Heavy quarks diffusion in hydrodynamics

or

“How confident can we be in the Fokker-Planck coefficients we extract from / constrain with exp. data?”

Heavy quarks production in heavy-ion collisions, W. Lafayette

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With J. Aichelin, M. Bluhm, Th. Gousset, H. van Hees, R. Rapp and S. Vogel,

I. Understanding (partly) the present RHIC data on HQ E-loss; the Nantes viewpoint

II. Influence of the medium on our understanding

Heavy quarks production in heavy ions collisions
Based on

- **Gluon Radiation at small kT and Radiative Energy Loss of Heavy Quarks; I. The Bethe-Heitler Regime**, J. Aichelin, P.B. Gossiaux & Th Gousset (In preparation)
I. “Understanding” the RHIC HQ-data

What is the dominant E loss mechanism @ RHIC ?

What can we extract from experimental data ?

Motivation: Even a fast parton with the largest momentum $P$ will undergo collisions with moderate $q$ exchange and large $\alpha_s(Q^2)$. The running aspect of the coupling constant has been “forgotten/neglected” in most of approaches.

Effective $\alpha_s(Q^2)$

(Dokshitzer 95, Brodsky 02)

$$\alpha_{\text{eff}}(Q^2, T=0)$$

Large values for intermediate momentum-transfer => larger cross section

IR safe. The detailed form very close to $Q^2 = 0$ is not important does not contribute to the energy loss

A model; not a renormalizable theory

Heavy quarks production in heavy ions collisions
Running $\alpha_s$ : some Energy-Loss values for purely collisional processes

\[
\frac{dE_{\text{coll}}(c/b)}{dx}
\]

<table>
<thead>
<tr>
<th>$T$(MeV) \ $p$(GeV/c)</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>1 / 0.65</td>
<td>1.2 / 0.9</td>
</tr>
<tr>
<td>400</td>
<td>2.1 / 1.4</td>
<td>2.4 / 2</td>
</tr>
</tbody>
</table>

\approx 10 \% \text{ of HQ energy}

Drag coefficient

Transp. Coef ...

... of expected magnitude to reproduce the data (we “explain” the transport coeff. in a rather parameter free approach).

Heavy quarks production in heavy ions collisions
Heavy quarks production in initial NN collisions + $k_T$ broad. (0.2 GeV²/coll)

Schematic view of the global framework

Bulk Evolution: non-viscous hydro (Heinz & Kolb) → $T(M)$ & $v(M)$

Evolution of HQ in bulk: Fokker-Planck or reaction rate + Boltzmann (no hadronic phase)

Quarkonia formation in QGP through c+c→Ψ+g fusion process

D/B formation at the boundary of QGP (or MP) through coalescence of c/b and light quark (low $p_T$) or fragmentation (high $p_T$)

Ψ suppression

MC@sHQ
Observables (Au-Au) vs (rescaled) Model

Best observable so far: $R_{AA}$ for single non-photonic electrons

One reproduces $R_{AA}$ on all $p_T$ range with cranking K-factor $\approx 2$
which permits to accommodate the “unknowns”
the QCD strong coupling constant from sum rules and spin structure function data

new data from CLAS@JLab give first evidence for leveling off at low $Q^2$


is QCD in the non-perturbative region a conformal field theory?
Heavy quarks production in heavy ions collisions

Basic (massive) Gunion-Bertsch

Radiation $\alpha$ deflection of current (semi-classical picture)

Eikonal limit (large $E$, moderate $q$)

$$\omega \frac{d^3 \sigma_{\text{rad}}^{x \ll 1}}{d\omega d^2 k_\perp dq_\perp} = \frac{N_c \alpha_s}{\pi^2} (1 - x) \times \frac{J_{\text{QCD}}^2}{\omega^2} \times \frac{d\sigma_{\text{el}}^{Qq}}{dq_\perp^2}$$

$$\frac{J_{\text{QCD}}^2}{\omega^2} = \left( \frac{\vec{k}_\perp}{k_\perp^2 + x^2 M^2 + (1 - x)m_g^2} - \frac{\vec{k}_\perp - \vec{q}_\perp}{\left(\vec{k}_\perp - \vec{q}_\perp\right)^2 + x^2 M^2 + (1 - x)m_g^2}\right)^2$$

Gluon thermal mass $\sim 2T$

(phenomenological: not in BDMPS)

Quark mass

Both cures the colinear divergences and influence the radiation spectra

Dominates as small $x$ as one “just” has to scatter off the virtual gluon $k'$
Radiation spectra

\[
\omega \frac{d^2 \sigma_{\text{rad}^{QCD}}^{x<1}}{d\omega dq_{\perp}^2} \approx \frac{2N_c \alpha_s}{\pi} \ln \left( 1 + \frac{q_{\perp}^2}{3\tilde{m}_g^2} \right) \times \frac{d\sigma_{el}^{Qq}}{dq_{\perp}^2}
\]

... to convolute with your favorite elastic cross section

For coulomb scattering:

- Light quark
- c-quark
- b-quark

Little mass dependence (especially from \(q \to c\))

If typical \(q_{\perp} \approx T\):

\[
\frac{d^2 I_{\text{GB}}^{x<1}}{dz d\omega} \sim \frac{2N_c \alpha_s}{3\pi} \times \frac{1}{m_g^2 + x^2 M^2} \times \frac{\langle q_{\perp}^2 \rangle}{\lambda q}
\]

Strong mass effect in the average Eloss (mostly dominated by region II), similar to AdS/CFT

Interesting per se, but not much connected to the quenching or \(R_{AA}\).

Heavy quarks production in heavy ions collisions
Results with (Coherent) Radiation Included

1. Coherence: Some moderate increase of $R_{AA}$ for D at large $p_T$.

2. No effect seen for B

Conclusions can vary a bit depending on the value of the transport coefficient

Indication that $R_{AA}$ at RHIC is mostly the physics of rather numerous but small E losses, not very sensitive to coherence.

Heavy quarks production in heavy ions collisions
Collisional vs \{Radiative + Coll\}

The present data cannot decipher between the 2 local microscopic E-loss scenarios

Heavy quarks production in heavy ions collisions
Interpretation

The heavy-quark physics at play for RHIC measured up to now ($R_{AA}$ and $v_2$) is the one of small (relative) E-loss (and thus of the Fokker-Planck equation)… even at the largest $p_T$

**What we need**

- D and B separately (in any case)
- tagged HQ jets and $I_{AA}$ (and other correlations)

**In our view,** it is nevertheless more plausible to describe the physics in terms of a rather strong collisional energy loss supplied with an even stronger radiative energy loss (at least for $\gamma \gg 1$).

**Bad control on the theory**

Heavy quarks production in heavy ions collisions
QGP properties: low momentum

As we reproduce experimental data with rescaled model:

Moore-Teaney:

\[
\frac{D}{\eta/(c+p)} \approx 6 \quad \Rightarrow \quad \eta/s \approx DT/6
\]

“robust” pQCD

\[
\approx \frac{2\pi T D}{3 \times 4\pi}
\]

\[
\Rightarrow \eta/s \approx \frac{0.5}{4\pi}
\]

at \( T_c \)

Strong coupling; AdS/CFT:

\[
\eta/s \approx \frac{DT}{2} \approx \frac{2\pi T D}{4\pi}
\]

\[
\Rightarrow \eta/s \approx \frac{1.5}{4\pi}
\]

at \( T_c \)

But diffusion constant of heavy quark is already an interesting quantity in itself and could be evaluated on the lattice !!!

Heavy quarks production in heavy ions collisions
QGP properties: stopping power

Gathering all rescaled models (coll. and radiative):

Seems “under control”

Challenge

Exp. cannot resolve between those various trends

quite consistent as the drag coefficient reflects the average momentum loss (per unit time) => large weight on $x \sim 1$
D & B meson: RHIC II (radiat + collisional)

$R_{AA}(B&D)$

$Au-Au; central$
$Boltzmann \rightarrow \epsilon_{trans,min}$
$run. \alpha (\kappa=0.2)$

Collis., rate x 2
Collis. + Rad (LPM), rate x 0.6
$\alpha_s(rad)=0.3$

... some small deviations for D spectra at large $p_T$

Z. Xu (sqm08)

Heavy quarks production in heavy ions collisions
Heavy quarks production in heavy ions collisions

D & B meson: LHC (radiat + collisional)

D spectra in Pb-Pb (5.5 TeV):
Some window to decipher between the various Energy-loss models, for $p_T > 20 \text{ GeV/c}$

B spectra in Pb-Pb (5.5 TeV):
Pretty independent of E-loss model (properly calibrated w.r.t. RHIC data)
QGV in pp at LHC?

Motivation: initial energy density $\varepsilon_0$ in pp (LHC) $\sim \varepsilon_0$ in CuCu (RHIC) $\Rightarrow$ possible quenching of (heavy flavours) jets

Combining MC@sHQ with EPOS

Centrality $\equiv$ number $\nu$ of initial individual parton-parton collisions

In EPOS: $N_{\text{ch}} \propto \nu$

First study, showing the possibility of an effect to be measured

Heavy quarks production in heavy ions collisions
II. Influence of the medium on heavy quarks phenomenology

Motivations:
I. We want to use experimental data to constrain the transport coefficient as much as possible.
II. 2 groups (Texas AM & Nantes) have proposed models compatible with the non-photonic single-electron data, although the drag coefficients differ by a significant amount… How can this be possible?

Role of the other ingredients?

Models ≡ microscopic model of HQ-QGP interactions (transport coefficients) + medium + initial distributions + hadronization + kinetic equation + …

Methodology: exchanging some ingredients of the models into a single framework (the Nantes MC@sHQ)

Need for a collaboration: big thanks to Ralf and Hendrik

Caution: The aim of this study is not to reproduce the data at all price.
Basic ingredients

Drag coefficient: \[ \frac{d\langle \vec{p} \rangle}{dt} = -A(p, T)\vec{p} \]

Diffusion coefficient $B$ tuned to satisfy Einstein relation: asymptotic thermal distribution

Heavy quarks production in heavy ions collisions
Basic ingredients

Transport: both solve FP equation through a Langevin Monte Carlo realization…

Back to the continuous stochastic process: \[ \dot{\xi} = -\gamma(\xi) + g(\xi)w(t) \]

• Spurious drift: \[ \frac{d\langle \xi \rangle}{dt} = -\gamma(\xi) + g(\xi)g'(\xi) \]

• Ambiguity: for a small dt, at which value of \( \xi \) should we consider \( g \)? Ito vs Stratonovich

But: a) we do not have a continuous process (smallest time scale: the collision)

b) we can evaluate the average momentum loss directly from the elastic cross section (fundamental blocks are not \( \gamma \) and \( g \) but \( A_p = -\gamma + g g' \) and \( B = 2g^2 \)): No ambiguity

However: Several realizations are possible

\[ dp_i^{(k)} = -A(\bar{p}^{(k)})p_i^{(k)} dt + \left( g(\bar{p}^{(k)}) \right)_{ij} w_j \sqrt{dt} \] Ito pre point (Nantes)

\[ dp_i^{(k)} = -\Gamma(\bar{p}^{(k)})p_i^{(k)} dt + \left( g(\bar{p}^{(k)}) + \xi \dot{p}^{(k)} \right)_{ij} w_j \sqrt{dt} \] Ito post point (Texas AM)

In principle equivalent if one chooses: \[ \Gamma p_i = A p_i + \frac{\partial g_{ik}}{\partial p_i} g_{lk} \]

Heavy quarks production in heavy ions collisions
Basic ingredients

The medium:

**Nantes:** Kolb-Heinz 2D+1 ideal hydrodynamics with Bjorken invariance (azhydro V2.0)

**Texas AM:** Fireball of the Landau-Type: $T(t)$, $s(t) = S/V(t)$, $\varepsilon(t)$

Transverse plane:

Volume: $V(t) = \pi a(t)b(b)(z0 + t)$

Heavy quarks production in heavy ions collisions
Basic ingredients

The medium:

Texas AM: Everything uniform but the velocity field

\[ \tau = 1.33 \text{ fm/c} \]

Calibration

Kolb-Sollfrank-Heinz (2000)

Heavy quarks production in heavy ions collisions
Basic ingredients

The medium:

**Texas AM**: Everything uniform but the *velocity field*

---

**Calibration**

Rather good agreement for the velocity

Good agreement for the bulk flow

Heavy quarks production in heavy ions collisions
Basic ingredients

The medium:

Temperature evolution along time:

![Graph showing temperature evolution](image)

- Pure QGP
- Mixed phase

EOS of the same type (QGP described within the MIT bag model, 1st order transition), with rather similar parameters.

Conclusion: Good confidence that the “bulk” fireball is calibrated at the best
Heavy quark evolution

All heavy quarks observables evaluated with the pre-point prescription

Nuclear Modification factor

\[ R_{AA}(c) \]

\begin{align*}
\text{Au–Au; central} & \quad \text{mod } \alpha_s \text{ run. Goss. & Aich.} \\
\text{\( \rightarrow \epsilon_{\text{trans min}} \)} & \quad \text{no } k_T \text{ broad.} \\
\text{mod reso–}\Gamma \text{ Van Hees & Rapf} & \quad \text{no } k_T \text{ broad.}
\end{align*}

\begin{itemize}
\item Crossed ingredients
\item original models (almost)
\end{itemize}

\[ P_T[\text{GeV}/c] \]

\downarrow: More coupling with the Nantes microscopic model (as seen from the drag coefficient)

\[ R_{AA} \text{ compatible} \]

How can we conclude?

Heavy quarks production in heavy ions collisions
Heavy quark evolution

All heavy quarks observables evaluated with the pre-point prescription

Nuclear Modification factor

$R_{AA}(c)$

$\text{Au-Au; central}$

$\rightarrow \epsilon_{\text{trans min}}$

mod $\alpha_s$ run. Goss. & Aich.

no $k_T$ broad.

mod reso-\Gamma Van Hees & Rap)

no $k_T$ broad.

$P_T[GeV/c]$

Crossed ingredients

original models (almost)

R$_{AA}$ compatible

How can we conclude?

↓: More quenching with the fireball then with the (original) KH hydro

Heavy quarks production in heavy ions collisions
Heavy quark evolution

Differential Elliptic flow

- More coupling with the Nantes microscopic model
- Higher elliptic flow from the fireball
Inclusive elliptic flow

Heavy quark evolution

Various gedanken decoupling energy densities for the c quarks

Large deviations at early times

Heavy quarks production in heavy ions collisions
Summary of HQ

<table>
<thead>
<tr>
<th>Degree of thermalization</th>
<th>Resonances</th>
<th>Running $\alpha_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KH hydro</td>
<td>Low</td>
<td>Intermediate</td>
</tr>
<tr>
<td>fireball</td>
<td>Intermediate</td>
<td>High</td>
</tr>
</tbody>
</table>

Is there something special with Heavy Quarks (due to their inertia) as compared to light ones?
Back on the bulk v2 and on calibration

Probes of the bulk v2:

I. So called momentum anisotropy

\[ \varepsilon_p := \left. \left( \frac{dT_{xx}}{d\eta} \right) \right|_{\eta=0} - \left. \left( \frac{dT_{yy}}{d\eta} \right) \right|_{\eta=0} \]

\[ \left. \left( \frac{dT_{xx}}{d\eta} \right) \right|_{\eta=0} + \left. \left( \frac{dT_{yy}}{d\eta} \right) \right|_{\eta=0} \]

directly evaluated from the energy-stress momentum; known to be closely related to the elliptic flow (Kolb, Sollfrank & Heinz 2000)

The fireball systematically overshoots the hydro, starting from early times !!!

HQ just inherits this larger anisotropy

Heavy quarks production in heavy ions collisions
Back on the bulk v2 and on calibration

II. Elliptic flow of light quarks and ensuing pions

Particle spectra:

\[ \frac{EdN}{d^3p} = \int d\sigma_{\mu} p^\mu f(p, T) \]

\[ f = e^{-\frac{p \cdot u}{T}} \]

Fireball: freeze out at constant lab time t

Asymptotic distribution in the VHR post-point

Langevin

\[ f = \frac{p \cdot u}{p^0} e^{-\frac{p \cdot u}{T}} \]

\[ f = e^{-\frac{p \cdot u}{T}} \]

Fireball with Cooper-Frye freeze out

fireball with Milekhin-like freeze out

\[ d\sigma_{\mu} = d\tilde{V} u_{\mu} \]
Back on the bulk $v_2$ and on calibration

several levels of subtle and somehow paradoxical conclusions:

KH with Cooper-Frye freeze out $\leftrightarrow$ fireball with Cooper-Frye freeze out

Confirms the excess of intrinsic $v_2$ in the fireball on the absolute level

KH with Cooper-Frye freeze out $\leftrightarrow$ fireball with Milekhin-like freeze out

$\epsilon = \frac{p \cdot u}{p_0} e^{-\frac{p \cdot u}{T}}$

Confirms the proper calibration of the fireball (5.5% $v_2$ for pions)

Comparing with 2 different freeze out prescriptions:
Calibration at a *relative* level

Bona fide argument:

\[
\begin{align*}
\text{HQ}_\text{lq} & \equiv \text{HQ}_\text{lq} \\
\text{HQ}_\text{lq} & \equiv \text{HQ}_\text{lq}
\end{align*}
\]

“One should calibrate the fireball using the asymptotic distributions (of light quarks) that results from Langevin transport one uses for the heavy quarks”

Remember: Several realizations are possible

Ito pre point (Nantes)

\[
dp^{(k)}_i = -A(p^{(k)})p^{(k)}_i dt + \left(g(p^{(k)})\right)_{ij} w_j \sqrt{dt}
\]

\[
f_{as} = e^{-\frac{p \cdot u}{T}}
\]

Ito post point (Texas AM)

\[
dp^{(k)}_i = -\Gamma(p^{(k)})p^{(k)}_i dt + \left(g(p^{(k)} + \xi dp^{(k)})\right)_{ij} w_j \sqrt{dt}
\]

\[
f_{as} = \frac{p \cdot u}{p^0} e^{-\frac{p \cdot u}{T}}
\]

- The medium calibration is intricately linked with the transport model one uses (exchanging the ingredient “medium” alone has no meaning)
- It is not legitimate to perform the HQ simulations in the fireball with the pre-point prescription.

Heavy quarks production in heavy ions collisions
Heavy quarks with pre-point vs post-point

All heavy quarks observables evaluated with the Nantes FP coefficients

The post-point vs pre-point prescription has little influence on the heavy quark observables (consequences of u.p/p\(^0\) are not mass independent).

In particular, the post-point prescription does not bring the “fireball results” in the range of the “pre-point KH hydro”
Summary and conclusions (1)

With respect to the Kolb Heinz hydro + Cooper-Frye reference:

- Larger intrinsic $v_2$
- Compensation due to the $u.p/p^0$ factor in equil. distribution
- Nearly negligible effect of $u.p/p^0$ factor; pre-point $\equiv$ post-point

Overall increase of $v_2(HQ)$

0 collectivity

Light quark sector | heavy quark sector

This all together explain why one is able to describe the heavy flavor data with smaller FP coefficients within the fireball than in the KH hydro, although they both reproduce the pions elliptic flow.

“Bona fide” is sometimes too optimistic
Summary and conclusions (2)

Third global level of interpretation:

After all, the KH hydro with a kinetic freeze out a given rather all energy density is a model among others…

- Good candidate for medium with a genuine Milekhin freeze out
- Large influence of the medium on the extraction of FP coefficient from the experimental data
- First study, not exhaustive and to be pursued

Diagram:

- fireball
  - Larger intrinsic \( v_2 \)
- hydro
  - Similar \( v_2(\pi) \)

- 0 collectivity
  - Light quark sector
  - heavy quark sector

- HQ rather insensitive to the freeze out assumptions
- \( v_2(\text{HQ}) \) for a given set of FP coefficient

Heavy quarks production in heavy ions collisions