Comment on "Matter-Wave Interferometry of a Levitated Thermal Nano-Oscillator Induced and Probed by a Spin"

The main point of Refs. [1,2] was to propose an experiment involving the center of mass (c.m.) position of a nanodiamond and the spin of its NV center to demonstrate "the interference between spatially separated states of the center of mass of a mesoscopic harmonic oscillator … by coupling it to a spin and performing solely spin manipulations." The creation and detection of *spatially* separated states of a levitated nano-oscillator would represent a major experimental advance. In this Comment, we argue that the proposed measurement does not achieve this goal. Instead, the measured signal results from the common *displacement* of the c.m. position of both ± 1 states, and hence, does not give information about the c.m. *separation* of these states.

The nanodiamond is held in a harmonic potential. A spatially varying magnetic field $\vec{B} = B_0(-x\hat{x} - y\hat{y} + 2z\hat{z})$ entangles the spin and c.m. degrees of freedom because the $|\pm 1>$ states have oppositely directed forces; it is this entanglement that Refs. [1,2] propose to measure. The conceptual problem is that the nanodiamond is not oscillating about the $z = 0$ point of the harmonic potential but about the shifted position, $-\Delta z_g$, due to gravity, leading to a nonzero average B field for both $|\pm 1>$ states. This gives a Zeeman phase difference that exactly reproduces the proposed signal [Eq. (11) in Ref. [1]] even if the nanodiamond is held fixed at $-\Delta z_g$, negating the interpretation that the signal results from "the interference between spatially separated states …" Another way to see that the interpretation of Refs. [1,2] is not correct is to note that they cancel the B field at $z = 0$ to eliminate a Zeeman phase difference; if the experimental proposal had canceled the B field at the shifted position, then their signal disappears. The orders of magnitude difference in distance scales make it clear that B should be canceled at $-\Delta z_g$. The parameters in Ref. [1] are $\omega_z \sim 10^5$ s^{-1}, $m \sim 1.25 \times 10^{-17}$ kg, and $B_0 \simeq 580$ T/m. Both $|\pm 1>$ states have the same spatial shift from gravity of order $g/\omega_z^2 \sim 10^{-9}$ m, much larger than the spatial width of the ground state $\sim\sqrt{\hbar/(m\omega_z)} \sim 10^{-11}$ m or, more importantly, the separation of the $|\pm 1>$ states due to the spatially varying B field $\sim 4B_0 g_{NV}\mu_B/(m\omega_z^2) \sim 3 \times 10^{-13}$ m.

This intuitive argument can be made precise. The Hamiltonian, Eq. (4) of Refs. [1,2], is rewritten using the *shifted* c.m. coordinate $\tilde{z} \equiv z + \Delta z_g$ as

$$H = DS_z^2 + \hbar\omega_z \tilde{c}^\dagger \tilde{c} - 2\lambda S_z(\tilde{c}^\dagger + \tilde{c}) + \sqrt{\frac{2m\omega_z}{\hbar}}\Delta z_g 2\lambda S_z - E_s, \tag{1}$$

where the parameters not defined in Ref. [1] are $\tilde{z} = \sqrt{\hbar/(2m\omega_z)}(\tilde{c}^\dagger + \tilde{c})$, $\Delta z_g \equiv g\cos(\theta)/\omega_z^2$, and $E_s = (1/2)m\omega_z^2\Delta z_g^2$. The first three terms of Eq. (1) will be grouped into H_1, the fourth term will be defined as H_2, and E_s is a constant and, thus, can be dropped.

The wave function can be written exactly as

$$\Psi(t) = \exp(-iH_2 t/\hbar)\exp(-iH_1 t/\hbar)\Psi(0), \tag{2}$$

where $\Psi(0) = \psi_0(\tilde{z})(|+1\rangle + |-1\rangle)/\sqrt{2}$ is an initial spatial function times the symmetric combination of spins $+1$ and -1. The H_1 is the only term that leads to separation of the $|\pm 1>$ states. After an integer N periods, $t = 2\pi N/\omega_z$, the

$$e^{-iH_1 t/\hbar}\Psi(0) = e^{iN\eta}\Psi(0), \tag{3}$$

where $\eta = 8\pi\lambda^2/(\hbar\omega_z)^2 - 2\pi D/(\hbar\omega_z)$. Thus, the part of the Hamiltonian that contains both the S_z and the $\tilde{z}$ operators, which is the only part of H that can entangle the spin and c.m. degrees of freedom, gives *no effect* on the wave function after an integer number of periods.

However, the term that results from the magnetic field at the shifted z, $H_2 = \sqrt{2m\omega_z/\hbar}\Delta z_g 2\lambda S_z$, gives

$$e^{-iH_2 t/\hbar}\Psi(0) = e^{-iN\phi/2}\psi_0(z)\frac{|+1\rangle + e^{iN\phi}|-1\rangle}{\sqrt{2}} \tag{4}$$

after N periods, where $\phi = 8\pi\lambda\Delta z_g\sqrt{2m\omega_z/\hbar}/(\hbar\omega_z)$. Evaluating this ϕ and $\Delta\phi_{\mathrm{grav}}$ in Eq. (10) of Ref. [1], one can show that $\phi = \Delta\phi_{\mathrm{grav}}$. Thus, the main result of Ref. [1], Eq. (9), is exactly obtained in Eq. (4) but Eq. (4) cannot contain information about the spatial evolution of the wave function since H_2 is proportional to S_z, has no dependence on $\tilde{z}$, and commutes with H_1. In fact, it is the Zeeman splitting of the $|\pm 1>$ states as discussed above.

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