Comment on “Matter-Wave Interferometry of a Levitated Thermal Nano-Oscillator Induced and Probed by a Spin”

The main point of Refs. [1,2] was to propose an experiment involving the center of mass (c.m.) position of a nanodiamond and the spin of its NV center to demonstrate “the interference between spatially separated states of the center of mass of a mesoscopic harmonic oscillator … by coupling it to a spin and performing solely spin manipulations.” The creation and detection of spatially separated states of a levitated nano-oscillator would represent a major experimental advance. In this Comment, we argue that the proposed measurement does not achieve this goal. Instead, the measured signal results from the common displacement of the c.m. position of both ±1 states, and hence, does not give information about the c.m. separation of these states.

The nanodiamond is held in a harmonic potential. A spatially varying magnetic field \( B = B_0(-x\lambda - y\lambda + 2z\lambda) \) entangles the spin and c.m. degrees of freedom because the \(|±1\rangle \) states have oppositely directed forces; it is this entanglement that Refs. [1,2] propose to measure. The conceptual problem is that the nanodiamond is not oscillating about the \( z = 0 \) point of the harmonic potential but about the shifted position, \(-\Delta z_g\), due to gravity, leading to a nonzero average \( B \) field for both \(|±1\rangle \) states. This gives a Zeeman phase difference that exactly reproduces the proposed signal [Eq. (11) in Ref. [1]] even if the nanodiamond is held fixed at \(-\Delta z_g\), negating the interpretation that the signal results from “the interference between spatially separated states …” Another way to see that the interpretation of Refs. [1,2] is not correct is to note that they cancel the \( B \) field at \( z = 0 \) to eliminate a Zeeman phase difference; if the experimental proposal had canceled the \( B \) field at the shifted position, then their signal disappears. The orders of magnitude difference in distance scales make it clear that \( B \) should be canceled at \(-\Delta z_g\). The parameters in Ref. [1] are \( \omega_c \sim 10^5 \) s\(^{-1} \), \( m \sim 1.25 \times 10^{-17} \) kg, and \( B_0 \approx 580 \) T/m. Both \(|±1\rangle \) states have the same spatial shift from gravity of order \( g/\omega_c^2 \sim 10^{-9} \) m, much larger than the spatial width of the ground state \( \sim \sqrt{\hbar/(\hbar \omega_c)} \sim 10^{-11} \) m or, more importantly, the separation of the \(|±1\rangle \) states due to the spatially varying \( B \) field \( \sim 4B_0 \delta_{NV} \mu_B/(\hbar \omega_c^2) \sim 3 \times 10^{-13} \) m.

This intuitive argument can be made precise. The Hamiltonian, Eq. (4) of Refs. [1,2], is rewritten using the shifted c.m. coordinate \( \tilde{z} \equiv z + \Delta z_g \) as

\[
H = DS_z^2 + \hbar \omega_c \tilde{z} \hat{c} - 2\lambda S_z(\hat{c}^\dagger + \hat{c}) + \frac{2m \omega_c}{\hbar} \Delta z_g 2\lambda S_z - E_s,
\]

where the parameters not defined in Ref. [1] are \( \Delta z_g \equiv g \cos(\theta)/\omega_c^2 \), and \( E_s = (1/2) m \omega_c^2 \Delta z_g^2 \). The first three terms of Eq. (1) will be grouped into \( H_1 \), the fourth term will be defined as \( H_2 \), and \( E_s \) is a constant and, thus, can be dropped.

The wave function can be written exactly as

\[
\Psi(t) = \exp(-iH_2t/\hbar)\exp(-iH_1t/\hbar)\Psi(0),
\]

where \( \Psi(0) = \psi_0(\tilde{z})(|+1\rangle + |-1\rangle)/\sqrt{2} \) is an initial spatial function times the symmetric combination of spins \(+1\rangle \) and \(-1\rangle \). The \( H_1 \) is the only term that leads to separation of the \(|±1\rangle \) states. After an integer \( N \) periods, \( \tau = 2\pi N/\omega_c \), the

\[
e^{-i\sha H_1t/\hbar}\Psi(0) = e^{i\eta \phi/2}\psi_0(\tilde{z})\left[|+1\rangle + e^{i\phi}|-1\rangle \right]/\sqrt{2}
\]

after \( N \) periods, where \( \phi = 8\pi \lambda \Delta z_g \sqrt{2m \omega_c /\hbar (\hbar \omega_c)} \). Evaluating this \( \phi \) and \( \Delta \phi_{grav} \) in Eq. (10) of Ref. [1], one can show that \( \phi = \Delta \phi_{grav} \). Thus, the main result of Ref. [1], Eq. (9), is exactly obtained in Eq. (4) but Eq. (4) cannot contain information about the spatial evolution of the wave function since \( H_2 \) is proportional to \( S_z \), has no dependence on \( \tilde{z} \), and commutes with \( H_1 \). In fact, it is the Zeeman splitting of the \(|±1\rangle \) states as discussed above.

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Received 29 August 2016; published 10 March 2017
DOI: 10.1103/PhysRevLett.118.108901

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