Electron Spin Decoherence in Quantum Dots due to Interaction with Nuclei

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We study the decoherence of a single electron spin in an isolated quantum dot induced by hyperfine interaction with nuclei. The decay is caused by the spatial variation of the electron wave function within the dot, leading to a nonuniform hyperfine coupling A. We evaluate the spin correlation function and find that the decay is not exponential but rather power (inverse logarithm) lawlike. For polarized nuclei we find an exact solution and show that the precession amplitude and the decay behavior can be tuned by the magnetic field. The decay time is given by $\hbar N/A$, where N is the number of nuclei inside the dot, and the amplitude of precession decays to a finite value. We show that there is a striking difference between the decoherence time for a single dot and the dephasing time for an ensemble of dots.

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The spin dynamics of electrons in semiconducting nanostructures has become of central interest in recent years [1]. The controlled manipulation of spin, and in particular of its phase, is the primary prerequisite needed for novel applications in conventional computer hardware as well as in quantum information processing. It is thus desirable to understand the mechanisms which limit the spin phase coherence of electrons, in particular in GaAs semiconductors, which have been shown to exhibit unusually long spin decoherence times T_2 exceeding 100 ns [2]. Since in GaAs each nucleus carries spin, the hyperfine interaction between electron and nuclear spins is unavoidable, and it is therefore important to understand its effect on the electron spin dynamics [3]. This is particularly so for electrons which are confined to a closed system such as a quantum dot with a spin $1/2$ ground state, since, besides fundamental interest, these systems are promising candidates for scalable spin qubits [4]. For recent work on spin relaxation (characterized by T_1 times) in GaAs nanostructures we refer to Refs. [5–7].

Motivated by this we investigate in the following the spin dynamics of a single electron confined to a quantum dot in the presence of nuclear spins. We treat the case of unpolarized nuclei perturbatively, while for the fully polarized case we present an exact solution for the spin dynamics and show that the decay is nonexponential and can be strongly influenced by external magnetic fields. We use the term "decoherence" to describe the case with a single dot, and the term "dephasing" for an ensemble of dots [8]. The typical fluctuating nuclear magnetic field seen by the electron spin via the hyperfine interaction is of the order of [9] $\sim A/\sqrt{N}\, g\mu_B$, with an associated electron precession frequency $\omega_N \simeq A/\sqrt{N}$, where A is a hyperfine constant, g the electron g factor, and μ_B the Bohr magneton. For a typical dot size the electron wave function covers approximately $N = 10^5$ nuclei, then this field is of the order of 100 G in a GaAs quantum dot. The nuclei in turn interact with each other via dipolar interaction, which does not conserve the total nuclear spin and thus leads to a change of a given nuclear spin configuration within the time $T_{n2} \approx 10^{-4}$ s, which is just the period of precession of a nuclear spin in the local magnetic field generated by its neighbors.

We note that there are two different regimes of interest, depending on the parameter $\omega_N\tau_c$, where τ_c is the correlation time of the nuclear field. The simplest case is given by the perturbative regime $\omega_N\tau_c \ll 1$, characterized by dynamical narrowing: different random nuclear configurations change quickly in time and, as a result, the spin dynamics is diffusive with a dephasing time $\simeq 1/\omega_N^2\tau_c$. A more difficult situation arises when $\omega_N\tau_c \gg 1$, requiring a nonperturbative approach. It is this regime which we will consider in this paper, i.e., the electron is localized in a quantum dot, and the correlation time is due to the internal nuclear spin dynamics, i.e., $\tau_c = T_{n2}$, giving $\omega_N\tau_c = 10^4$. Next, we need to address the important issue of averaging over different nuclear spin configurations in a single dot. Without internal nuclear spin dynamics, i.e., $T_{n2} \to \infty$, no averaging is indicated. However, each flip-flop process (due to hyperfine interaction) creates a different nuclear configuration, and because of the spatial variation of the hyperfine coupling constants inside the dot, this leads to a different value of the nuclear field seen by the electron spin and thus to its decoherence. Below we will find that this decoherence is nonexponential, but still we can indicate a characteristic time given by $(A/\hbar N)^{-1}$ [8]. Moreover, we shall find that $T_{n2} \gg (A/\hbar N)^{-1}$, and thus still no averaging over the nuclear configurations is indicated (and dipolar interactions will be neglected henceforth). To underline the importance of this point, we will contrast below the unaveraged correlator with its average.

Unpolarized nuclei.—We consider a single electron confined to a quantum dot whose spin $\mathbf{S}$ couples to an external magnetic field $\mathbf{B}$ and to nuclear spins $\{\mathbf{I}^i\}$ via hyperfine contact interaction, described by the Hamiltonian

$$\hat{\mathcal{H}} = g\mu_B \mathbf{S}\cdot\mathbf{B} + \mathbf{S}\cdot\mathbf{h}_N, \qquad \mathbf{h}_N = \sum_i A_i \mathbf{I}^i, \quad (1)$$

where $\mathbf{h}_N/(g\mu_B)$ is the nuclear field. The coupling constant with the ith nucleus, $A_i = Av_0|\Psi(\mathbf{r}_i)|^2$, contains the electron envelope wave function $\Psi(\mathbf{r}_i)$ at the nuclear site $\mathbf{r}_i$, and v_0 is the volume of the crystal cell. We start with the case $\mathbf{B} = 0$, and for simplicity we consider nuclear spin 1/2. Neglecting dipolar interactions between the nuclei, we can consider only some particular nuclear configuration, described in the $\hat{I}_z^i$ eigenbasis as $|\{I_z^i\}\rangle$, with $I_z^i = \pm 1/2$. Moreover, we assume an unpolarized configuration with a typical net nuclear magnetic field $A/(\sqrt{N}\, g\mu_B)$, being much less than $A/(g\mu_B)$ (fully polarized case). We study now the decay of the electron spin from its initial ($t = 0$) $\hat{S}_z$ eigenstate $|\Uparrow\rangle$. For this we evaluate the correlator $C_n(t) = \langle n|\delta\hat{S}_z(t)\hat{S}_z|n\rangle$, where $\delta\hat{S}_z(t) = \hat{S}_z(t) - \hat{S}_z$, and $\hat{S}_z(t) = \exp(it\hat{\mathcal{H}})\hat{S}_z\exp(-it\hat{\mathcal{H}})$. This correlator is proportional to $\langle n|\hat{S}_z(t) - \hat{S}_z(0)|n\rangle$. Since at $t = 0$ the total (electron and nuclear) state $|n\rangle = |\Uparrow, \{I_z^i\}\rangle$ is an eigenstate of $\hat{\mathcal{H}}_0 = \hat{S}_z\hat{h}_{Nz}$ (with eigenergy ϵ_n), we can expand in the perturbation $\hat{V} = (1/2)(\hat{S}_+\hat{h}_{N-} + \hat{S}_-\hat{h}_{N+})$ (with $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{V}$). Going over to the interaction picture, we obtain in leading order

$$C_n(t) = \sum_k \frac{|V_{nk}|^2}{\omega_{nk}^2}[\cos(\omega_{nk}t) - 1], \qquad (2)$$

where $V_{nk} = \langle n|\hat{V}|k\rangle$ is the matrix element between initial state $n = \Uparrow, \{\ldots, I_z^k = -1/2, \ldots\}$ and intermediate state $k = \Downarrow, \{\ldots, I_z^k = +1/2, \ldots\}$, and $\omega_{nk} = \epsilon_n - \epsilon_k$. Using $|V_{nk}|^2 = A_k^2\langle n|1/2 - \hat{I}_z^k|n\rangle/4$, and $\omega_{nk} = (h_z)_n + A_k/2$, where $(h_z)_n = \langle n|\hat{h}_{Nz}|n\rangle$, we obtain for the typical nuclear configuration, for which $(h_z)_n^2 \simeq \omega_N^2 \gg A_k^2$,

$$\begin{aligned} C_n(t) &\simeq \gamma\{I_0 - I_1(\tau)\cos[(h_z)_n t] + I_2(\tau)\sin[(h_z)_n t]\}, \\ I_i(\tau) &= \int_{-\infty}^{+\infty} dz\, \chi_0^4(z) F_i[\tau\chi_0^2(z)], \qquad i = 0,1,2, \end{aligned} \qquad (3)$$

where $\gamma = -A^2/[8\pi N(h_z)_n^2]$, $F_0 = 1/2$, $F_1(\eta) = \sin\eta/\eta + (\cos\eta - 1)/\eta^2$, and $F_2(\eta) = \sin\eta/\eta^2 - \cos\eta/\eta$. Here, $N = a_z a^2/v_0 \gg 1$ is the number of nuclei inside the dot, and $\tau = At/2\pi N$. We have replaced the sums over k (which run over the entire space) by integrals and used that $|\Psi(\rho, z)|^2 = (1/\pi a^2 a_z) \times \exp(-\rho^2/a^2)\chi_0^2(z)$. Here a, a_z are the dot sizes in the lateral and transverse (perpendicular to the 2D plane) directions, respectively, and the transverse wave function is normalized, i.e., $\int_{-\infty}^{+\infty} dz\, \chi_0^2(z) = 1$. For any analytic function with expansion $\chi_0^2(z) = \chi_0^2(0) - z^2(\chi_0^2)''/2$ near its maximum, we have $I_{1,2}(\tau \gg 1) = \pm[\chi_0^2(0)/\tau^{3/2}] \times \sqrt{\pi/(\chi_0^2)''}\{\sin[\tau\chi_0^2(0)] \mp \cos[\tau\chi_0^2(0)]\}$. Thus, $C_n(t) - C_n(\infty) \propto 1/\tau^{3/2}$, i.e., the spin decay follows a *power law* for times $\tau \gg 1$, i.e., $t \gg (A/N)^{-1}$. Note that for the typical nuclear configuration the quantity $A^2/N(h_z)_n^2$ is of order unity, thus the part of the electron spin state which decays is of the order of the initial value. Hence the same holds for the spin part which survives at $\tau \gg 1$. For the fully polarized nuclear state the result (3) should be multiplied by 2, and $(h_z)_n$ should be replaced by $A/2$. Moreover, in the presence of a large Zeeman field, $\epsilon_z = g\mu_B B_z \gg \omega_N$, we should substitute ϵ_z for $(h_z)_n$. Thus, the asymptotic behavior of $C_n(t)$ is not changed, the only difference being that the decaying part of the initial spin state is small now, being of the order of $(\omega_N/\epsilon_z)^2 \ll 1$.

We note that $C_n(t)$ in (2) is quasiperiodic in t, and, thus, it will decay only up to the Poincaré recurrence time τ_P. This time can be found from the condition that the terms omitted when converting sums to integrals become comparable with the integral itself. This will happen at $\tau \simeq N$, giving $\tau_P = 0.1$–1 s.

In next order, $\hat{V}^4$, we face the problem of "resonances," i.e., the corrections will contain zero denominators. This gives rise to linearly growing terms $\propto \omega_N t$, even for $t \ll (A/N)^{-1}$. In higher order the degree of the divergence will increase. This means that the decay law we found can, in principle, change after proper resummation, because no small expansion parameter exists, which, strictly speaking, would justify a perturbative approach. Still, the result found in lowest order remains qualitatively correct in that it shows that a nonuniform hyperfine coupling leads to a nonexponential decay of the spin. This conclusion is confirmed by an exactly solvable case to which we turn next.

Polarized nuclei: exactly solvable case.—In this section we consider the exactly solvable case where the initial nuclear spin configuration is fully polarized. We also allow for a magnetic field but neglect its effect on the nuclear spins. With the initial wave function $\Psi_0 = |\Downarrow; \uparrow, \uparrow, \uparrow, \ldots\rangle$ we can construct the *exact* wave function of the system for $t > 0$,

$$\Psi(t) = \alpha(t)\Psi_0 + \sum_k \beta_k(t)|\Uparrow; \uparrow, \uparrow, \downarrow_k, \uparrow, \ldots\rangle, \qquad (4)$$

with normalization $|\alpha(t)|^2 + \sum_k |\beta_k(t)|^2 = 1$, and we assume that $\alpha(t = 0^+) = 1$, $\alpha(t < 0) = 0$. Then, inserting $\Psi(t)$ into the Schrödinger equation we obtain

$$\begin{aligned} i\frac{d\alpha(t)}{dt} &= -\frac{1}{4}A\alpha(t) + \sum_k \frac{A_k}{2}\beta_k(t) - \frac{\epsilon_z\alpha(t)}{2}, \\ i\frac{d\beta_l(t)}{dt} &= \left(\frac{A}{4} - \frac{A_l}{2}\right)\beta_l(t) + \frac{A_l}{2}\alpha(t) + \frac{\epsilon_z\beta_l(t)}{2}, \end{aligned} \qquad (5)$$

where $A = \sum_k A_k$. Laplace transforming (5), we obtain

$$\alpha(t) = \frac{1}{2\pi i}\int_\Gamma \frac{d\omega\, i\exp[(\omega - iA'/4)t]}{i\omega + \epsilon_z + \pi N i\omega\int dz\ln(1 - \frac{iA\chi_0^2(z)}{2\pi N\omega})}, \qquad (6)$$

where $A' = A + 2\epsilon_z$. We have replaced the sum $\sum_k \frac{A_k^2}{i\omega - A'/4 + A_k/2}$ over the xy plane by an integral and calculated it using $|\Psi(\mathbf{r}_k)|^2$ given above. As usual, the integration contour Γ in Eq. (6) is the vertical line in the complex ω plane so that all singularities of the integrand lie to its left. These singularities are the following: two branch points [$\omega = 0$, $\omega_0 = iA\chi_0^2(0)/2\pi N$], and first order poles which lie on the imaginary axis ($\omega = i\nu$). For $\epsilon_z > 0$ there is one pole, while for $\epsilon_z < 0$ there are two poles, and for $\epsilon_z = 0$ there is one first order pole at $\omega_1 \approx iA/2 + iA \int dz\, \chi_0^4(z)/4\pi N$. For the contribution from the branch cut between $\omega = 0$ and $\omega = \omega_0$ we obtain

$$\tilde{\alpha}(t) = \frac{e^{-iA't/4}}{\pi N} \int_0^1 d\kappa\, 2z_0 \kappa e^{i\tau'\kappa} \left\{ \left[\kappa \int dz \ln\left| 1 - \frac{\chi_0^2(z)}{\chi_0^2(0)\kappa} \right| + \kappa/\pi N - 2\epsilon_z/A\chi_0^2(0) \right]^2 + (2\pi z_0)^2 \kappa^2 \right\}^{-1}, \quad (7)$$

where $\tau' = \tau\chi_0^2(0)$, and $z_0 = z_0(\kappa)$ is defined through $\chi_0^2(z_0) = \chi_0^2(0)\kappa$. Let us consider first the case $\epsilon_z = 0$. The asymptotic behavior of the integral (7) for $\tau \gg 1$ is determined by $\kappa \ll 1$. For example, for $\chi_0^2(z)/\chi_0^2(0) = \exp(-z^2)$ we find $\tilde{\alpha} \propto 1/\ln^{3/2}\tau$. Thus, the decay of $|\alpha(t)|$ starts at $\tau > 1$, i.e., at $t > A^{-1}N$, as in the unpolarized case. Note that the magnitude of $\tilde{\alpha}$ is of order $1/N$, thus the decaying part of the initial spin state has this smallness as well, in contrast to the unpolarized case above where this part is of order unity [10]. For large Zeeman field ($|\epsilon_z| \gg A$) and for $\tau \gg 1$, the main contribution in (7) is given for $\kappa \to 1$. Expanding $\chi_0^2(z)$ for small z (see above), we obtain $z_0^2 = 2\chi_0^2(0)(1 - \kappa)/(\chi_0^2)''$. Then from Eq. (7) we have for $|\epsilon_z| \gg A$

$$\tilde{\alpha}(\tau \gg 1) = \frac{-e^{i\tau' - iA't/4}}{4\sqrt{\pi}\, N} \frac{\chi_0^2(0)}{\sqrt{(\chi_0^2)''}} \frac{A^2}{\epsilon_z^2} \frac{(1 + i)}{\tau^{3/2}}. \quad (8)$$

From this we find then that the correlator $C_0(t) = -\langle\Psi_0|\delta\hat{S}_z(t)\hat{S}_z|\Psi_0\rangle = (1 - |\alpha(t)|^2)/2$ agrees with the perturbative result (3) for the fully polarized state, i.e., $C_0(t) - C_0(\infty) \propto 1/t^{3/2}$. (Note that the asymptotic behavior of the correlator is given by the term which is the cross product of the pole contribution and the one [Eq. (8)] from the branch cut.) This agreement is to be expected, since for large Zeeman field, the perturbative treatment with a small parameter $A/|\epsilon_z| \ll 1$ is meaningful [11]. However, at zero Zeeman field, when the system cannot be treated perturbatively, we find $C_0(t) - C_0(\infty) \propto 1/\ln^{3/2}t$, and the agreement with (3) breaks down. Nevertheless, the characteristic time scale for the onset of the nonexponential decay is the same for all cases and given by $(A/N)^{-1}$. We have also checked that $\langle\hat{S}_x(t) - \hat{S}_x(0)\rangle$ has the same behavior as $\langle\hat{S}_z(t) - \hat{S}_z(0)\rangle$ with the same characteristic time $-N/A$.

There are several interesting features which we can observe for the fully polarized state. In an external Zeeman field, the effective gap seen by the electron spin is $A'/2 = A/2 + \epsilon_z$. Thus, when ϵ_z is made negative this gap decreases and even vanishes at $|\epsilon_z| = A/2$. From Eq. (6) we find that the two poles are symmetric in this case, and the system resonates between the two frequencies $\omega_\pm = \pm iA(\int \chi_0^4(z)\, dz)^{1/2}/\sqrt{8\pi N}$. Note that the residual gap is of order $A/\sqrt{N}$ (and not A/N, as one might naively expect). Near this Zeeman field we have $|\alpha(t)|^2 = \cos^2(\omega_+ t)$ (up to small corrections of order $1/N$), and, as a result, $|\alpha|^2$ averaged over time is $1/2$, i.e., the up and down states of the electron spin are strongly coupled via the nuclei. In contrast, outside this resonance regime the value of $|\alpha|^2$ is close to 1 (again with small $1/N$ corrections), i.e., $\langle\hat{S}_z(t)\rangle = 1/2 - |\alpha|^2$ is close to $-1/2$ at any time. The width of the resonance is $\sim A/\sqrt{N}$, i.e., small compared to the initial gap $A/2$. We note that this behavior represents periodic (Rabi) oscillations with a single well-defined frequency and is not related to decoherence. [The latter is described by the branch cut contribution $\tilde{\alpha}$ which remains small (order $1/N$) even near the resonance.] This abrupt change in the amplitude of oscillations of $\langle\hat{S}_z(t)\rangle$ (when changing ϵ_z in a narrow interval around $A/2$) can be used for an experimental detection of the fully polarized state. Note that the weight of the upper pole alone (i.e., that which exists at $\epsilon_z = 0$) also drops abruptly from a value close to 1 to a value much smaller than 1 in the same narrow interval, which can be experimentally checked by Fourier analysis. Another special value of Zeeman field corresponds to the case when the upper pole is close to ω_0 ($\kappa = 1$)—the upper edge of the branch cut. This occurs [see Eq. (6)] at the critical value $\epsilon_z^\star = bA/2 < 0$, where $b = \chi_0^2(0)\int dz \ln|1 - \chi_0^2(z)/\chi_0^2(0)| < -1$ is a nonuniversal number which depends on the dot shape. Since at finite Zeeman field the asymptotics in t is determined by κ's close to 1, we see from Eq. (7) that for $\epsilon \approx \epsilon_z^\star$ the asymptotics changes abruptly. Indeed, for $[(\epsilon_z - \epsilon_z^\star)/A]^2 \ll 1$, we find $\tilde{\alpha} \propto 1/\sqrt{\tau}$, for $1 \ll \tau \ll [(\epsilon_z - \epsilon_z^\star)/A]^{-2}$, and $\tilde{\alpha} \propto 1/\tau^{3/2}$, for $\tau \gg [(\epsilon_z - \epsilon_z^\star)/A]^{-2}$. Thus, when approaching the critical Zeeman field $\epsilon_z^\star$ there is a *slow down* of the asymptotics from $1/\tau^{3/2}$ to $1/\tau^{1/2}$. It is interesting that this slow down is related to a strong modulation of the density of states (DOS) of the excitations within the continuum band (branch cut) near its edge when $\epsilon_z \to \epsilon_z^\star$. In the subspace of none or one nuclear spin flipped [see Eq. (4)], the DOS becomes $\nu(u) = \mathrm{Im}[G_0(u) + d/du \ln D(u)]$, where $u = i\omega$, $G_0(u) = \sum_k 1/(u + A_k/2)$ is the "unperturbed Green's function," and $D(u)$ is the denominator of $\alpha(\omega)$; see Eq. (6). One can then show that for $\epsilon_z \to \epsilon_z^\star$ (i.e., the upper pole approaches the continuum edge), the DOS develops a square root singularity: $\nu(u) \propto 1/\sqrt{\omega_0 - u}$. Simultaneously, the weight of the upper pole vanishes linearly in ϵ_z as $\epsilon_z^\star - \epsilon_z \to 0$.

Finally, the nuclear state is characterized by $\beta_k(t)$, which allows for similar evaluation as α. Here we just note that its branch cut part, $\tilde{\beta}_k(t)$, is nonmonotonic in time, particularly pronounced at $\epsilon_z \to \epsilon_z^\star$: First $\tilde{\beta}_k(t)$ grows like $\sqrt{\tau}$, until τ reaches $\sim 1/(1 - a_k) \gg 1$, and then it decays like $1/[\sqrt{\tau}(1 - a_k)]$, with $a_k = A_k/A_0 \to 1$. Thus, β_k is maximal for A_k close to A_0, i.e., the nuclei near the dot center are affected most by the hyperfine interaction with the electron spin.

Averaging over nuclear configurations.—We have seen that the decay of $C_n(t)$ occurs in the time interval $N/A \ll t \ll N^2/A$, with $N/A \simeq 10^{-6}$ s in GaAs dots. On the other hand, the electron spin precesses in the net nuclear field [see Eq. (3)] with the characteristic period $(h_z)_n^{-1} \simeq \omega_N^{-1} \simeq 10^{-8}$–$10^{-9}$ s. Thus, $\omega_N^{-1} \ll N/A$, and we see that the electron spin undergoes many precessions in a given nuclear field before decoherence sets in due to the nonuniform hyperfine couplings A_k. This behavior changes dramatically when we average over nuclear configurations [8]. For that purpose we consider high temperatures, $k_B T \gg \hbar\omega_N$, and average $C_n(t)$ in Eq. (2) over all nuclear configurations, i.e., $C(t) = \sum_n C_n(t)/\sum_n$. We then find

$$C(t) = \sum_k \frac{-A_k^2}{8} \int_0^t dt_1 \int_0^t dt_2 \prod_{i \neq k} \cos\left[\frac{A_i}{2}(t_1 - t_2)\right]. \tag{9}$$

For $\tau \ll 1$, we get $\prod_{i\neq k}\cos(A_i t/2) = \exp[-NC(At/2\pi N)^2]$, where $C = \pi \int dz\, \chi_0^4(z)/4$. Thus, the averaged spin correlator $C(t)$ (9) is of order $-\int_0^{\omega_N t} dx\, \Phi(x)$, with Φ being the error function. Thus, $C(t)$ grows without bound as $\omega_N t$ for $\omega_N t \gg 1$ (the condition $\tau \ll 1$ can still be satisfied). Consequently, the perturbative approach breaks down even in leading order in $\hat{V}$ (we recall that *without* averaging the divergences occur in all higher but not in lowest order). To treat this case properly, we need a nonperturbative approach. For that purpose we calculate now the correlator $C(t)$ exactly by treating the nuclear field purely classically, i.e., as a c number. Then we obtain

$$C_n(t) = -\frac{h_{N\perp}^2}{4h_N^2}(1 - \cos h_N t), \tag{10}$$

where $h_N = \sqrt{h_{Nz}^2 + h_{N\perp}^2}$ is the nuclear field, with $h_{N\perp}^2 = h_{Nx}^2 + h_{Ny}^2$. The value of h_N corresponds to a given nuclear configuration n. To make contact with the perturbation procedure which we used before in the quantum case we go to the regime $h_{N\perp}^2 \ll h_{Nz}^2$, where h_N can be replaced by h_{Nz} in Eq. (10). Then we average the resulting expression $(h_{N\perp}^2/h_{Nz}^2)(1 - \cos h_{Nz} t)$ over a Gaussian distribution for h_N, i.e., over $P(h_N) \propto \exp(-3h_N^2/2\omega_N^2)$. The result becomes proportional to $\int_0^{+\infty} dz \exp(-z^2/2) \times [1 - \cos(\gamma z)]/z^2 \propto \int_0^\gamma dx\, \Phi(x)$, where $\gamma = \omega_N t/\sqrt{3}$. Thus, we see that we obtain exactly the same functional form as before from Eq. (9) with the same divergencies in t. This reassures us that the treatment of the nuclear field as a classical field is not essential. On the other hand, the same Gaussian averaging procedure can now be applied to the nonperturbative form Eq. (10). Defining $C_{cl}(t) = \int dh_N\, P(h_N) C_n(t)$, we obtain

$$C_{cl}(t) = -\frac{1}{6}\left[1 + \left(\frac{\omega_N^2 t^2}{3} - 1\right)e^{-\omega_N^2 t^2/6}\right]. \tag{11}$$

Thus we get rapid (Gaussian) decay of the correlator for $t \gg \omega_N^{-1}$, giving the dephasing time $\omega_N^{-1} = \sqrt{N}/A$. This means that $\langle \hat{S}_z(t) S_z\rangle$ saturates at $1/3$ of its initial value of $1/4$. Finally, it seems likely that for the case of nuclear quantum spins a nonperturbative treatment of the averaged correlator $C(t)$ will lead to a similar rapid time decay as found for the classical case in Eq. (11).

In conclusion, we have studied the spin decoherence of an electron confined to a single quantum dot in the presence of hyperfine interaction with nuclear spins. The decoherence is due to a nonuniform coupling of the electron spin to nuclei located at different sites. The decoherence time is given by $\hbar N/A$ and is of the order of several μs. It is shown that in a weak external Zeeman field the perturbative treatment of the electron spin decoherence is impossible. Moreover, the decay of the electron spin correlator in time does not have an exponential character, instead it is given by a power or inverse logarithm law. We have shown that there is a strong difference between the decoherence time for a single dot, $\hbar N/A$, and the dephasing time for an ensemble of dots, $\hbar\sqrt{N}/A$.

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[8] We note that in cases with exponential decay the spin decoherence (dephasing) time is usually denoted by T_2 (T_2^*) (e.g., in the Bloch equations). E.g., in Ref. [2] it is $T_2^\star$ which is measured (due to an ensemble average over many spins). In general, $T_2^\star < T_2$. Since the spin decay found in the present work turns out to be nonexponential, we will not use this notation (i.e., T_2 or T_2^*) to avoid confusion.

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[11] The same is true for any model with a small expansion parameter. E.g., for a system with anisotropy, where the hyperfine constants in perpendicular and transverse directions are different, i.e., $A_i^z = A_z v_0|\Psi(\mathbf{r}_i)|^2 \neq A_i^\perp = A_\perp v_0|\Psi(\mathbf{r}_i)|^2$, one obtains again a change in the asymptotics from $1/\ln^{3/2}\tau$ (zero anisotropy) to $1/\tau^{3/2}$ (large anisotropy). The anisotropy plays the role of ϵ_z.