Electron spin relaxation in InSb at high magnetic fields

1980 J. Phys. C: Solid State Phys. 13 3933

(http://iopscience.iop.org/0022-3719/13/20/015)

J. Phys. C: Solid St. Phys., 13 (1980) 3933–50. Printed in Great Britain

Electron spin relaxation in InSb at high magnetic fields

P Bogusławski and W Zawadzki
Institute of Physics, Polish Academy of Sciences, 02-668, Warsaw, Poland

Received 15 February 1980

Abstract. Spin relaxation of conduction electrons in InSb at high magnetic fields is considered theoretically and compared with existing experiments. A three-level model of band structure is used, which takes into account the main peculiarities of InSb: narrow energy gap and strong spin–orbit interaction. Spin mixing in the electron wavefunctions allows for spin-flip transitions due to electric-type perturbations (Elliott mechanism). Times of spin relaxation between the two lowest spin sub-bands are calculated for scattering by ionised impurities, acoustic phonons (deformation potential and piezoacoustic interactions), optic phonons (polar and deformation potential interactions) as well as for scattering by localised magnetic moments (paramagnetic ions, hyperfine interaction with nuclei). It is shown that the spin relaxation in existing experimental conditions is mainly due to the scattering by ionised impurities and acoustic phonons (deformation potential interaction). The theory well describes some experiments performed with a spin-flip Raman laser, while in others the observed spin relaxation times are shorter than the calculated ones. Possible sources of these discrepancies are considered.

1. Introduction

Spin properties of conduction electrons in narrow-gap semiconductors have been known to be highly anomalous due to the main features of their band structure: narrow energy gap and strong spin–orbit interaction. The resulting electron g-factors are very large and depend on energy. These peculiarities became of practical importance in InSb when the spin-flip Raman laser was constructed and developed into a tunable source of infrared radiation.

It has been realised for some time that the above features of band structure can affect not only spin energies but also possible transitions between the spin states. Elliott (1954) was the first to realise that the spin–orbit interaction can allow for spin-flip transitions due to electric-type perturbations. In InSb-type semiconductors this mechanism can be effective in the conduction band only through the strong interband coupling due to the small energy gap. This was first demonstrated by Rashba and Sheka (1961) and Sheka (1964), who proposed a new type of optical spin-flip excitations which are due to the electric component of electromagnetic radiation, and by Yafet (1963) who considered spin relaxation due to phonons.

Spin–lattice relaxation has also been considered by Pavlov and Firsov (1965, 1967). Explicit forms of the electron wavefunctions in InSb-type materials (Bowers and Yafet 1959, Zawadzki 1973) and in HgTe-type materials (Kacman and Zawadzki 1971) have been employed more recently for theoretical investigation of spin-flip transitions in the magnetophonon effect (Zawadzki *et al* 1975), the Shubnikov–de Haas effect (Zawadzki *et al* 1977) and the resonant polaron effect (Kacman and Zawadzki 1976, Zawadzki 1978). Similar treatment was proposed to account for the first experiments with a spin-flip Raman laser in which reliable direct measurements of the spin relaxation time at high magnetic fields were carried out (Grisar *et al* 1976). More extensive measurements by Brueck and Mooradian (1976) and by Pascher *et al* (1976) have made it desirable to carry out calculations of the spin relaxation times in InSb in more detail, to try for a quantitative description of existing experiments. This is the purpose of the present work.

2. Three-level model of band structure. Spin-flip transitions

We consider a three-level model for the band structure of InSb at $k = 0$: the Γ_6 s-like conduction level separated by the energy gap E_g from the degenerate Γ_8 p-like valence level, which is in turn split off by the spin–orbit energy Δ from the Γ_7 p-like valence level. It is assumed that the conduction band is adequately described by this model, neglecting all distant bands. The three-level $\boldsymbol{kp}$ scheme including an external magnetic field can then be solved exactly, both for the energies and the wavefunctions (Bowers and Yafet 1959, Zawadzki 1973). We are interested in two spin sub-bands of the lowest Landau level, $n = 0$. For the magnetic field oriented in the z direction the energies of these two sub-bands are given by

$$E_{\pm}(k_z) = -E_g/2 + [(E_g/2)^2 + E_g D_{\pm}(k_z)]^{1/2} \tag{2.1}$$

$$D_{\pm}(k_z) = \hbar\omega_c/2 + E_z \pm (1/2)\, g_0^* \mu_B H \tag{2.2}$$

where m_0^* and g_0^* are the effective mass and the effective g-factor at the bottom of the conduction band. $\omega_c = eH/m_0^* c$ is the cyclotron frequency, and $E_z = \hbar^2 k_z^2/2m_0^*$ is the electron kinetic energy along the magnetic field. In the assymetric (Landau) gauge $A = [0, Hx, 0]$. The corresponding electron wavefunctions are

$$\begin{aligned}\psi_- = A\Bigg[&\left(ia_-\phi_0 S + \frac{b_- + \sqrt{2}c_-}{2}\left(\frac{\hbar\omega_c}{D_-}\right)^{1/2}\phi_1 R_- + c_-\frac{\hbar k_z}{(2m_0^* D_-)^{1/2}}\phi_0 Z\right)\downarrow\\ &+ b_-\left(\frac{\hbar k_z}{(2m_0^* D_-)^{1/2}}\phi_0 R_-\right)\uparrow\Bigg]\end{aligned} \tag{2.3}$$

$$\begin{aligned}\psi_+ = A\Bigg[&\left\{ia_+\phi_0 S - \frac{b_+ - \sqrt{2}c_+}{2}\left(\frac{\hbar\omega_c}{D_+}\right)^{1/2}\phi_1 R_- + c_+\frac{\hbar k_z}{(2m_0^* D_+)^{1/2}}\phi_0 Z\right\}\uparrow\\ &- b_+\left\{\frac{\hbar k_z}{(2m_0^* D_+)^{1/2}}\phi_0 R_+ - \left(\frac{\hbar\omega_c}{D_+}\right)^{1/2}\phi_1\frac{Z}{\sqrt{2}}\right\}\downarrow\Bigg]\end{aligned} \tag{2.4}$$

with

$$a_{\pm}^2 = \frac{E_g + E_{\pm}}{E_g + 2E_{\pm}}, \qquad b_{\pm}^2 = \frac{\beta^2}{3}\frac{E_{\pm}}{E_g + 2E_{\pm}}, \qquad c_{\pm}^2 = \frac{2\gamma^2}{3}\frac{E_{\pm}}{E_g + 2E_{\pm}} \tag{2.5}$$

and

$$\beta^2 = \frac{\Delta^2}{(\Delta + E_g)\,(\Delta + 3E_g/2)}, \qquad \gamma^2 = \frac{\Delta + 3E_g/2}{\Delta + E_g}\,. \tag{2.6}$$

Here $A = \exp[\mathrm{i}(k_y y + k_z z)]/L$, and $V = L^3$ is the crystal volume. S and X, Y, Z are periodic functions of Γ_1 and Γ_{15} symmetry, respectively, and $R_{\pm} = (X \pm \mathrm{i}Y)/\sqrt{2}$. $\phi_n(x - X)$ are harmonic oscillator functions centred at $X = -l_H^2 k_y$, $l_H = (\hbar c/eH)^{1/2}$ being the magnetic length. The '+' and '−' signs in equations (2.3)–(2.4) denote effective spin, while ↑ and ↓ denote the true spin-up and spin-down functions. Clearly, in the presence of spin–orbit interaction spin is not a good quantum number. In the following we call transitions between '+' and '−' states the spin-flip transitions.

Among perturbations causing spin-flip transitions in the above sense we consider the potentials of ionised impurities and phonons. These perturbations are spin independent, and the spin-flip transitions are allowed because of the spin mixing of the electron wavefunctions. We consider also the interaction with localised magnetic moments of nuclei and of paramagnetic ions. There exists also a precessional-type mechanism for spin relaxation due to spin splitting of the conduction band related to inversion asymmetry of III–V compounds (Dyakonov and Perel 1971). However, this mechanism is strongly suppressed by an external magnetic field (Ivchenko 1973) and we will neglect it in the following considerations.

When an amount of electrons is excited from the lower to the upper sub-band, its time behaviour is characterised by three relaxation times. The first is the time τ_{temp} of establishing the temperature T_{e} of excited electrons. The second is the energy relaxation time τ_{en}, describing the relaxation of T_{e} to the temperature of the crystal lattice T. The third is the spin relaxation time T_1, defined below. In the following we will assume that $\tau_{\mathrm{temp}}, \tau_{\mathrm{en}} < T_1$, i.e. that $T_{\mathrm{e}} = T$, and the electron distribution is given by the Fermi–Dirac distribution function $f_{\pm}$.

In the quantum limit, the spin relaxation time can be obtained in the form

$$1/T_1 = \int \mathrm{d}E\, \rho_-(E)\, f_-(E)\, W(E)/N_- \tag{2.7}$$

where ρ_- and N_- are the density of states and the electron concentration in the upper sub-band, respectively. $W(E)$ is the probability of the spin-flip transition per unit time. Notice that T_1 is also the electron lifetime in the upper sub-band.

The distribution function f_- depends on N_-. For this reason the relaxation time T_1 depends also on N_-. Consequently, one obtains non-exponential decay of the excited electron population. The exponential decay occurs in two cases. First, when the transition probability is energy independent, $W(E) = W = \mathrm{const}$, and the spin relaxation time is $T_1 = 1/W$. Second, for the non-degenerate electron statistics, the probability of occupation of a given state does not depend on electron concentration, therefore T_1 is independent of N_- for any $W(E)$. Since the magnetic field always tends to make conditions less degenerate (Blakemore 1962), one deals with the non-degenerate statistics even at relatively low temperatures.

3. Spin relaxation by ionised impurities

The scattering potential given by ionised impurities in the crystal is

$$U(\boldsymbol{r}) = \sum_I v(\boldsymbol{r} - \boldsymbol{R}_I) \tag{3.1}$$

where the summation runs over impurity atoms situated at $\boldsymbol{R}_I$. The potential of a single impurity is assumed to be the Coulomb potential in the medium with dielectric constant ϵ_0 screened by free carriers: $v(r) = -e^2 \exp(-q_s r)/\epsilon_0 r$. The value of the screening radius

q_s is discussed in Appendix 1. The matrix element of the potential (3.1) for spin-flip transitions calculated using electron wavefunctions (2.3–2.4) has the form

$$\langle \boldsymbol{k}', +|U(\boldsymbol{r})|\boldsymbol{k}, -\rangle = C\frac{1}{L^2}\int \mathrm{d}^3r \exp[-\mathrm{i}(\Delta k_y y + \Delta k_z z)]\, \phi_0(x - X)\, \phi_1(x - X')\, U(\boldsymbol{r}) \tag{3.2}$$

with

$$C = \frac{\hbar k_z}{(2m_0^* D_-)^{1/2}}\left(\frac{\hbar\omega_c}{D_+}\right)^{1/2}\left[\frac{b_+c_-}{\sqrt{2}} - \frac{b_-(b_+ - \sqrt{2}c_+)}{2}\right]. \tag{3.3}$$

It follows from equation (3.3) that spin-flip transitions from the bottom of the upper sub-band are forbidden, as the matrix element (3.2) is proportional to k_z. After expanding $U(\boldsymbol{r})$ into the Fourier series, one obtains the square of the modulus of the matrix element averaged over random impurity positions in the form

$$\langle|\langle \boldsymbol{k}', +|U|\boldsymbol{k}, -\rangle|^2\rangle_I = (1/V)\, N_I C^2 \sum_{q_x} |v(q_x, \Delta k_y, \Delta k_z)|^2\, |J_{0\,1}(X, q_x, X')|^2 \tag{3.4}$$

where N_I is the concentration of impurities, and $\Delta\boldsymbol{k} = \boldsymbol{k}' - \boldsymbol{k}$. We have used the standard notation

$$J_{mn}(X, q_x, X') = \int \mathrm{d}x \phi_m(x - X) \exp(\mathrm{i}q_x x)\, \phi_n(x - X'). \tag{3.5}$$

The required properties of J_{mn} are given in Appendix 2. Using equation (3.4), one obtains the following expression for the spin relaxation time

$$\frac{1}{T_{ii}} = \frac{2\pi}{\hbar}\frac{N_I C^2}{V}\sum_{k'_y k'_z}\sum_{q_x} |v(q_x, \Delta k_y, \Delta k_z)|^2 |J_{0\,1}|^2\, \delta(E_+ - E_-). \tag{3.6}$$

The summations in equation (3.6) are typical for our problem and will appear in the next sections. They can be transformed in the following way. The sum over k'_z runs over two possible values due to the energy conservation

$$\sum_{k_z} \delta(E_+ - E_-)\, f(k'_z) = \sum_{i=1}^{2} \frac{L}{2\pi} f(k_i) \frac{m_0^*}{\hbar^2 |k_i|} \tag{3.7}$$

where $k_{1,2} = \pm(k_z^2 + k_s^2)^{1/2}$, with $\hbar^2 k_s^2/2m_0^* = -g_0^* \mu_B H$, and f is a function of k_z. The sum over k'_y and q_x can be transformed into the integral

$$\sum_{q_x, k'_y} = \sum_{q_x, q_y} = \frac{L^2}{2\pi}\frac{1}{l_H^2}\int \mathrm{d}\zeta \tag{3.8}$$

where $\zeta = l_H^2 q_\perp^2/2$, with $q_\perp^2 = q_x^2 + q_y^2$.

Applying equations (3.7), (3.8) and (3.3) to the expression (3.6) we have

$$\frac{1}{T_{ii}} = \frac{\pi e^4 N_I}{2\hbar\epsilon_0^2}\frac{E_z}{(E_g + 2D_-)^2}\frac{\alpha}{(k_s^2 + k_z^2)^{1/2}}\sum_{i=1}^{2} I(\zeta_i) \tag{3.9}$$

where

$$I(x) = -1 - (1 + x)\exp(x)\,\mathrm{Ei}(-x). \tag{3.10}$$

Ei is the exponential integral function. We have also defined

$$\zeta_{1,2} = (l_H^2/2)\{(k_z \pm (k_z^2 + k_s^2)^{1/2}]^2 + q_s^2\} \tag{3.11}$$

and

$$\alpha = \frac{\Delta}{\Delta + E_g}\frac{\Delta + 2E_g}{\Delta + 3E_g/2}. \tag{3.12}$$

In the corresponding expression of Brueck and Mooradian (1976) the functions $I(\zeta_i)$ are replaced by $1/(1 + \zeta_i)$.

One can see from equation (3.12) that the spin relaxation due to impurity scattering vanishes with vanishing spin–orbit interaction, i.e. for $\Delta = 0$. The essential feature of this mechanism is its dependence on k_z^2. This is related to the slow spatial dependence of the Coulomb potential, which acts only on the slowly varying envelopes in the electron wavefunctions (2.3–2.4).

4. Spin–lattice relaxation. General

The electron–phonon interaction will be described within the method of localised potentials (Ziman 1963, Ginter and Mycielski 1971, Bir and Pikus 1960). In the presence of a phonon the jth nucleus in the cell $\boldsymbol{n}$ is displaced from its position $\boldsymbol{R}_n^j$ by ξ_n^j, and the change of the crystal potential is

$$\delta V(\boldsymbol{r}) = \sum_{j,n} W^j(\boldsymbol{r} - \boldsymbol{R}_n^j)\,\xi_n^j \tag{4.1}$$

where $W^j(\boldsymbol{r} - \boldsymbol{R}_n^j)$ is a vector field centred at $\boldsymbol{R}_n^j$. One can split W^j into a short-range and a long-range part (Ginter and Mycielski 1971). The short-range part describes the change of the crystal potential in the vicinity of a displaced nucleus and leads to the deformation potential interaction with acoustic or optic phonons. The long-range part describes a slowly varying macroscopic electric field induced by a phonon and leads to the piezoelectric coupling with acoustic phonons or to the Fröhlich polar coupling with longitudinal optical (LO) phonons.

The change of the crystal potential δV leads to the spin-dependent electron–phonon interaction

$$\mathscr{H}_{\text{el-ph}} = \delta V + (\hbar/4m_0^2c^2)\ (\boldsymbol{\sigma} \times \nabla\delta V).\boldsymbol{p}. \tag{4.2}$$

It can be shown that for the slowly varying potential of the macroscopic electric field the second term is much smaller than the first one. For the quickly varying deformation potential the spin–orbit contribution may be not negligible, but is usually omitted. In the following we will also omit the spin–orbit term of equation (4.2).

Free carriers present in a crystal screen the initial phonon potential. Screening of the macroscopic slowly varying electric field of a phonon will be treated in the Thomas–Fermi approximation (cf. Ziman 1972). Screening of the quickly varying deformation potential is more complicated but, as shown by Bogusławski and Mycielski (1977), it can also be treated in a similar way. One notices however that the matrix element of the deformation potential is proportional to the momentum transfer, so that the main contribution to the momentum relaxation is given by transitions with large momentum transfers. Since the screening of these transitions is not effective, we will neglect the screening of the deformation potential.

In the experimental conditions of spin-flip Raman scattering the energy of an acoustic phonon causing the splin-flip process is smaller from both the spin splitting energy $|g_0^*|\mu_B H$, and $k_B T$. This is no longer true in the case of optical phonons, which has two important consequences. First, as $\hbar\omega_{op} \gg k_B T$, even at $T = 30\,\mathrm{K}$ we have $N_{qv} < 1$. Therefore the only possible process of spin flip is a spontaneous emission, while the phonon absorption can be neglected. Secondly, it follows from the energy conservation that the emission process can take place only at sufficiently high magnetic fields, when the condition $\hbar\omega_{op} < |g_0^*|\mu_B H$ is fulfilled. The minimum value of the magnetic field satisfying this is about 83 kG.

5. Deformation potential interaction with acoustic phonons

We will consider first the spin relaxation time due to the deformation potential interaction with acoustic phonons. It was shown by Bir and Pikus (1960) that within the effective mass approximation the interaction potential can be transformed into the form

$$\delta V_{ac} = \sum_{mn} D_{mn}\varepsilon_{mn} \tag{5.1}$$

where m, n are cartesian coordinates. $\varepsilon_{mn} = \frac{1}{2}[\partial\xi_m/\partial x_n + \partial\xi_n/\partial x_m]$ is the deformation tensor in the continuum approximation, with

$$\xi(r) = \left(\frac{\hbar}{2V\rho\omega_v(q)}\right)^{1/2} \hat{e}^v[b_{qv}\exp(\mathrm{i}q.r)+\mathrm{HC}]. \tag{5.2}$$

ρ is the crystal density, b_{qv}, $\omega_v(q)$ and $\hat{e}^v$ are annihilation operator, frequency and polarisation versor of a phonon mode, respectively. q is the wavevector from the vth phonon branch. D_{mn} is the deformation potential

$$D_{mn} = -\frac{p_m p_n}{m_0} + V_{mn} \tag{5.3}$$

where V_{mn} represents the derivative of the crystal potential with respect to strain ε_{mn} (Bir and Pikus 1972), and p_m is the momentum operator.

To calculate spin-flip transition probabilities we employ a procedure used by Szymańska *et al* (1974); a similar approach was used by Grisar *et al* (1976) and Zawadzki *et al* (1977). Both longitudinal and transverse phonons interact with electrons and can cause the spin-flip processes. We denote the directional cosines of a phonon wavevector q as e. Then the polarisation versor of longitudinal branch is $\hat{e}^L = e = (e_1, e_2, e_3)$, and for the transverse branches we take $\hat{e}^{T1} = (e_1^2 + e_2^2)^{-1/2}(-e_2, e_1, 0)$ and $\hat{e}^{T2} = (e_1^2 + e_2^2)^{-1/2}(-e_1e_3, -e_2e_3, e_1^2 + e_2^2)$. Polarisations T1 and T2 correspond to the slow transverse and fast transverse modes, respectively. As follows from equations (5.1) and (5.2) the spin-flip matrix elements are

$$
\begin{aligned}
I^{\nu} = \langle \boldsymbol{k}', +|\sum_{mn} D_{mn}\varepsilon_{mn}\, e^{iqr}\, |\boldsymbol{k}, -\rangle \\
= \Big\{ & \hat{\varepsilon}_{xx}\left[A_1 J_{01}\frac{l-m}{2} + A_2 m J_{10} + A_3 J_{10}\frac{l+m}{2}\right] \\
& + \hat{\varepsilon}_{yy}\left[-A_1 J_{01}\frac{l-m}{2} + A_2 J_{10} m + A_3 J_{10}\frac{l+m}{2}\right] \\
& + \hat{\varepsilon}_{zz}[A_2 l + A_3 m]\, J_{10} \\
& + \hat{\varepsilon}_{xy} A_1 J_{10} 2\mathrm{i}n + (\hat{\varepsilon}_{xz} + \mathrm{i}\hat{\varepsilon}_{yz})\, A_4 J_{11}\sqrt{2}n \\
& + [\hat{\varepsilon}_{xy} J_{10} A_5 + (\varepsilon_{yz} - \mathrm{i}\hat{\varepsilon}_{xz})\, J_{00} A_6]\, s \Big\}\, \delta_{q_y,\Delta k_y}\delta_{q_z \Delta k_z}
\end{aligned}
\tag{5.4}
$$

with

$$
\begin{aligned}
A_1 &= -b_+ \frac{b_- + \sqrt{2}c_-}{2}\frac{\hbar k_z'(\hbar\omega_c)^{1/2}}{(2m_0^* D_+ D_-)^{1/2}} \\
A_2 &= \frac{b_+ c_-}{\sqrt{2}}\frac{\hbar k_z(\hbar\omega_c)^{1/2}}{(2m_0^* D_+ D_-)^{1/2}}, \\
A_3 &= b_- c_+ \frac{\hbar^2 k_z k_z'}{2m_0^*(D_+ D_-)^{1/2}}, \\
A_4 &= \frac{b_+}{\sqrt{2}}\frac{b_- + \sqrt{2}c_-}{2}\frac{\hbar\omega_c}{(D_+ D_-)^{1/2}}, \\
A_5 &= a_- b_+ \left(\frac{2\hbar\omega_c}{D_+}\right)^{1/2}, \\
A_6 &= a_+ b_- \frac{\hbar k_z}{(m_0^* D_-)^{1/2}} - a_- b_+ \frac{\hbar k_z'}{(m_0^* D_+)^{1/2}}.
\end{aligned}
\tag{5.5}
$$

Primes in equation (5.5) refer to the final electron state. We have defined $\hat{\varepsilon}_{mn} = \mathrm{i}\hat{e}_m q_n$, and only the phonon absorption term has been worked out.

The spin-flip transitions can occur because the electron wavefunction is not a pure spin function, as it contains the admixture of the valence states. They are a consequence of the spin–orbit coupling in the initial Hamiltonian, and not of the spin-dependent perturbation.

The deformation potential constants are defined as

$$
l = (X|D_{xx}|X), \qquad m = (X|D_{yy}|X), \qquad n = (X|D_{xy}|Y) \qquad s = (S|D_{xy}|Z) \tag{5.6}
$$

or their cyclic equivalents. $(\quad|\quad) = 1/\Omega \int \mathrm{d}\Omega$ denotes integration over a unit cell of volume Ω.

It follows from equations (5.4–5.5) that the probability of the spin-flip process does not vanish for $k_z = 0$. As in general there is $k_z \ll k_z'$, in the following we will put $A_2 = A_3 = 0$, which is a good approximation for low electron concentrations and high magnetic fields.

The transition probability is proportional to $|I^{\nu}|^2$. After averaging $|I^{\nu}|^2$ over the possible q directions it turns out that the final expression is separated into 'intraband' part (related to l, m, n constants) and 'interband' part (related to s interband constant). We consider the two parts separately, which makes the final expressions more lucid.

The intraband part is

$$|J^{\mathrm{L}}|^2 = (1 - e_3^2)\{\tfrac{1}{2}(1 - e_3^2)\, C_1 A_1^2 |J_{01}|^2 + 4e_3^2 C_2 A^2 |J_{11}|^2\}\, q^2 \tag{5.7}$$

$$|J^{\mathrm{T1}}|^2 = \{\tfrac{1}{2}(1 - e_3^2)\, C_1 A^2 |J_{10}|^2 + e_3^2 C_2 A^2 |J_{11}|^2\}\, q^2 \tag{5.8}$$

$$|J^{\mathrm{T2}}|^2 = \{\tfrac{1}{2}e_3^2(1 - e_3^2)\, C_1 A^2 |J_{01}|^2 + (1 - 2e_3^2)\, C_2 A^2 |J_{11}|^2\}\, q^2 \tag{5.9}$$

where

$$C_1 = \left(\frac{l - m}{2}\right)^2 + n^2, \qquad C_2 = n^2/2. \tag{5.10}$$

The transition probability is given by

$$P = \frac{2\pi}{\hbar}\frac{\hbar}{2V\rho v_\nu q}|J^\nu|^2\{N_{\boldsymbol{q}\nu}\delta(E_- - E_+ - \hbar\omega_{\boldsymbol{q}\nu}) + (N_{\boldsymbol{q}\nu} + 1)\,\delta(E_- - E_+ + \hbar\omega_{\boldsymbol{q}\nu})\}$$
$$\times\, \delta_{q_z, \Delta k_z}\delta_{q_y, \Delta k_y} \tag{5.11}$$

where v_ν is the sound velocity of the νth mode.

We can assume that

$$N_{\boldsymbol{q}\nu} = 1/[\exp(\hbar\omega_\nu(\boldsymbol{q})/k_{\mathrm{B}}T) - 1] \approx k_{\mathrm{B}}T/\hbar\omega_\nu(\boldsymbol{q}) \gg 1$$

hence $N_{\boldsymbol{q}\nu} + 1 \approx N_{\boldsymbol{q}\nu}$. We also treat the transitions as quasi-elastic, i.e. neglect the phonon energies in the delta functions. The spin relaxation time is obtained by summing the transition probability over all possible final states, which are assumed to be unoccupied. After some algebra one finds

$$\frac{1}{T_{\mathrm{ac}}^{\mathrm{intra}}} = \frac{k_{\mathrm{B}}T}{16\pi\rho}\frac{m_0^*}{(k_s^2 + k_z^2)^{1/2}}\left(\frac{\omega_{\mathrm{c}}}{\hbar l_{\mathrm{H}} E_{\mathrm{g}}}\right)^2 \sum_\nu \sum_{i=1}^{2} \frac{1}{v_\nu^2} M^\nu(z_i), \tag{5.12}$$

where $z_{1,2} = (l_H^2/2)\,[k_z \pm (k_z^2 + k_s^2)^{1/2}]^2$, and

$$M^{\mathrm{L}}(z) = [C_1 - 8C_2 - z(2C_1 + 16C_2) - z^2(C_1 + 4C_2)]\, z$$
$$+ [-4C_2 - z20C_2 - z^2(20C_2 + 3C_1) - z^3(C_1 + 4C_2)]\, F(z) \tag{5.13a}$$

$$M^{\mathrm{T1}}(z) = [C_1 - C_2 - z(C_1 + C_2)]\, z$$
$$+ [-C_2 - 2zC_2 - (C_1 + C_2)\, z^2]\, F(z) \tag{5.13b}$$

$$M^{\mathrm{T2}}(z) = [C_2 + 8zC_2 + z^2(C_1 + 16C_2) + z^3(C_1 + 4C_2)$$
$$+ 4C_2 + 10zC_2 + z^2(2C_1 + 20C_2)$$
$$+ z^3(C_1 + 4C_2)]\, F(z). \tag{5.13c}$$

Here $F(z) = -z\exp(z)\,\mathrm{Ei}(-z)$.

The interband part is obtained similarly in the form

$$\frac{1}{T_{\mathrm{ac}}^{\mathrm{inter}}} = \frac{1}{3\pi\hbar}\frac{\beta^2 s^2}{l_H^2 \rho E_{\mathrm{g}}}\frac{k_{\mathrm{B}}T}{(k_s^2 + k_z^2)^{1/2}} \sum_\nu \sum_{i=1}^{2} \frac{1}{v_\nu^2} N(z_i) \tag{5.14}$$

with

$$N^{\mathrm{L}} = \tfrac{1}{8}(1 - 2z - 9z^2) - \tfrac{1}{8}z(11 + 9z)\,F(z), \tag{5.15a}$$

$$N^{\mathrm{T1}} = \tfrac{1}{2}(1 - z) - \tfrac{3}{2}zF(z), \tag{5.15b}$$

$$N^{\mathrm{T2}} = \tfrac{3}{2}z(1 + 3z) + (5 + \tfrac{9}{2}z)\,zF(z). \tag{5.15c}$$

β is defined in equation (2.6). Notice that intraband and interband parts have somewhat different dependences on magnetic field but, as remarked above, the total relaxation is the sum of both. The dependence of $1/T_{\mathrm{ac}}^{\mathrm{intra}}$ on the magnetic field is given mainly by the factor $\omega_c^2/l_H^2(k_s^2 + k_z^2)^{1/2} \approx \omega_c^2/l_H^2 k_s$, which is proportional to $H^{5/2}$ (Yafet 1963). In this way T_{ac} is inversely proportional to the magnetic field strength.

It should be noticed that, contrary to spin relaxation due to ionised impurities, the spin-flip probability due to deformation potential interaction with acoustic phonons does not have the k_z^2 dependence. This is related to the fact that this interaction has also a quickly varying component and the resulting matrix elements have different structure than that for slowly varying scattering potentials.

It is well known that the perturbation (5.1) describes not only the electron–phonon interaction, but also the energy shifts at the centre of the Brillouin zone caused by uniaxial or hydrostatic stress of the crystal. In particular, the splitting of the Γ_8 valence bands under uniaxial stress is given by two terms: $b = (l - m)/3$ and $d = 2n/\sqrt{3}$, where l, m, n are defined in equation (5.6) (Bir and Pikus 1972). The values of b and d have been measured by several authors (Seiler 1974). The constants C_1 and C_2 are combinations of b and d, and thus they can be determined from deformation experiments. In numerical calculations we take $b = -1{\cdot}8$ eV, $d = -5{\cdot}4$ eV.

The interband part is described in terms of the one deformation potential constant s. In InSb this constant is non-zero due to lack of inversion symmetry. In calculations we take $s = 1$ eV, which is probably an overestimation.

6. Piezo-acoustic interaction

The piezo-acoustic (PA) interaction results from the long-range electron-displaced nucleus interaction. Within the continuum approximation for ξ_n^j given by equation (5.2), the coupling potential (4.1) can be transformed into the commonly used form (Zawadzki and Szymańska 1971)

$$\delta V_{\mathrm{PA}} = -\frac{8\pi eP}{\epsilon_0}\left(\frac{\hbar}{2V\rho\omega_\nu(q)}\right)^{1/2} K_\nu[b_{q\nu}\exp(\mathrm{i}qr) - b_{q\nu}^{+}\exp(-\mathrm{i}qr)] \tag{6.1}$$

for a single phonon mode $q\nu$. Here $P = e_{123}$ is the piezoelectric constant of the crystal, and

$$K_\nu = \hat{e}_1^\nu e_2 e_3 + \hat{e}_2^\nu e_1 e_3 + \hat{e}_3^\nu e_1 e_2$$

describes the angular dependence of the interaction. Free carriers screen the macroscopic potential (6.1), which is taken into account by dividing δV_{PA} by the free carrier dielectric function. The matrix element for the spin-flip transition is

$$\langle \boldsymbol{k}', + \left| \frac{1}{\epsilon(q)} \delta V_{\mathrm{PA}} \right| \boldsymbol{k}, - \rangle = \frac{8\pi Pe}{\epsilon(q)\epsilon_0} C K_{\mathrm{v}} \times \left(\frac{\hbar N_{q\mathrm{v}}}{2V\rho\omega_{\mathrm{v}}(q)} \right)^{1/2} J_{10}(X, q_x, X') \, \delta_{q_y, \Delta k_y} \delta_{q_z, \Delta k_z} + \mathrm{em} \tag{6.2}$$

where em stands for the emission term ($\boldsymbol{q} \to -\boldsymbol{q}$, $N_{q\mathrm{v}} \to N_{q\mathrm{v}} + 1$), and C is given by equation (2.3).

To simplify the evaluation of the spin relaxation time we again assume that the transitions are quasi-elastic and $N_{q\mathrm{v}} \gg 1$. We also replace the strongly anisotropic quantity $K_{\mathrm{v}}^2/v_{\mathrm{v}}^2$ by its averaged value $(K_{\mathrm{v}}^2/v_{\mathrm{v}}^2)_{\mathrm{av}}$, and obtain

$$\frac{1}{T_{\mathrm{PA}}} = 36\pi \left(\frac{Pe\alpha}{E_{\mathrm{g}}\epsilon_0} \right)^2 \frac{k_{\mathrm{B}} T \omega_{\mathrm{c}} k_z^2}{(k_s^2 + k_z^2)^{1/2}} \left(\frac{K^2}{v^2} \right)_{\mathrm{av}} \sum_{i=1}^{2} \mathscr{P}(\zeta_i), \tag{6.3}$$

where $(K^2/v^2)_{\mathrm{av}} = \Sigma_{\mathrm{v}} (K_{\mathrm{v}}^2/v_{\mathrm{v}}^2)_{\mathrm{av}}$, and

$$\mathscr{P}(\zeta) = 1 + s + (\zeta + s + \zeta s) \exp(\zeta) \operatorname{Ei}(-\zeta). \tag{6.4}$$

ζ_i and α are defined by equations (3.11) and (3.12), and $s = l_H^2 q_s^2/2$.

7. Deformation potential interaction with optic phonons

The non-polar optical (NPO) scattering (Bir and Pikus 1960, 1972) in small-gap semiconductors in the absence of a magnetic field has been investigated by Bogusławski (1975). The coupling potential is given by equation (4.1), but we have to consider only the short-range part W_{SR}^j of the W^j. The polarisation versors can be chosen as in the case of acoustic phonons. The coupling potential can be written in the form

$$\delta V_{\mathrm{NPO}}(\boldsymbol{r}) = \xi^0(\boldsymbol{r}) \, W^0(\boldsymbol{r}) \tag{7.1}$$

where

$$\xi^0(\boldsymbol{r}) = \left(\frac{\hbar}{2V\rho_{\mathrm{r}}\omega_{\mathrm{v}}} \right)^{1/2} \hat{e}^{\mathrm{v}} [b_{q\mathrm{v}} \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}) - \mathrm{HC}]. \tag{7.2}$$

The reduced density is $\rho_{\mathrm{r}} = M/\Omega$, where M is the reduced mass of the unit cell, and

$$W^0(\boldsymbol{r}) = \frac{1}{m_1 + m_2} \sum_{\boldsymbol{n}} [m_2 W_{\mathrm{SR}}^1(\boldsymbol{r} - \boldsymbol{R}_{\boldsymbol{n}}^1) - m_1 W_{\mathrm{SR}}^2(\boldsymbol{r} - \boldsymbol{R}_{\boldsymbol{n}}^2)] \tag{7.3}$$

is a vector field of Γ_{15} symmetry.

The transition probability is described by the matrix element of the potential (7.1), proportional to

$$I^{\mathrm{v}} = \langle \boldsymbol{k}', + | \hat{e}^{\mathrm{v}} W^0(\boldsymbol{r}) \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}) | \boldsymbol{k}, - \rangle = -\mathrm{i}[A_1 J_{10} \hat{e}_z^{\mathrm{v}} + 1/\sqrt{2} \, A_4 J_{11} (\hat{e}_x^{\mathrm{v}} + \mathrm{i}\hat{e}_y^{\mathrm{v}})] \, d_0 \delta_{q_y, \Delta k_y} \delta_{q_z, \Delta k_z}, \tag{7.4}$$

where A_1 and A_4 are defined in (5.5). The deformation-potential constant d_0 is

$$d_0 = a_0 (X | W_y^0 | Z) \tag{7.5}$$

where a_0 is the lattice constant. The value of d_0 in InSb is about 35 eV (Bogusławski 1975). In I^ν we have omitted the weak interband coupling. One can see that all phonon modes can cause the spin-flip process. The transition probability is proportional to

$$|I^{\mathrm{L}}|^2 = d_0^2[e_3^2 A_1^2 |J_{10}|^2 + \tfrac{1}{2}(1 - e_3^2)\, A_4^2 |J_{11}|^2] \tag{7.6}$$

$$|I^{\mathrm{T1}}|^2 = d_0^2 \tfrac{1}{2} A_4^2 |J_{11}|^2 \tag{7.7}$$

$$|I^{\mathrm{T2}}|^2 = d_0^2[(1 - e_3^2)\, A_1^2 |J_{10}|^2 + \tfrac{1}{2} e_3^2 A_4^2 |J_{11}|^2]. \tag{7.8}$$

In the evaluation of the relaxation time we neglect the phonon dispersion, and assume that the final electron states are unoccupied. After some algebra one finds

$$\frac{1}{T_{\mathrm{NPO}}} = \frac{1}{8\pi\rho_r} \frac{\Delta}{\Delta + 3E_{\mathrm{g}}/2} \left(\frac{m_0^* d_0 \omega_{\mathrm{c}}}{\hbar a_0 E_{\mathrm{g}}}\right)^2 \sum_\nu \sum_{i=1}^{2} [\omega_\nu^{-1}(k_s^2 + k_z^2 - k_\nu^2)^{-1/2}]\theta^\nu(z_i) \tag{7.9}$$

where

$$k_\nu^2 = 2m_0^*\omega_\nu/\hbar, \qquad z_{1,2} = l_H^2[k_z \pm (k_s^2 + k_z^2 - k^2)^{1/2}]^2/2$$

and

$$\theta^{\mathrm{L}} = zE_z'[1 + zF(z)] + \tfrac{1}{4}\hbar\omega_{\mathrm{c}}[(1 + z)^2\,(1 + zF(z)) - z] \tag{7.10a}$$

$$\theta^{\mathrm{T1}} = \tfrac{1}{4}\hbar\omega_{\mathrm{c}} \tag{7.10b}$$

$$\theta^{\mathrm{T2}} = E_z'[1 - z - z^2F(z)] + \tfrac{1}{4}\hbar\omega_{\mathrm{c}} z[-z + z^2 - (1 - z^2)\,F(z)]. \tag{7.10c}$$

8. Polar interaction with optic phonons

We now turn to the long-range part of the $\boldsymbol{W}^j$ fields, describing the macroscopic electric field induced by an optical phonon. Only the longitudinal phonons produce a macroscopic field, and its potential (4.1) is given by

$$\delta V_{\mathrm{PO}} = \mathrm{i}\frac{4\pi e^* e}{\Omega}\left(\frac{\hbar}{2V\rho_{\mathrm{r}}\omega_{\mathrm{LO}}}\right)^{1/2} \frac{1}{q}\,[b_q^+ \exp(\mathrm{i}qr) + b_q \exp(-\mathrm{i}qr)] \tag{8.1}$$

where e^* is the Callen effective charge. We take the screening into account by dividing δV_{PO} by the free carrier dielectric function. The dispersion in the $\omega(\boldsymbol{q})$ dependence will be neglected. We assume that the potential (8.1) is slowly varying in space (small q values) so that it acts only on the slowly varying envelope components of the electron wavefunctions. The spin-flip matrix element is obtained in the form (cf. Zawadzki *et al* 1975)

$$\left\langle \boldsymbol{k}', + \left| \frac{1}{\epsilon(q)} \delta V_{\mathrm{PO}} \right| \boldsymbol{k}, - \right\rangle = -\mathrm{i}CD \frac{1}{q\epsilon(q)} J_{01}(X, q_x, X')\, \delta_{q_y, \Delta k_y} \delta_{q_z, \Delta k_z} \tag{8.2}$$

where

$$D = \frac{4\pi e^* e}{\Omega} \left(\frac{\hbar}{2V\rho_{\mathrm{r}}\omega_{\mathrm{LO}}}\right)^{1/2}. \tag{8.3}$$

C is given by equation (3.3), and only the spontaneous emission is taken into account. The spin relaxation time is given by

$$\frac{1}{T_{\mathrm{PO}}} = \frac{\pi}{4}\left(\frac{e^* e\alpha}{\Omega E_{\mathrm{g}}}\right)^2 \frac{\hbar\omega_{\mathrm{c}}}{\rho_{\mathrm{r}}\omega_{\mathrm{LO}}} \frac{k_z^2}{(k_s^2 + k_z^2 - k_{\mathrm{LO}}^2)^{1/2}} \sum_{i=1}^{2} \mathscr{P}(z_i). \tag{8.4}$$

The function $\mathscr{P}$ is defined in equation (6.4), but the reduced momentum transfer is the same as in equation (7.9) with $k_\nu = k_{\mathrm{LO}}$, which again indicates explictly that the spin relaxation due to optical phonons is possible only for spin splittings larger than the phonon energy. The spin relaxation time (8.4) has the characteristic k_z^2 dependence due to assumed slow spatial variation of the Fröhlich Hamiltonian (8.1).

Pavlov and Firsov (1965, 1967) have also considered spin-flip transitions due to deformation potential and polar interactions with optical phonons. Both their procedure and final results differ from ours.

9. Interaction with localised magnetic moments

One of the possible processes of spin relaxation is due to the presence of localised magnetic moments in the crystal. We will consider two types of magnetic moments: the nuclei of the host crystal and the magnetic moments of electronic clouds of the impurity atoms. Although the origin of interaction is different in both cases, their formal treatment is very similar.

We assume the contact interaction of the electron spin $\boldsymbol{\sigma}/2$ with the localised spin $\boldsymbol{S}$ situated at $\boldsymbol{r}_s$

$$\mathscr{H} = J_0 \delta(r - r_s) \sigma . S. \tag{9.1}$$

The main contribution to the spin-flip matrix element is given by the s-like component of the wavefunctions (2.3)–(2.4). We shall neglect the p-like component which introduces the error of the order of one per cent to the final results. The eigenfunctions of S are denoted by $|j, m\rangle$, j being the total momentum and m its z component. Decomposing

$$\boldsymbol{\sigma S} = \sigma_+ S_- + \sigma_- S_+ + \sigma_z S_z$$

one finds the spin-flip matrix element in the form

$$\langle j, m-1; \boldsymbol{k}', +|\mathscr{H}|j, m; \boldsymbol{k}, -\rangle = L^{-2}\langle j, m-1|S_-|j, m\rangle \langle +|\sigma_+|-\rangle \times \exp[\mathrm{i}(y_s \Delta k_y + z_s \Delta k_z)]\, J_0 \phi_0(x_s - X)\, \phi_0(x_s - X'). \tag{9.2}$$

The electron spin relaxation time is obtained by summing the contribution of all magnetic moments. This gives

$$\frac{1}{T_s} = \frac{2\pi}{\hbar}\langle \sum_s \sum_m \sum_{k'_z} p_m |\langle j, m-1; \boldsymbol{k}', +|\mathscr{H}|j, m; \boldsymbol{k}, -\rangle|^2\, \delta(E_- - E_+)\rangle_s, \tag{9.3}$$

where p_m is the probability of occupation of the state $|j, m\rangle$ and $\langle\ \ \rangle_s$ denotes the average over possible r_s. We have assumed elastic transitions. Introducing equation (9.2) into the expression (9.3) one obtains

$$\frac{1}{T_s} = \frac{2}{\pi} N_s J_0^2 \langle S_+ S_- \rangle \frac{m_0^*}{\hbar^3 l_H^2 (k_s^2 + k_z^2)^{1/2}} \tag{9.4}$$

where N_s is the concentration of the magnetic moments, and the angular bracket implies the thermal average†.

---

† A similar expression was obtained by Kossut and Walukiewicz (1976). Their formula omits the factor $\pi/4$.

9.1. *Hyperfine interaction*

The hyperfine interaction with crystal nuclei was first studied by Overhauser (1953) in metals, and recently by Chazalviel (1975) and Fishman and Lampel (1977) in III–V semiconductors in the absence of magnetic field. The interaction is given by equation (9.1), where $S = I$ is the nuclear spin, and

$$J_\nu = (8\pi/3)/\mu_B\mu_N g_N^\nu. \tag{9.5}$$

Here μ_N is the nuclear magneton, and g_N^ν is the Landé factor of the nucleus of the νth kind. In this case, we have to sum the contributions of all nuclei of kind ν and concentration N_ν. This leads to

$$\frac{1}{T_{\mathrm{hf}}} = \frac{1}{6\pi}\frac{1}{a_0^3\hbar^3}\frac{m_0^*}{l_H^2(k_s^2 + k_z^2)^{1/2}} \sum_\nu J_\nu^2 S_\nu^4 N_\nu I_\nu(I_\nu + 1) \tag{9.6}$$

where S_ν is the value of the Γ_6 s-like function at the nucleus of νth kind. As the nuclear magnetic moments are very small, there is $g_N\mu_N H \ll k_B T$ for the temperatures of experiment. For this reason the nuclear polarisation vanishes, and the probability that a nucleus occupies a state $|I, m\rangle$ is $1/(2I + 1)$. In other words, the nuclear spins are uncorrelated, which was used in the derivation of equation (9.6).

9.2. *Paramagnetic centres*

As mentioned above, one can introduce the localised magnetic moments to the crystal by doping. For example, a Mn ion with the d-shell half filled acts as a paramagnetic centre. The exchange-coupling potential is of the form (9.1), where J_0 is the exchange integral, and the spin relaxation time is given by equation (9.4). At low temperatures the magnetic field can partially order the spin of paramagnetic centres, as $g_P\mu_B H \sim k_B T$ where g_P is the impurity g-factor. We describe this effect by using the thermal average $\langle S_+S_-\rangle$. In this way, the dependence of the relaxation time (9.4) on the magnetic field is more involved than that due to the hyperfine interaction. First, let us observe that the order of S_+ and S_- operators in equation (9.4) is important as they do not commute. This order is governed by the sign of the conduction electron g-factor: if $g_0^* < 0$ then we are interested in the transitions between '−' and '+' states due to the operator σ_+. This process is accompanied by a transition from $|j, m\rangle$ to $|j, m - 1\rangle$ given by S_-, and a quotient S_+S_- appears in equation (9.4). If $g_0^* > 0$, one would get the inverse order.

Secondly, the increase of a magnetic field can additionally increase or decrease the spin relaxation time; this influence depends on the relative signs of g-factors of paramagnetic centres and that of the conduction electrons. This can be easily understood if one considers an example of a Mn ion ($g_P = +2$) in InSb ($g_0^* = -50$). At $T \approx 0$ and for very large H the paramagnetic centres are completely polarised antiparallel to the field direction. In such a situation the electron spin-flip transitions '−' → '+' are forbidden, and the relaxation time (9.4) becomes infinite. In general, the electron spin relaxation time additionally increases when g_P and g_0^* have inverse signs. A similar expression for spin relaxation has been independently obtained by Walukiewicz (1980).

10. Comparison with experiment

The first direct determination of T_1 in InSb was performed by Grisar *et al* (1976) who measured the time-resolved conductivity following a stimulated SFR pulse. Next, T_1 was investigated by Brueck and Mooradian (1976) and by Pascher *et al* (1976). The experimental conditions and the determined values of T_1 are presented in table 1, together with the theoretical results for the spin relaxation times due to scattering by ionised impurities and acoustic phonons (deformation potential interaction).

Table 1. Experimentally determined T_1^{exp} and calculated spin relaxation times. (a) Pascher *et al* (1976), (b) Brueck and Mooradian (1976), (c) Grisar *et al* (1976).

$N(\text{cm}^{-3})$	$H(\text{G})$	$T(\text{K})$	$T_1^{\text{exp}}(\text{ns})$	$T_{ii}(\text{ns})$	$T_{ii}^{h}(\text{ns})$	$T_{\text{ac}}^{\text{inter}}(\text{ns})$	$T_{\text{ac}}^{\text{intra}}(\text{ns})$
$1{\cdot}35 \times 10^{15}$	$1{\cdot}0 \times 10^4$	1·8	$110 \pm 20^{(a)}$	$1200\ (T_e = 5\ \text{K})$ $600\ (T_e = 10\ \text{K})$	165	20000	8000
$3{\cdot}00 \times 10^{15}$	$2{\cdot}0 \times 10^4$	1·8	$100 \pm 20^{(a)}$		24		
$3{\cdot}00 \times 10^{15}$	$1{\cdot}3 \times 10^4$	1·8	$185 \pm 20^{(a)}$		44		
$1{\cdot}20 \times 10^{16}$	$6{\cdot}0 \times 10^4$	20·0	$60 \pm 20^{(b)}$	120		85	112
$1{\cdot}60 \times 10^{16}$	$5{\cdot}0 \times 10^4$	24·0	$250 \pm 50^{(c)}$	55		102	100
$1{\cdot}60 \times 10^{16}$	$13{\cdot}5 \times 10^4$	24·0	$150^{(c)}$	220		16·5	17

In the numerical calculations we have used the following values of material parameters for InSb: $m_0^* = 0{\cdot}014\, m_0, g_0^* = -50, E_g = 0{\cdot}23\ \text{eV}, \Delta = 0{\cdot}9\ \text{eV}, \epsilon_0 = 17{\cdot}8, \rho = 5{\cdot}8\ \text{g cm}^{-3}$, $v_L = 3{\cdot}8 \times 10^5 \text{m s}^{-1}$, $v_{T_1} = 1{\cdot}63 \times 10^5 \text{cm s}^{-1}$, $v_{T_2} = 2{\cdot}3 \times 10^5 \text{cm s}^{-1}$, $\omega_{LO} = 3{\cdot}5 \times 10^{13}\ \text{s}^{-1}, a_0 = 6{\cdot}5\ \text{Å}, e_{123}^2 = 5{\cdot}62 \times 10^8\ \text{dyn cm}^{-2}, e_c^* = 0{\cdot}13\, e$.

For the spin relaxation due to acoustic phonons, both interband and intraband spin relaxation rates are overestimated because of the assumed elasticity of electronic transitions. Among various phonon branches the $T1$ mode is most effective, and the L mode is least effective. In the experimental conditions in consideration, the piezo-acoustic scattering is weaker than the deformation potential scattering by at least one order of magnitude.

In the ionised impurity scattering the free carrier screening is of essential importance (cf. Appendix 1). In the numerical calculations the general formula (A1.2) has been used.

The hyperfine interaction leads to the relaxation time 2000 ns at 10^4 G, and 600 ns at 10^5 G. The interaction with paramagnetic centres may be important only at very high concentrations of these centres. Such a situation occurs for example in mixed semiconducting compounds of the type of $\text{Cd}_x\text{Mn}_{1-x}\text{Te}$.

As can be seen, for magnetic fields not high enough to allow for relaxation due to optic phonons the most effective spin-flip modes are ionised impurities and acoustic phonons. It is then worthwhile to compare these mechanisms. First, as a result of the structure of the electron wavefunctions (2.3, 2.4) $T_{\text{imp}}^{-1} \propto E_z$ whereas T_{ac}^{-1} does not vanish at the bottom of the spin sub-band. Second, the matrix element for the ionised impurity scattering is inversely proportional to the momentum transfer $\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$, and for acoustic phonon scattering the matrix element is proportional to the momentum transfer. The minimal value of the momentum transfer, given by $k_s = (2m_0^* g_0^* \mu_B H)^{1/2}/\hbar$, is increasing with magnetic field, therefore the role of the acoustic phonon scattering also increases.

Describing more specifically for different data we notice that in the experiment of Pascher *et al* (1976) performed at low temperatures only ionised impurities are of importance. The calculated relaxation times are longer by one order of magnitude than those determined in the experiment. We propose two possible explanations of this discrepancy. The first one is the heating of electrons by the laser radiation. One can expect that this effect takes place, as the energy relaxation rate is comparable to the spin relaxation rate (Brueck and Mooradian 1976). In this case the shortening of $T_{\mathrm{imp}}(\propto 1/E_z)$ is due to the increase of the temperature of electron gas, i.e. to the increase of the mean value of $E_z\ (=k_{\mathrm{B}}T/2)$. The experimental data cannot be accounted for even if one assumes $T_{\mathrm{e}} = 10\ \mathrm{K}$ (the temperature of the sample is 1·8 K).

Another possible explanation takes into account the influence of ionised impurities on electronic states close to the bottom of the Landau levels. Approximate treatment of this problem is presented in Appendix 3. The calculated values of T_1 are of the order of experimental ones; this approach however is very sensitive to the value of the screening radius q_{s}, which is difficult to evaluate correctly.

The experiment of Brueck and Mooradian (1976) is described quite well by our theory. The contributions of the ionised impurity mode and of the phonon mode to the spin relaxation are comparable.

The experiment of Grisar *et al* (1976) was performed at highest magnetic fields and temperatures, where the phonon scattering may dominate. The agreement with experiment is poor. However, the observed T_1 decreases with increasing magnetic field, this feature being characteristic of acoustic phonon scattering.

Appendix 1

Free electron screening in high magnetic fields has been investigated by Wallace (1974) within the random phase approximation. Both inter- and intra-Landau level transitions have been taken into account. In the experimental conditions of interest the inter-level contribution is negligible. The intra-level contribution has a complicated form, which can be well approximated by the expression given by Krupski (1975)

$$q_{\mathrm{s}}^2 = \frac{e^4 m_0^*}{\pi^3 \hbar^4 c^2 \epsilon_0} \frac{H^2}{N}. \tag{A1.1}$$

Equation (A1.1) holds for $T = 0\ \mathrm{K}$ and at the ultra-quantum limit. The non-sphericity of the screening induced by the magnetic field can be neglected. At finite temperatures we have

$$q_{\mathrm{s}}^2 = \frac{e^2 H}{\hbar^2 c \epsilon_0} \left(\frac{2m_0^*}{\pi k_{\mathrm{B}} T}\right)^{1/2} \mathscr{F}_{3/2}(\mu), \tag{A1.2}$$

where $\mathscr{F}_{-3/2}$ is the Fermi integral, and μ the reduced Fermi level.

When considering the SFR scattering we have to take into account the electron spin, neglected until now. If the electron concentration in the lower (upper) spin sub-band is $N_+(N_-)$, $1/N$ in equation (A1.1) should be replaced by

$$1/N \to 1/N_+ + 1/N_-. \tag{A1.3}$$

It follows from equation (A1.1) and (A1.3) that when the electron population in the upper sub-band approaches zero due to spin-flip transitions, the screening becomes infinite. This non-physical result comes from the fact that, in the derivation of equation (A1.1), the infinite density of states has been assumed at the bottom of the Landau sub-band. In the real crystal this divergence is removed by damping due to impurities or other scattering processes.

Appendix 2

For n and m of interest, the squares of J_{nm} functions defined in equation (3.5) are

$$|J_{00}|^2 = \mathrm{e}^{-\zeta}, \tag{A2.1}$$

$$|J_{10}|^2 = |J_{01}|^2 = \zeta\,\mathrm{e}^{-\zeta} \tag{A2.2}$$

$$|J_{11}|^2 = (1-\zeta)^2\,\mathrm{e}^{-\zeta} \tag{A2.3}$$

where $\zeta = l_H^2(q_x^2 + q_y^2)/2$.

Appendix 3

As we have pointed out in § 10, our approach does not describe the data of Pascher *et al* (1976). It seems however that at very low temperatures and low electron concentrations the ionised impurities provide the main channel of spin relaxation. On the other hand, it is well known (Kubo *et al* 1965) that the presence of ionised impurities strongly affects the electronic states close to the bottom of the Landau levels. In particular, the density of states becomes broadened and the electron kinetic energy is finite even for the lowest-lying states.

Let us try to estimate the modification of the spin-flip scattering caused by the spin-conserving transitions. For this purpose we replace the unperturbed density of states by the perturbed one. We would like to replace the kinetic energy $E_z = \hbar^2k_z^2/2m_0^*$ appearing in the simple theory presented in § 3 by the perturbed kinetic energy K along the magnetic field. The first assumption seems quite natural, the second is more doubtful.

Both the kinetic energy $K(\epsilon)$ and the density of states $D(\epsilon)$ can be obtained by using the method of the resolvent operator. The final results obtained by Davies (1973) and Yuen *et al* (1974) for the upper sub-band are of the form

$$D(\epsilon) = \frac{2}{\pi N_I J(\kappa)}\left(\frac{\epsilon_0 q_s}{4\pi e^2 l_H}\right)^2 \Gamma(\epsilon), \tag{A3.1}$$

$$K(\epsilon) = \frac{1}{D(\epsilon)}\frac{1}{(2\pi l_H)^2}\frac{(2m_0^*)^{1/2}}{\hbar}\cos\left[\tfrac{1}{2}\tan^{-1}\frac{\Gamma(\epsilon)}{\epsilon-\Delta(\epsilon)}\right] \tag{A3.2}$$

where ϵ is the energy counted from the bottom of the unperturbed sub-band. The functions Γ and Δ are the line broadening and the level shift, respectively. They are given by

$$\Gamma(\epsilon) = (\surd 3/2)(Z_+ - Z_-), \tag{A3.3}$$

$$\Delta(\epsilon) = \tfrac{1}{3}\epsilon - \tfrac{1}{2}(Z_+ + Z_-), \tag{A3.4}$$

where

$$Z_\pm = \{(\epsilon/3)^3 + \xi^2/2 + \xi[(\epsilon/3)^3 + \xi^2/4]^{1/2}\}^{1/3} \tag{A3.5}$$

Here ξ is the electron impurities effective coupling constant,

$$\xi = N_I\left(\frac{4\pi e^2}{\epsilon_0}\right)^2\frac{(2m_0^*)^{1/2}}{\hbar}\frac{1}{8\pi q_s^2}J(\kappa). \tag{A3.6}$$

Taking into account only the forward scattering, the above authors obtained

$$J(\kappa) = 1 + \kappa \exp(\kappa)\,\mathrm{Ei}(-\kappa), \tag{A3.7}$$

with $\kappa = l_H^2 q_s^2/2$. For states close to the bottom of the sub-band one has to consider also the backward scattering contribution which increases the value of $J(\kappa)$ by a factor of two approximately.

Note that the above approach is self-consistent in the following sense. The density of states (A3.1) is determined by the screening radius q_s, which in turn is given by the density of states. Since the solution is difficult to obtain, one can alternatively treat q_s as a free parameter (Heuser and Hajdu 1976).

The numerical results are obtained by using the screening radius (A1.1). One can see from table 1 that the calculated spin relaxation times T_{ii}^h are now shorter than the observed ones. The heuristic version, with E_z replaced by K, leads to the shortening of T_{ii}, since the spin-flip transitions are now allowed for all electron energies, and $K(\epsilon) > \epsilon(= E_z)$.

References

Bir G L and Pikus G E 1960 *Fiz. Tverd. Tela* **2** 2287
—— 1972 *Symmetry and Deformation Effects in Semiconductors* (Moscow: Nauka)
Blakemore J S 1962 *Semiconductor Statistics* (Oxford: Pergamon)
Bogusławski P 1975 *Phys. Stat. Solidi* (b) **70** 53
Bogusławski P and Mycielski J 1977 *J. Phys. C: Solid St. Phys.* **10** 2413
Bowers R and Yafet Y 1959 *Phys. Rev.* **115** 1165
Brueck S R J and Mooradian A 1976 *Optics Commun.* **18** 539
Chazalviel J-N 1975 *Phys. Rev.* **B11** 1555
Davies R W 1973 *Phys. Rev.* **B7** 3731
Dyakonov M I and Perel V I 1971 *Fiz. Tverd. Tela* **13** 3581
Elliott R J 1954 *Phys. Rev.* **96** 266
Fishman G and Lampel G 1977 *Phys. Rev.* B **16** 820
Ginter J and Mycielski J 1971 *J. Phys. C: Solid St. Phys.* **4** 1540
Grisar R, Wachering H, Bauer G and Zawadzki W 1976 *Infrared Physics* **16** 149
Heuser H and Hajdu J 1976 *Solid St. Commun.* **20** 313
Ivchenko E L 1973 *Fiz. Tverd. Tela* **15** 1566
Kacman P and Zawadzki W 1971 *Phys. Stat. Solidi* (b) **47** 629
—— 1976 *Solid St. Commun.* **18** 945
Kossut J and Walukiewicz W 1976 *Solid St. Commun.* **18** 343
Krupski J 1975 *J. Phys. C: Solid St. Phys.* **8** 4171
Kubo R, Miyake S J and Hashitsume N 1965 *Solid St. Phys.* **17** (New York: Academic Press)
Overhauser A W 1953 *Phys. Rev.* **89** 689
Pascher H, Appold G, Ebert R and Häfele H G 1976 *Opt. Commun.* **19** 104
Pavlov S T and Firsov Yu A 1965 *Fiz. Tverd. Tela* **7** 2634
—— 1967 *Fiz. Tverd. Tela* **9** 1780
Rashba E I and Sheka V I 1961 *Fiz. Tverd. Tela* **3** 1735, 1863
Seiler D G 1974 *Lectures of International Conference, Wurzburg* p 492
Sheka V I 1964 *Fiz. Tverd. Tela* **6** 3099
Szymańska W, Bogusławski P and Zawadzki W 1974 *Phys. Stat. Solidi* (b) **65** 641
Wallace P R 1974 *J. Phys. C: Solid St. Phys.* **7** 1136
Walukiewicz W 1980 *Phys. Rev.* B to be published
Yafet Y 1963 *Solid St. Phys.* **14** (New York: Academic Press)
Yuen S Y, Wolff P A and Lax B 1974 *Phys. Rev.* **B9** 3394
Zawadzki W 1973 *New Developments in Semiconductors* ed P R Wallace (Leyden: Noordhoof)
—— 1978 *Physics of Narrow-gap Semiconductors* (Warsaw: Polish Scien. Publ.) p 281

Zawadzki W, Bauer G, Racek W and Kahlert H 1975 *Phys. Rev. Lett.* **35** 1098
Zawadzki W, Mauger A, Otmezguine S and Vérié C 1977 *Phys. Rev.* B **15** 1035
Zawadzki W and Szymańska W 1971 *Phys. Stat. Solidi* (b) **45** 415
Ziman J 1963 *Electrons and Phonons* (Oxford: Clarendon)
—— 1972 *Principles of the Theory of Solids* (Cambridge: CUP)