Heavy quarkonia and collective dynamics of strongly interacting matter

D. Kharzeev
Outline

Lecture 1: Heavy quarkonia in QCD matter

Lecture 2: New developments in relativistic hydrodynamics of strongly interacting matter (the role of quantum anomalies)
Heavy quarkonia in QCD matter

- Why heavy quarks?
- Quarkonium: the “hydrogen atom of QCD”
- Quarkonium production in strong color fields (cold nuclear matter effects in pA, AA)
- Quarkonium in quark-gluon plasma
- Confinement, holography and entanglement
Why heavy quarks?

Heavy quark masses $M_H$ are generated at the electroweak scale, and are external parameters in QCD;

Heavy quarks are “heavy” because their masses are large on the typical QCD scale of $\Lambda_{\text{QCD}}$:

$$M_H \gg \Lambda_{\text{QCD}}$$

\[
\alpha_s(M_H) \ll 1
\]
\[
\frac{\langle \alpha_s G^2 \rangle}{M_H^4} \ll 1
\]
Why heavy quarks?

QCD matter is characterized by dimensionful parameters: saturation scale $Q_S$, density, transport coefficient, …

$$M_H \leftrightarrow Q_S, \frac{Q_S^2}{\Lambda_{QCD}}, \rho^{1/3}, T, \sqrt{\hat{q}L}, \ldots$$

depending on their values, “heavy” quarks can behave either as heavy or as light!

⇒ Use heavy quarks to extract information about the properties of QCD matter
November revolution:
Why is heavy quarkonium so narrow?
The annihilation width of heavy quarkonia is small due to the asymptotic freedom.

Heavy quark masses $M_H$ are generated at the electroweak scale, and are external parameters in QCD;

Heavy quarks are “heavy” because their masses are large on the typical QCD scale of $\Lambda_{QCD}$:

$$M_H \gg \Lambda_{QCD}$$

$$\alpha_s(M_H) \ll 1$$

$$\left\langle \alpha_s G^2 \right\rangle \ll 1$$

$$\frac{M_H^4}{\langle \alpha_s G^2 \rangle} \ll 1$$
Quarkonium and asymptotic freedom (Coulomb potential)

Spectral representation in the t-channel:

\[ V(R) = \sum_{m} \sigma(m^2) \frac{\exp(-mR)}{R} \]

If physical particles can be produced (positive spectral density), then unitarity implies screening

\[
\left\{ \frac{d \alpha_s^{-1}(R)}{d \ln R} \right\}_{\text{phys}} \propto \frac{1}{3} N + \frac{2}{3} n_f
\]

Gluons Quarks
(transverse)
Coulomb potential in QCD - II

Missing non-Abelian effect: instantaneous Coulomb exchange dressed by (zero modes of) transverse gluons

\[ \sum_n \left[ 0 + \perp \rightarrow 0 \right]^n = \begin{array}{c}
\begin{array}{c}
0 \bullet 0 \\
\downarrow \\
0 \bullet 0 \bullet 0 \\
\downarrow \\
0 \bullet 0 \bullet 0 \bullet 0 \end{array}
\end{array} + \cdots \]

Negative sign
(the shift of the ground level due to perturbations - unstable vacuum!):

\[ \delta E \equiv E - E_0 = \sum_n \frac{|\langle 0 | \delta V | n \rangle|^2}{E_0 - E_n} < 0 \]

Anti-screening

\[ \left\{ \frac{d \alpha_s^{-1}(R)}{d \ln R} \right\}^{\text{stat}} \propto -4N \]
Asymptotic freedom and the instability of the perturbative vacuum

The effective potential: sum over 2D Landau levels

\[ V_{\text{pert}}(H) = \frac{gH}{4\pi^2} \int dp_z \sum_{n=0}^{\infty} \sum_{s_z=\pm 1} \sqrt{2gH(n + 1/2 - s_z) + p_z^2}. \]

Paramagnetic response of the vacuum:

\[ \text{Re} V_{\text{pert}}(H) = \frac{1}{2} H^2 + (gH)^2 \frac{b}{32\pi^2} \left( \ln \frac{gH}{\mu^2} - \frac{1}{2} \right) \]

1. The lowest level \( n=0 \) of radius \( \sim (gH)^{-1/2} \) is unstable!

2. Strong fields \( \leftrightarrow \) Short distances

Instability of perturbative QCD vacuum; What is the true ground state?
Quarkonia: the potential problems...

\[ D_{\mu\nu}^{ab} = i \delta_{ab} \frac{1}{k^2} \]

\[ = i \delta_{ab} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{1}{k^2 + i\epsilon} \quad \mu = i, \nu = j. \]

Coulomb gauge

Screening Anti-Screening

THE STATIC POTENTIAL IN QUANTUM CHROMODYNAMICS

Thomas APPELQUIST\textsuperscript{1,2} and Michael DINE\textsuperscript{1}

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Quarkonia: the potential problems...

Fig. 4. A three loop graph with a singular static limit.

Higher Fock states; classified in NRQCD

- energy of transverse gluons - constant per unit of length (planar, large N limit);
- ... a string?

... can be seen as parts of the QCD string?

Bodwin, Braaten, Lepage; ... e.g., the “gluon chain” model

Greensite and Thorn
Consider a rectangular Wilson loop: 

\[ W(C) = \exp \left( ig \int_C A_\mu dx^\mu \right) \]

It is related to the potential \( V(R) \) acting between the charges \( Q \) and \( \bar{Q} \):

\[ W(C) \rightarrow \exp (-TV(R)) \]

Scale transformation: \( T \rightarrow \lambda T; \quad R \rightarrow \lambda R; \)

the only solution: Coulomb potential

\[ V(R) \sim \frac{1}{R} \]
Insights from holography

Of course, no confinement in N=4 SYM: scale invariance dictates that the expectation value of $W$ must scale with $T/L$ - thus Coulomb potential

$$V(L) \sim 1/L$$

Nevertheless, an interesting lesson: solving for the minimal surface in super-gravity and evaluating its area, one finds

$$V(L) = - \frac{4\pi^2 \sqrt{\lambda}}{\Gamma^4(1/4) L}.$$ 

Why square root? “not your grandfather’s Coulomb potential”, a weaker one! screening?
Insights from holography

Indeed, the square root of the coupling (but not the pre-factor) can be reproduced on the Field Theory side by resummation of planar diagrams:

$$\alpha(\lambda) = \begin{cases} \frac{\lambda}{4\pi} + \cdots, & \text{for } \lambda \to 0; \\ \frac{4\pi^2 \sqrt{2\lambda}}{\Gamma^4(1/4)} + O(1), & \text{for } \lambda \to \infty. \end{cases}$$

$$V(L) = -\frac{\alpha(L)}{L}$$

Erickson, Semenoff, Szabo, Zarembo, hep-th/9911088

Indeed, the square root of the coupling (but not the pre-factor) can be reproduced on the Field Theory side by resummation of planar diagrams:

$$\ln \left\{ 1 + \sum \text{ladders} + \cdots \right\} = \frac{\lambda T}{4\pi L} - \frac{\lambda^2}{(2\pi)^3} \frac{T}{L} \ln \frac{T}{L} + \cdots.$$
Quarkonium production in nuclear collisions

T. Ludlam, L. McLerran, Physics Today October 2003

Tuesday, May 15, 2012
Heavy quarks and the Color Glass Condensate

Lecture by L. McLerran

In CGC, heavy quarks can behave either as “light” or “heavy”

Naïve consideration:

CGC is characterized by the chromo-electric field

\[ E \sim \frac{Q_s^2}{g} \]

when the strength of the field is

\[ gE \sim \frac{M}{1/M} = M^2 \]

or

\[ Q_s^2 \geq M^2 \]

heavy quarks no longer decouple \( \Rightarrow \) they are not really “heavy”
J/Ψ production in strong color fields

At high parton density, when this diagram is the leading one

\[ \alpha_s^6 A^{2/3} \sim \alpha_s^2 \]

\[ \alpha_s^5 A^{1/3} \sim \alpha_s^3 \]

DK, K.Tuchin, hep-ph/0510358
$J/\Psi$ production in strong color fields

DK, K.Tuchin, hep-ph/0510358
J/Ψ production in strong color fields

At high enough energy (LHC?), high parton density is reached even in pp collisions - expect the change in the scaling of J/psi yield with associated multiplicity:

\[ \alpha_s^5 A^{1/3} \sim \alpha_s^3 \]

\[ \frac{dn_J}{dy} \sim Q_s^2 S \sim \frac{dn}{dy} \]

\[ \alpha_s^6 A^{2/3} \sim \alpha_s^2 \]

\[ \frac{dn_J}{dy} \sim Q_s^4 S \sim \left(\frac{dn}{dy}\right)^{4/3} \]
J/\Psi$ production as a function of associated multiplicity in pp collisions: the ALICE data

\[ \frac{dn_J}{dy} \sim Q_s^4 S \sim \left( \frac{dn}{dy} \right)^{4/3} \]

Indication for high parton density in high multiplicity pp collisions?

\[ Q_s^2 \quad m_c^2 \]

Lecture by P.Braun-Munzinger; ALICE Coll., arxiv:1202.5864;
J/Ψ production in strong color fields

Sample diagram contributing to J/Ψ production in pA collisions

\[
\frac{d\sigma_{pA\rightarrow J/\psi X}}{dyd^2b} = x_1 G(x_1, m_c^2) \int_0^1 dz \int \frac{d^2r}{4\pi} \Phi(r, z) \int_0^1 dz' \int \frac{d^2r'}{4\pi} \Phi^*(r', z') \times \frac{4r \cdot r'}{(r + r')^2} \left( e^{-\frac{Q^2_{s}}{16}(r-r')^2} - e^{-\frac{Q^2_{s}}{8}(r^2+r'^2)} \right).
\]

F. Dominguez,\textsuperscript{1} D.E. Kharzeev,\textsuperscript{2,3} E.M. Levin,\textsuperscript{4,5} A.H. Mueller,\textsuperscript{1} and K. Tuchin\textsuperscript{6}

J/Ψ production in strong color fields

Reasonable agreement with p(d)A data

but the AA data indicate additional suppression, stronger at forward rapidity!

F. Dominguez,1 D.E. Kharzeev,2,3 E.M. Levin,4,5 A.H. Mueller,1 and K. Tuchin6


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The transverse momentum spectra of $J/\Psi$

DK, E. Levin, K. Tuchin, 
arxiv: 1205.1554

$$\frac{d\sigma_{A_1 A_2 \rightarrow J/\psi X}}{d^2 p_\perp dy d^2 B_1 d^2 B_2} = \int_0^1 dz \int \frac{d^2 r}{4\pi} \Phi(r, z) \int_0^1 dz' \int \frac{d^2 r'}{4\pi} \Phi(r', z') \int d^2 k_\perp \int_0^1 d\xi'$$

$$\times \frac{C_F}{\alpha_s \pi^2} \frac{Q_{s1}^2 Q_{s2}^2}{Q_{s1}^2 + Q_{s2}^2} F_k \frac{r \cdot r'}{16\pi \xi'} e^{-\frac{p_\perp^2 + k_\perp^2}{(Q_{s1}^2 + Q_{s2}^2)\xi'}}$$

$$\times I_0 \left( \frac{2p_\perp k_\perp}{(Q_{s1}^2 + Q_{s2}^2)\xi'} \right) e^{-\frac{1}{4} (Q_{s1}^2 + Q_{s2}^2) \frac{1}{4} (r - r')^2 \xi'} e^{-\frac{1}{8} (Q_{s1}^2 + Q_{s2}^2)(r^2 + r'^2)(1 - \xi')}$$
The transverse momentum spectra of J/Ψ comparison to the RHIC data: $y=0$

Large discrepancy at semi-hard transverse momenta

AuAu

dAu

PHENIX data

DK, E. Levin, K. Tuchin, arxiv: 1205.1554
The transverse momentum spectra of $J/\Psi$ comparison to the RHIC data: $y=1.7$

Discrepancy increases at forward $y$!

dAu data described well:

PHENIX data

DK, E. Levin, K. Tuchin, arxiv: 1205.1554
The transverse momentum spectra of $J/\Psi$: predictions for LHC

![Graphs showing nuclear modification factor $R_{AA}$ for $J/\psi$ vs. $p_T$ at $\sqrt{s} = 7$ TeV in $PbPb$ for rapidities (a) $y = 0$ and (b) $y = 3.25$. Each line corresponds to a different centrality bin: from bottom to top: 0-10% (solid red), 10-20% (dashed green), 20-30% (dash-dotted blue), 30-50% (solid purple), 50-80% (dashed magenta) and in minbias $pPb$ (solid brown).]

FIG. 6: (Color online). Nuclear modification factor for $J/\psi$'s vs $p_\perp$ in GeV at $\sqrt{s} = 7$ TeV in $PpPb$ for rapidities (a) $y = 0$ and (b) $y = 3.25$. Each line corresponds to a different centrality bin; from bottom to top: 0-10% (solid red), 10-20% (dashed green), 20-30% (dash-dotted blue), 30-50% (solid purple), 50-80% (dashed magenta) and in minbias $pPb$ (solid brown).

DK, E.Levin, K.Tuchin, arxiv: 1205.1554
The J/Ψ production at LHC

Lecture by P. Braun-Munzinger

Weaker suppression at LHC than at RHIC !?

ALICE Coll.,
arxiv: 1202.1383
J/ψ as a probe of QGP

J/ψ SUPPRESSION BY QUARK–GLUON PLASMA FORMATION *

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Received 17 July 1986

If high energy heavy ion collisions lead to the formation of a hot quark–gluon plasma, then colour screening prevents c ¯ c binding in the deconfined interior of the interaction region. To study this effect, the temperature dependence of the screening radius, as obtained from lattice QCD, is compared with the J/ψ radius calculated in charmonium models. The feasibility to detect this effect clearly in the dilepton mass spectrum is examined. It is concluded that J/ψ suppression in nuclear collisions should provide an unambiguous signature of quark–gluon plasma formation.
Helmut Satz and Tetsuo Matsui at BNL
T. Matsui & H. Satz:

We thus conclude, that there appears to be no mechanism for $J/\psi$ suppression in nuclear collisions except the formation of a deconfining plasma, and if such a plasma is produced, there seems to be no way to avoid $J/\psi$ suppression.

NA38: the first observation of $J/\Psi$ suppression

6 April 1989

THE PRODUCTION OF $J/\Psi$
IN 200 GeV/NUCLEON OXYGEN-URANIUM INTERACTIONS

NA38 Collaboration
Experimental information

CERN: NA38/50/60 experiments

NA50 Coll., hep-ex/0412036
Heavy quarkonia are very sensitive to the properties of QCD matter; when Debye length becomes smaller than the size of quarkonium,

\[ R_{\text{Debye}}(T) \sim \frac{1}{(gT)} < R_{\text{Quarkonium}} \sim \frac{1}{(\alpha_s M_H)}, \]

quarkonia are screened out of existence. This happens when \( T \sim g M_H \)

(what is the corresponding formula for strong coupling?)

However, even before that, when \( T \sim \varepsilon \sim \alpha_s^2 M_H \), quarkonia can be dissociated due to thermal activation.
What is the mechanism of dissociation?

In cold matter, dissociation rate is relatively small due to the softness of gluon distributions in confined matter, but it is large, $O(1 \text{ fm}^{-1})$, in hot QCD matter.

DK & H. Satz ‘94

Dissociation mechanism - gluo-effect

E. Shuryak ‘78
G. Bhanot, M. Peskin ‘79

dominates if $\frac{\epsilon}{T} \gg 1$ (strong coupling regime)

Screening dominates if $\frac{\epsilon}{T} \ll 1$ (weak coupling)
What mechanism is more important?

DK, L.McLerran, H.Satz
hep-ph/9504338

Weak coupling:

$$R_{act} = \frac{1}{Z(T)} \frac{V}{L} \left( \frac{c}{\pi^2} MT^2 \right) e^{-\frac{E_{J/\psi}}{T}}$$

Strong coupling:

$$R_{act} = \frac{4}{L} \sqrt{\frac{T}{2\pi M}} = \frac{v(T)}{L}$$

$$R_{act} = \frac{(LT)^2}{3\pi} Me^{-\frac{E_{J/\psi}}{T}}$$

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Color screening and J/Ψ suppression: lattice QCD results

Interaction energy exceeds the temperature - strong coupling!

O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, hep-lat/0309121
Screening in QGP

Strong force is screened by the presence of thermal gluons and quarks.

T-dependence of the running coupling develops in the non-perturbative region at $T < 3 T_c$; $\Delta E/T > 1$

“cold” plasma

O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow, hep-lat/0309121
Heavy quark internal energy above $T$

O.Kaczmarek, F. Karsch, P.Petreczky, F. Zantow, hep-lat/0309121
J/Ψ above $T_c$: lattice QCD

Lectures by H.-T. Ding, F.Karsch

H.T.Ding et al, arxiv:1204.4945

dissociation at $\sim 1.4 \ T_c$?

M.Asakawa, T.Hatsuda

Tuesday, May 15, 2012
Entropy: coupling to real states $\Rightarrow$ non-instantaneous, non-local effective potential

String contribution to the black hole entropy: a hint for understanding better quarkonia in the QCD plasma?

\[ S_{\infty} = -\frac{\partial F_{\infty}}{\partial T} \]

\[ F = U - TS \]

\[ \delta F = \left( \frac{\partial U}{\partial l} - T \frac{\partial S}{\partial l} \right) \delta l = 0 \]

Kosterlitz-Thouless phase transition: Frautschi, Polyakov
AdS/CFT Correspondence: 
Conformal gauge theory is dual to gravity

The chain of ideas:

1. String model pre-dates QCD as a description of hadron spectrum and hadron scattering amplitudes
   (Regge poles, Veneziano amplitude, Hagedorn spectrum,…)

2. Quantum theory of strings cannot be made consistent in (3+1) dimensions; the minimum number of dimensions is 5.
   Also: massless spin 2 state - graviton?

3. It has long been expected that QCD in the strong coupling and large N is described by string theory
   (at large N, planar diagrams dominate; they describe worldsheets of strings)
4. How to make this string description consistent on the quantum level?
Add a 5th dimension to QCD and let gravity live there
BUT: what is the metric?

\[ ds^2 = R^2 w^2(z) \left( dx^2_{3+1} + dz^2 \right) \]

what is the form of \( w(z) \)?

5. Consider instead a conformal theory, such as maximally super-symmetric \( N=4 \) Yang-Mills; then the requirement of conformal invariance fixes the metric of the 5th dimension uniquely - it is an Anti- de Sitter space \( \text{AdS}_5 \):
invariance w.r.t. \( x \to \lambda x \) fixes \( w(z) = \frac{1}{z} \)

supersymmetry - \( S_5 \), so the theory lives in \( \text{AdS}_5 \times S_5 \)
$N = 4$ SU(N) Yang-Mills theory $\equiv$ String theory on $\text{AdS}_5 \times S^5$

Radius of curvature

$$R_{S^5} = R_{\text{AdS}_5} = \left( g_{YM}^2 N \right)^{1/4} l_s$$

Duality:

$g^2 N \text{ small} \rightarrow R \text{ small} \rightarrow$ perturbation theory

$g^2 N \text{ large} \rightarrow R \text{ large} \rightarrow$ classical gravity $T_{\mu\nu}$
Color screening and black holes?

One new idea: use a mathematical correspondence between a conformal gauge theory and gravity in AdS space

- Gauge theory
- Gravity

Anti de Sitter space - solution of Einstein’s equations with a negative cosmological constant $\Lambda$
(de Sitter space - solution with a positive $\Lambda$ (inflation))

Finite T - black hole in AdS$_5 \times$S$_3 \times$T space

Maldacena
Witten; Polyakov;
Gubser, Klebanov;
Son, Starinets, Kovtun…

Tuesday, May 15, 2012
One new idea: use a mathematical correspondence between a conformal gauge theory and gravity in AdS space

Anti de Sitter space - solution of Einstein’s equations with a negative cosmological constant $\Lambda$
(de Sitter space - solution with a positive $\Lambda$ (inflation))

Finite T - black hole in AdS$_5$ x S$_3$ x T space

boundary

Witten; Polyakov; Gubser, Klebanov; Son, Starinets, Kovtun…

Color screening and black holes?

Gauge theory Gravity

Maldacena
The metaphor of the cave, 380 B.C.

"Physical objects and physical events are only "shadows" of their ideal or perfect forms, and exist only to the extent that they instantiate the perfect versions of themselves."

Socrates, in Plato’s “Republic.”

“The prisoners would take the shadows to be real things and the echoes to be real sounds, not just reflections of reality, since they are all they had ever seen or heard.”

Tuesday, May 15, 2012
“The prisoners would take the shadows to be real things and the echoes to be real sounds, not just reflections of reality, since they are all they had ever seen or heard.”

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Entropy and color screening: an insight from holography

e.g., Shuryak, Zahed
Color screening and entropy

Internal energy

Entropy

Kaczmarek, Zantow, arxiv:0506019
Confinement and entanglement entropy

\[ \rho_A = \text{Tr}_B \rho, \text{ where } \rho = |\Psi\rangle\langle\Psi| \]

Entanglement entropy:

\[ S_E = -\text{Tr} (\rho_A \ln \rho_A) \]
Entanglement entropy and holography

Strings lead to quantum entanglement at large distances

S. Sachdev, arxiv:1203.4565
Entanglement entropy and holography

Ryu-Takanayagi formula
Entanglement entropy and holography
Entanglement entropy and color screening

\[ C(l) \equiv \frac{l^d}{V} \frac{dS_A(l)}{dl} \]

Nishioka, Ryu, Takayanagi, arxiv:0905.0932
Entanglement entropy and deconfinement

Nakagawa, Nakamura, Motoki, Zakharov, arxiv:0911.2596

also:
Buividovich, Polikarpov;
Velytsky; ....
Entanglement entropy and deconfinement
Summary

• Using the heavy quarkonium as a probe of QCD matter appeared to be a very productive idea

• 25 years later, still a very active field of research
Summary

• Using the heavy quarkonium as a probe of QCD matter appeared to be a very productive idea
• 25 years later, still a very active field of research

In other words, we still do not understand a thing.... but we are not losing hope!