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Investigation of Decoherence of Superconducting Charge Qubit Entangled with the Environment

Ying-Hua Ji

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Abstract In the Born-Markov approximation, a method that calculates the energy relaxation time T_1 and the decoherence time T_2 of superconducting qubits is given by solving the set of Bloch-Redfield equations and considering the results of decoherence of a superconducting charge qubit. Compared to the spin-boson model, it not only contains the decoherence being caused by the dissipative environment, but also includes the decoherence being generated by the dissipative elements in a superconducting electronic circuit. Hence, it is good for studying the decoherence of superconducting qubits comprehensively.

Keywords Charge qubit · Decoherence · The Born-Markov approximation

1 Introduction

In the quest for practical systems for carrying out quantum computations, solid-state systems that make use of the Josephson effect are viable candidates [1–3]. Presently, three prototypes of superconducting qubits are studied experimentally [4–6]. The charge ($E_C \gg E_J$) and the flux ($E_C \ll E_J$) qubits are distinguished by their Josephson junction's relative magnitude of charging energy E_C and Josephson energy E_J. A third type, the phase qubit, operates in the same regime as the flux qubit, but consists of a single Josephson. In all of these systems, the quantum state of the superconducting phase differences across the Josephson junctions in the circuit contains the quantum information, i.e., the state of the qubit.

The interaction of quantum system and environment will cause two demolishment processes, e.g., quantum dissipation and quantum decoherence. The former will cause energy dissipation, and the latter will make the system degenerate from coherent state to classical state [7, 8]. For a true two-level qubit, decoherence occurs due to the coupling of the

Y.-H. Ji (✉)
Department of Physics, Jiangxi Normal University, Nanchang, Jiangxi 330022, China
e-mail: jyh2006@jxnu.edu.cn

Y.-H. Ji
Key Laboratory of Optoelectronic & Telecommunication of Jiangxi, Nanchang, Jiangxi 330022, China

qubit to its environment. Compared with other qubit candidates (such as trapped ions, nuclear spins, and cavity QED), decoherence presents a much more formidable challenge to superconducting qubits. All of the proposed superconducting qubits have multiple energy levels which result in adverse effects on quantum gate operations. A number of decoherence mechanisms can be important, being both intrinsic to the Josephson junctions (e.g., oxide barrier defects or vortex motion), and external (e.g., charge fluctuations from the external control circuits such as voltage sources). Here, we concentrate on the latter effect, i.e., charge fluctuations. Decoherence in charge qubits has previously been investigated using the spin-boson model [9, 10].

The spin-boson model, one of the most basic models of describing the quantum system, is simple and practical in studying the quantum dissipation characteristic, and can be easily used to describe the dissipation nature in the quantum system. Based on this model, the evolution of the quantum systems is described as a two-level dynamics process. Previous theoretical works on the decoherence of superconducting qubits have typically relied on the widely used spin-boson model that postulates a purely two-level dynamics, therefore neglecting leakage effects. Combining network graph theory with the Caldeira-Leggett model for dissipative elements [11], Refs. [12, 13] present a multilevel quantum theory of decoherence for a general circuit with superconducting flux qubits. In this paper, in Born-Markov approximation, adopting the multilevel quantum description method, we derive the Hamiltonian of a circuit with Josephson junction, and study a concrete decoherence of a quantum circuit with superconducting charge qubit.

2 Model and Equations of Motion

In this section, we investigate the decoherence property of superconducting charge qubits coupled the environment which is shown in Fig. 1. In the circuit, a superconducting block is connected to a superconducting pole through a tunneling knot with capacitance C_J and the Josephson energy E_J, and the gate voltage V_g is coupled to the system through the gate capacitor C_g. The charge energy of single electron is $E_{C\Sigma} = e^2/2(C_g + C_J)$, which is related to the gate voltage; choosing the superconducting material with large energy level spacing, we can stop the transitions of electronics. We use an impedance $Z(\omega)$ to imitate the phase relaxation which arises from the fluctuations of the voltage being caused by the charge fluctuations. Or, we primarily consider the decoherence of superconducting charge qubits arising from the fluctuations of the charge in the environment [14].

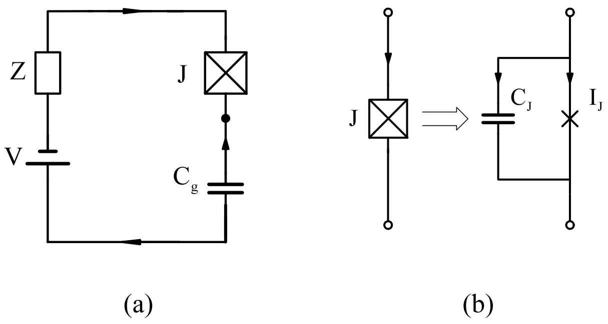

Fig. 1 Circuit of a single voltage-biased charge box (**a**); A Josephson consists of two branches: a Josephson junction (*cross*) and shunt capacitor C_J and a shunt resistor R is neglected (**b**)

The constraint relation between the current moving through Josephson junction and voltage (flux) is [15, 16]

$$I_J = I_C \sin\varphi, \tag{1}$$

$$\frac{d\Phi(t)}{dt} = V(t). \tag{2}$$

Where, φ is the superconducting phase differences cross the junction; I_J is the super-current of Josephson junction; I_C is the critical current of the junction.

$$\Phi(t) = \varphi\frac{2\pi}{\Phi_0}, \tag{3}$$

$\Phi_0 \equiv h/2e$ is the superconducting flux quantum. The current-voltage relations for the capacitor is

$$Q = CV. \tag{4}$$

Because of $V_J(t) = V_{CJ}$, we get

$$\frac{\Phi_0}{2\pi}\dot{\varphi} = C^{-1}Q_C. \tag{5}$$

In linear approximation, the relation between current and voltage of bias impedance $Z(t)$ is

$$V_Z(t) = \int_{-\infty}^{t} Z(t-\tau)I_Z(\tau)d\tau \equiv (Z * I_Z)(t). \tag{6}$$

$Z(t)$, the external impedance, is a response function being lingering, which means $Z(t-\tau)$ to be nonzero only for $t > \tau$. From physical aspect, $Z(t)$ should be limited within the whole integral region. Make Fourier transform to $V_Z(t)$ and $I_Z(t-\tau)$ and we get the expression of response function $Z(t)$ in frequency domain

$$Z(\omega) = \int_{-\infty}^{\infty} Z(t)\exp(i\omega)dt = \int_{0}^{\infty} Z(t)\exp(i\omega)dt, \tag{7}$$

$$V_Z(\omega) = Z(\omega)I_Z(\omega). \tag{8}$$

$V_Z(\omega)$ and $I_Z(\omega)$ are the Fourier transforms of $V_Z(t)$ and $I_Z(t)$ respectively. According to Kirchhoff's current (or voltage) law, from Fig. 1, we can obtain

$$\dot{\Phi} = V_g - V - V_Z, \tag{9}$$

$$I_{CJ} + I_g + I_J = 0. \tag{10}$$

Assuming all charges and fluxes are zero at initial time, from (1)–(10),we arrive at the classical equation of motion for the superconducting phase Φ in Fig. 1

$$\begin{aligned} C_\Sigma\ddot{\Phi} &= -\frac{\partial U}{\partial \Phi} + \frac{i\omega C_g^2 Z(\omega)}{C_\Sigma + i\omega C_g Z(\omega)(C_\Sigma + C_g)} * \dot{Q}, \\ U(\Phi) &= E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) \end{aligned} \tag{11}$$

where $C_\Sigma = C_J + C_g$.

Next, the Hamiltonian of the total system is

$$H(t) = H_S + H_B + H_{SB}. \tag{12}$$

The Hamiltonian of the circuit

$$H_S = \frac{(Q + C_g V)^2}{2C_\Sigma} + E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right), \tag{13}$$

giving rise to the equation of motion (11) without dissipation $Z(\omega) = 0$, can readily be quantized with the commutator rule

$$[\Phi, Q] = i\hbar.$$

H_B is given by

$$H_B = \frac{1}{2}\sum_\alpha \left(\frac{1}{m_\alpha} p_\alpha^2 + m_\alpha \omega_\alpha^2 x_\alpha^2\right). \tag{14}$$

H_B is the Hamiltonian describing a bath of harmonic oscillators (fictitious) position and momentum operators x_α and p_α with $[x_\alpha, p_\beta] = i\hbar\delta_{\alpha\beta}$, masses m_α, and oscillator frequencies ω_α. The H_{SB} describes the coupling between the system and bath degrees of freedom

$$H_{SB} = Q \cdot \sum_\alpha c_\alpha x_\alpha, \tag{15}$$

c_α is the coupling coefficient. From $H(t)$, we can derive the Hamilton equations for the bath and the system coordinates,

$$\dot{x}_\alpha = \frac{\partial H}{\partial p_\alpha} = \frac{1}{m_\alpha} p_\alpha, \tag{16}$$

$$\dot{p}_\alpha = -\frac{\partial H}{\partial x_\alpha} = -m_\alpha \omega_\alpha^2 x_\alpha - c_\alpha Q, \tag{17}$$

$$\dot{\Phi} = \frac{\partial H}{\partial Q} = Q + \sum_\alpha c_\alpha x_\alpha, \tag{18}$$

$$\dot{Q} = -\frac{\partial H}{\partial \Phi} = -\frac{\partial U}{\partial \Phi}. \tag{19}$$

From (16)–(19), take their derivative with respect to time, we obtain

$$C_\Sigma \ddot{\Phi} = -\frac{\partial U}{\partial \Phi} + \sum_\alpha c_\alpha \dot{x}_\alpha. \tag{20}$$

Set

$$K(\omega) = \sum_\alpha \frac{c_\alpha^2}{m_\alpha(\omega^2 - \omega_\alpha^2)}. \tag{21}$$

And the spectral density of a bath of harmonic oscillators is defined as

$$J(\omega) = \frac{\pi}{2} \sum_\alpha \frac{c_\alpha^2}{m_\alpha \omega_\alpha} \delta(\omega - \omega_\alpha). \tag{22}$$

We use the replacement $K(\omega) \to K(\omega + i\varepsilon)$. The way to make it passing the limit is

$$\frac{1}{\omega - \omega'} = \lim_{\varepsilon \to 0} \frac{1}{\omega - \omega' + i\varepsilon} = P \frac{1}{(\omega - \omega')} - i\pi\delta(\omega' - \omega).$$

Where P denotes the principal value of $\frac{1}{\omega - \omega'}$. Combining the representation (22) of the spectral density $J(\omega)$, we get

$$K(\omega) = \frac{2}{\pi} P \int_0^\infty d\omega' \frac{\omega' J(\omega')}{(\omega^2 - \omega'^2)} - iJ(\omega).$$

Then, we have

$$J(\omega) = -\operatorname{Im} K(\omega), \tag{23}$$

$$\operatorname{Re} K(\omega) = -\frac{2}{\pi} P \int_0^\infty d\omega' \frac{\omega' \operatorname{Im} K(\omega')}{(\omega^2 - \omega'^2)}. \tag{24}$$

To take the Fourier transform of (11) and (20), under the low-order approximation, we have

$$K(\omega) = -i\omega Z(\omega) \left(\frac{C_g}{C_J + C_g} \right)^2. \tag{25}$$

Then, we obtain

$$J(\omega) = \omega \operatorname{Re} Z(\omega) \left(\frac{C_g}{C_J + C_g} \right)^2. \tag{26}$$

3 Bloch-Redfield Formalism

From the Hamiltonian $H(t)$, the master equation for the evolution of the system density matrix can be derived. In Born-Markon approximation, the eigenbasis vector should be the eigenstate $|n\rangle$ of Hamiltonian H_S, with the reduced density matrix element $\rho_{nm} = \langle n|\rho|m\rangle$, $H_S|n\rangle = \omega_n|n\rangle$, which obeys Redfield equation [17, 18]

$$\dot{\rho}_{nm}(t) = -i\omega_{nm}\rho_{nm}(t) - \sum_{kl} R_{nmkl}\rho_{kl}(t), \tag{27}$$

where $\omega_{nm} = \omega_n - \omega_m$ and with the Redfield relaxation tensor

$$R_{nmkl} = \delta_{lm} \sum_r \Gamma^{(+)}_{nrrk} + \delta_{nk} \sum_r \Gamma^{(-)}_{lmnk} - \Gamma^{(+)}_{lmnk} - \Gamma^{(-)}_{lmnk}, \tag{28}$$

where

$$\Gamma^{(+)}_{lmnk} = \int_0^\infty dt \exp(-i\omega_{nk}t) Tr_B \tilde{H}_{SB}(t)_{lm} \tilde{H}_{SB}(0)_{nk} \rho_B, \tag{29}$$

$$\Gamma^{(-)}_{lmnk} = \int_0^\infty dt \exp(-i\omega_{lm}t) Tr_B \tilde{H}_{SB}(0)_{lm} \tilde{H}_{SZ}(t)_{nk} \rho_B, \tag{30}$$

$$\tilde{H}_{SB}(t) = \exp(iH_Bt) H_{SB} \exp(-iH_Bt). \tag{31}$$

We also obtain

$$\mathrm{Re}\,\Gamma^{(+)}_{lmnk} = (Q)_{lm}\cdot(q)_{nk}\frac{J(|\omega_{nk}|)}{\hbar}\frac{\exp(-\beta\omega_{nk}/2)}{\sinh(\beta|\omega_{nk}|/2)}, \tag{32}$$

$$\mathrm{Im}\,\Gamma^{(+)}_{lmnk} = \frac{1}{\hbar}(Q)_{lm}\cdot(Q)_{nk}\frac{2}{\pi}P\int_0^\infty d\omega\frac{J(\omega)}{\omega^2-\omega_{nk}^2}\left(\omega_{nk}\coth\frac{\hbar\beta\omega}{2}-\omega\right). \tag{33}$$

The Redfield equation (27) can be derived for arbitrary superconducting circuits. The superconducting circuit can represent a single qubit or number of qubits. In very low temperature, we assume that the circuit system is initially in one of the two lowest level eigenstates ($|0\rangle$ or $|1\rangle$), then compared to R_{nmkl} at $n,m,k,l=0,1$, all the R_{nmkl} at $k,l=0,1,n,m\neq 0,1$ can be neglected ($\beta\omega_{12}\gg 1$). In this case, the Redfield equation (27) turns into a Bloch equation with the energy relaxation (T_1) and decoherence (T_2) times

$$\frac{1}{T_1} = 2\,\mathrm{Re}(\Gamma^{(+)}_{0110}+\Gamma^{(+)}_{1001}), \tag{34}$$

$$\frac{1}{T_\phi} = \mathrm{Re}(\Gamma^{(+)}_{0000}+\Gamma^{(+)}_{1111}-2\Gamma^{(+)}_{0011}), \tag{35}$$

$$\frac{1}{T_2} = R'_{0101} = \frac{1}{2T_1}+\frac{1}{T_\phi}, \tag{36}$$

where $R'_{nmkl}=\mathrm{Re}\,R_{nmkl}$, T_ϕ is dephasing time. Using (32) and (33), we obtain

$$\frac{1}{T_1} = \frac{4}{\hbar}|\langle 0|Q|1\rangle|^2 J(\omega_{01})\coth\left(\frac{\omega_{01}}{2k_BT}\right), \tag{37}$$

$$\frac{1}{T_\phi} = \frac{1}{\hbar}|\langle 0|Q|0\rangle-\langle 1|Q|1\rangle|^2\left.\frac{J(\omega)}{\omega}\right|_{\omega\to 0}(2k_BT). \tag{38}$$

4 Semiclassical Approximation

The potential energy $V(q)$ will form a asymmetry double-well potential under a certain condition, the left and right well has a minimum value, respectively. Under semiclassical approximation, we only consider two-level question. When the potential barrier is very high, there exist bind states ψ_L and ψ_R, respectively. However, when the potential barrier is limited, there will exist a potential barrier tunneling, with the two states being combined. The Hibert space spanned by the ground state of the two wells is not convenient since the Hamiltonian of (12) is not diagonal in the basis $\{|L\rangle,|R\rangle\}$. We represent the two-state system in new basis $\{|0\rangle,|1\rangle\}$ given by

$$|0\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1+\frac{\varepsilon}{\omega_{01}}}|L\rangle+\sqrt{1-\frac{\varepsilon}{\omega_{01}}}|R\rangle\right), \tag{39}$$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{\varepsilon}{\omega_{01}}}|L\rangle-\sqrt{1+\frac{\varepsilon}{\omega_{01}}}|R\rangle\right). \tag{40}$$

ε is the classical energy difference and $\Delta=\langle L|H_S|R\rangle$ are the tunneling amplitude between the two wells, and $\omega_{01}=\sqrt{\Delta^2+\varepsilon^2}$ is the energy splitting between the two quantum eigenstates in this energy double well. Because ψ_L and ψ_R are local track,we can do approximation as follows

$$\langle L|Q|R\rangle \approx 0, \qquad \langle L|Q|L\rangle \approx Q_L, \qquad \langle R|Q|R\rangle \approx Q_R.$$

We can easily conclude the following eigen-matrix element

$$\langle 0|Q|1\rangle \approx \frac{1}{2}\frac{\Delta}{\omega_{01}}(\delta Q), \tag{41}$$

$$\langle 0|Q|0\rangle - \langle 1|Q|1\rangle = \frac{\varepsilon}{\omega_{01}}(\delta Q). \tag{42}$$

$(\delta Q) = Q_0 - Q_1$ is the "distance" between two localized low-energy classical charge states Q_0 and Q_1. Finally, under semiclassical approximation, the relaxation and decoherence time are given by

$$\frac{1}{T_1} = \frac{\Delta^2(\delta Q)^2}{\hbar}\left(\frac{C_g}{C_J + C_g}\right)^2 \frac{\operatorname{Re} Z(\omega_{01})}{\omega_{01}} \coth\left(\frac{\omega_{01}}{2k_B T}\right), \tag{43}$$

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{(\delta Q)^2}{\hbar}\left(\frac{\varepsilon}{\omega_{01}}\right)^2\left(\frac{C_g}{C_J + C_g}\right)^2 \frac{\operatorname{Re} Z(0)}{\hbar}(2k_B T). \tag{44}$$

We briefly analyse the results mentioned above: (1) The construction of the circuit affects directly on the energy relaxation time T_1 and the decoherence time T_2. Only coming from the external impedance $Z(\omega)$, these results are obtained. The external impedance $Z(\omega)$ is taken as the fluctuation of the external voltage. (2) From (41), we realize that Δ reflects the behavior of the coupled matrix element $Q_{01} = \langle 0|Q|1\rangle$ for two levels. Combining (43) and (44), the energy relaxation time T_1 and the decoherence time T_2 decrease as the coupled matrix element $Q_{01} = \langle 0|Q|1\rangle$ increases. To make sure the correctness of quantum computation on the given decoherence time, we must enhance the operating velocity. (3) For the charge qubit described by the circuit shown in Fig. 1, supposed that $Z(\omega)$ is pure resistor, it is quite apparent that the dephasing time T_ϕ is proportional to the $1/\operatorname{Re} Z(\omega)$; the energy relaxation time T_1 is also proportional to the $1/\operatorname{Re} Z(\omega)$ which transits from high level to low level and the decoherence time T_2 has an identical property. (4) The energy relaxation time T_1 and the decoherence time T_2 are sensitive to the environmental temperature. In fact, at low temperature, the dephasing time T_ϕ is inverse proportional to the temperature. The energy relaxation time T_1 reduces as the temperature rises and the decoherence time T_2 also decreases as the temperature goes up.

What should be pointed out is that the state $|L\rangle$ and $|R\rangle$ in the left and the right wells actually overlap locally, that is the overlap integral between the left and right states $\langle L|R\rangle \neq 0$. In semiclassical approximation, we think $\langle L|R\rangle = 0$ without considering the effect of such overlap locally on the decoherence of the system. In the following we are going to have quantum corrections on the above results.

Taking into account the overlap effect of the potential well we introduce parameters β as the overlap integral between the left and right states

$$\beta = \langle L|R\rangle. \tag{45}$$

The normalization of the wave function is

$$|\Psi\rangle = \frac{1}{\sqrt{1 + 2\beta\lambda + \lambda^2}}(|L\rangle + \lambda|R\rangle). \tag{46}$$

Supposing $\langle L|H_S|L\rangle = \langle R|H_S|R\rangle$, $\langle L|H_S|R\rangle = \langle L|H_S|R\rangle$, the average energy of the system is

$$\langle E\rangle = \langle \Psi|H_S|\Psi\rangle = \frac{(1+\lambda^2)\langle L|H_S|L\rangle + 2\lambda\langle L|H_S|R\rangle}{1+2\beta\lambda+\lambda^2}. \tag{47}$$

The variation parameters λ can be given from

$$\frac{\partial\langle E\rangle}{\partial\lambda} = 0. \tag{48}$$

Obviously, when the two wells are far from each other, the variation parameters tends to one, $\lambda \to 1$ and the overlap integral tends to zero, $\beta \to 0$; when the two wells completely overlap, the variation parameters tends to zero, $\lambda \to 0$ and the overlap integral tends to one, $\beta \to 1$. For the issues discussed in this paper, overlap integral β is a very small number. The values of λ depends on the characteristics of the potential wells. In this paper we adopt the transformation in Ref. [10]

$$|\tilde{L}\rangle = \frac{1}{\sqrt{1-2\beta\lambda+\lambda^2}}(|L\rangle - \lambda|R\rangle), \tag{49}$$

$$|\tilde{R}\rangle = \frac{1}{\sqrt{1-2\beta\lambda+\lambda^2}}(|R\rangle - \lambda|L\rangle). \tag{50}$$

From the orthonormality, we can get

$$\lambda = \frac{1-\sqrt{1-\beta^2}}{\beta}. \tag{51}$$

In the above transformation, we have

$$|0\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1+\frac{\varepsilon}{\omega_{01}}}|\tilde{L}\rangle + \sqrt{1-\frac{\varepsilon}{\omega_{01}}}|\tilde{R}\rangle\right), \tag{52}$$

$$|1\rangle = \frac{1}{\sqrt{2}}\left(\sqrt{1-\frac{\varepsilon}{\omega_{01}}}|\tilde{L}\rangle - \sqrt{1+\frac{\varepsilon}{\omega_{01}}}|\tilde{R}\rangle\right). \tag{53}$$

We can calculate that

$$\langle\tilde{L}|Q|\tilde{L}\rangle - \langle\tilde{L}|Q|\tilde{L}\rangle = \frac{1-\lambda^2}{1-2\beta\lambda+\lambda^2}(\delta Q) \approx \frac{2+\beta^2}{2}(\delta Q), \tag{54}$$

$$\langle 0|Q|1\rangle \approx \frac{1}{2}\frac{\Delta}{\omega_{01}}\left(1+\frac{\beta^2}{2}\right)(\delta Q), \tag{55}$$

$$\langle 0|Q|0\rangle - \langle 1|Q|1\rangle = \frac{\varepsilon}{\omega_{01}}\left(1+\frac{\beta^2}{2}\right)(\delta Q). \tag{56}$$

Considering the overlap effect, the relaxation and dephasing time are given by

$$\frac{1}{T_1} = \frac{\Delta^2(\delta Q)^2}{\hbar}\left(1+\frac{\beta^2}{2}\right)\left(\frac{C_g}{C_J+C_g}\right)^2\frac{\mathrm{Re}\,Z(\omega_{01})}{\omega_{01}}\coth\left(\frac{\hbar\omega_{01}}{2k_BT}\right), \tag{57}$$

$$\frac{1}{T_\phi} = \frac{(\delta Q)^2}{\hbar}\left(\frac{\varepsilon}{\omega_{01}}\right)^2\left(1+\frac{\beta^2}{2}\right)\left(\frac{C_g}{C_J+C_g}\right)^2\frac{\mathrm{Re}\,Z(0)}{\hbar}(2k_BT). \tag{58}$$

5 Conclusions

Under the Born-Markov approximation, the energy relaxation time T_1 and the decoherence time T_2 of superconducting charge qubits of Fig. 1 is given using multilevel quantum theory. The multilevel quantum theory, compared to the spin-boson model, investigate not only decoherence being caused by the dissipative environment, but also decoherence being generated by the dissipative elements in the superconducting circuit and the coupling effect between the elements in the circuit. Thus, it is good for estimating quantitatively the energy relaxation time and decoherence time which is resulted from an inevitable interaction between the circuit system and the environment.

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