electron from the dot, thereby turning off the hyperfine interaction entirely, or by using ESR spin-echo techniques to reverse the phase evolution [14].

The leading decoherence process for the stored state is nuclear-spin diffusion, with dephasing rates in the kHz domain. Thermal effects associated with spin-lattice coupling are on the order of minutes even for room-temperature samples [15]. As for spin-diffusion, techniques from NMR can be used to mitigate its effect [15,16]. By applying fast NMR pulse sequences [17] to rotate the nuclear spins the time average of the leading terms in the dipole-dipole Hamiltonian can be reduced to zero, leaving only residual dephasing due to second-order dipolar effects, pulse imperfections, and terms due to the finite length of the averaging sequence. These phenomena have been studied for several decades for solid-state NMR systems and well-developed techniques routinely reduce T_2 by several orders of magnitude [15], down to sub-Hz levels. Hence, coherent qubit storage on the time scale of seconds seems feasible.

To evaluate the effects of partial polarization on the storage fidelity, we use the Heisenberg picture. In the homogeneous case ($a_i = a = A/N$), the Dicke-like collective operators defined above yield $\hat{I}^2$ and $\hat{J}_z = \hat{S}_z + \hat{I}_z$ as the constants of motion. We consider operator equations of motion $\dot{\hat{A}} = i[\hat{A}, \hat{H}]$, for the three operators $\hat{S}_z$, $\hat{S}_+\hat{I}_-$, and $\hat{S}_-\hat{I}_+$, which commute with the constants of motion. We replace $\hat{I}_z$ terms in the resulting equations with $\hat{J}_z - \hat{S}_z$ and use the identity $\hat{I}_+\hat{I}_- = \hat{I}^2 - \hat{J}_z^2 - [\hat{I}_-, \hat{I}_+]/2$ to put the equations in terms of constants of motion and the three operators we look to solve:

$$\frac{d}{dt}\hat{S}_z = a\frac{\hat{S}_+\hat{I}_- - (\hat{S}_+\hat{I}_-)^\dagger}{2i}, \tag{3}$$

$$\frac{d}{dt}(\hat{S}_+\hat{I}_-) = i[(g^*\mu_B - g_n\mu_n)B_0 + a(\hat{J}_z - 1)](\hat{S}_+\hat{I}_-) - ia(\hat{I}^2 - \hat{J}_z^2 + 1/4)\hat{S}_z. \tag{4}$$

It is convenient to choose new constants of motion, $\hat{\delta} = (g^*\mu_B - g_n\mu_n)B_0 + a(\hat{J}_z - 1)$ and $\hat{\Omega} = a(\hat{I}^2 - \hat{J}_z^2 + 1/4)^{1/2}$. These commute with each other and with $\hat{S}_z$, $\hat{S}_+\hat{I}_-$. As these equations are similar to those for two-level atoms in a field, we make the Bloch vector identifications:

$$\hat{U} = -a/\hat{\Omega}\frac{\hat{S}_+\hat{I}_- + (\hat{S}_+\hat{I}_-)^\dagger}{2},$$
$$\hat{V} = -a/\hat{\Omega}\frac{\hat{S}_+\hat{I}_- - (\hat{S}_+\hat{I}_-)^\dagger}{2i}, \tag{5}$$
$$\hat{W} = \hat{S}_z,$$

and Eqs. (3) and (4) become $\dot{\hat{\vec{M}}} = \hat{\vec{M}} \times \hat{\vec{\omega}}$ with $\hat{\vec{\omega}} = (\hat{\Omega}, 0, -\hat{\delta})$. The Bloch vector operator, $\hat{\vec{M}}$, will rotate about the axis defined by $\hat{\vec{\omega}}$ at a frequency $\hat{\omega}_0 = \sqrt{\hat{\delta}^2 + \hat{\Omega}^2}$. For no initial electron spin-nuclear spin correlation, we can easily solve for $\hat{S}_z$ and find

$$\langle \hat{S}_z(t)\rangle = \left\langle \frac{\hat{\delta}^2 + \hat{\Omega}^2\cos(\hat{\omega}_0 t)}{\hat{\omega}_0^2} S_z(0)\right\rangle. \tag{6}$$

For the perfectly polarized nuclear state, Eq. (6) gives $\langle \hat{S}_z\rangle = -1/2\cos(A\sqrt{2I_0/N}t)$ for the spin-down species and $1/2$ for the spin-up species, exactly replicating the fully polarized behavior. In the case of partial polarization, even though we will set $\langle\hat{\delta}\rangle = 0$, $\langle\hat{\delta}^2\rangle$ remains finite. The first part of Eq. (6) prevents complete transfer, to order $\langle\hat{\delta}^2\rangle/\langle\hat{\omega}_0^2\rangle$. For partial polarization $P < 1$, $\langle\hat{\omega}_0^2\rangle \approx a^2N[2I_0 + O(1-P)]$. In a thermal state, all two-particle expectation values factor, so $\langle\hat{\delta}\rangle = 0$ and

$$\langle\hat{\delta}^2\rangle = a^2(\langle\hat{J}_z^2\rangle - \langle\hat{J}_z\rangle^2) \approx Na^2(1-P)2I_0. \tag{7}$$

Accordingly, the error scales as

$$\frac{\langle\hat{\delta}^2\rangle}{\langle\hat{\omega}_0^2\rangle} \approx 1 - P. \tag{8}$$

Remarkably, this result demonstrates that efficient transfer is possible even with many nuclei in the "wrong" state as long as the average polarization per nuclei is high.

We modeled this system numerically with an initial thermal nuclear state. Oscillation of $\langle\hat{S}_z\rangle$ for several polar-

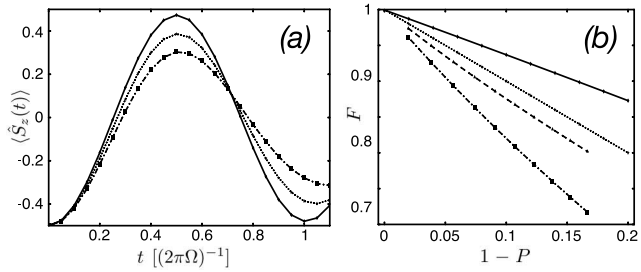

FIG. 2. Transfer characteristics for different nuclear polarizations. (a) The evolution of $\langle S_z(t)\rangle$ for three nuclear polarizations, $P = 0.96$ (solid curve), 0.82 (dotted), and 0.67 (dot-dashed) versus time in units of the $P = 1$ Rabi time. (b) Solid line: Storage fidelity F for inhomogeneous effects ($N = 10^4$) versus error in nuclear polarization, $1 - P$. Dotted line: analytical estimate of thermal effects. Dashed line: numerical simulations of thermal effects. Dot-dashed line: product of inhomogeneous and thermal results.

izations is shown in Fig. 2(a). The effect of partial polarization is immediately apparent; the transfer peak is less than 1/2, and the Rabi frequency decreases. We define minimal fidelity as $F = \mathrm{Tr}\{\rho_0[1/2 + \hat{S}_z(\pi/\langle\hat{\omega}_0\rangle)]^2\}$ with a spin-down initial state. Figure 2(b) shows that the analytical estimate is close to the numerically calculated values; thus, the transfer error can, to a large extent, be explained by the thermal uncertainty in $\hat{\delta}$. The residual effect most likely stems from phase mismatching, as measured by the broadening of ω_0, such that $\langle\cos(\pi\omega_0/\langle\omega_0\rangle)\rangle = 1 - O(1 - P)$.

Inhomogeneous broadening for a thermal initial nuclear state is somewhat more pronounced than the fully polarized case first considered. The $\hat{V}_D$ inhomogeneity causes slow dephasing of the stored state, and the inhomogeneous coupling in $\hat{V}_\Omega$ results in leakage in readout of the stored state into a set of states orthogonal to the original nuclear state. The results shown in Fig. 2(b) were calculated for a distribution of a_i's corresponding to a Gaussian $|\psi(\vec{r})|^2$. We plot the estimated fidelity for a complete storage and readout cycle as a function of the initial polarization of nuclear spins. The total expected fidelity, is approximated by the product of these two results [Fig. 2(b)]. Hence, only modest nuclear polarizations are required to obtain a high fidelity of storage.

In summary, we have demonstrated that it is possible to reliably map the quantum state of a spin qubit onto long-lived collective nuclear-spin states. The resulting states have long coherence times, and can be retrieved on demand. Furthermore, the stored states can be efficiently manipulated and similar techniques can be used for quantum state engineering of collective nuclear states.

Experimental implementation of these ideas will require strong nuclear-spin polarization in the vicinity of confined electrons. Optical pumping of nuclear spins has demonstrated polarizations in GaAs 2D electron gases on the order of 20% [18] and 65% in self-assembled dots [19], and forced spin flips through quantum hall edge states [20] has a claim of a similarly high polarization ability (85%). The methods outlined in this Letter can be used to further increase the nuclear polarization. A current of spin-polarized electrons passing through the quantum dot with a dwell time $\tau_{\mathrm{dwell}} < \pi/\Omega_0$ will result in spin transfer, thereby increasing nuclear polarization. By keeping B_{eff} tuned to zero with increasing nuclear polarization the spin flip-flop remains resonant and, when combined with dephasing to prevent saturation, leads to efficient cooling, similar to a recent proposal [21].

Coherence properties of the spin-exchange process could be probed in transport measurements. For example, sending spin-polarized currents through the quantum dot in which the spin-exchange interaction is tuned to resonance will result in collapses and revivals of the electron-spin polarization that will be a periodic function of the dwell time in the dot. Those can be measured using spin-filter techniques. For a given polarization P and τ_{dwell}, the spin will be rotated by $\Omega(P)\tau_{\mathrm{dwell}}/\pi$.

Practical applications of the storage and retrieval techniques and manipulation of stored states requires time-varying control over the spin-exchange coupling. This can be accomplished by using a pulsed magnetic field of order 50 mT for a few ns, by engineering the electron g factors [22,23], or by optical ac Stark shifts [18]. These techniques can be combined with a number of avenues for entanglement and manipulation of the electronic spin and charge states currently under exploration [1,2]. Finally, quantum memory can facilitate implementation and reduce scaling problems for more ambitious tasks such as quantum error correction [24] or quantum repeaters [9].

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