Torsional Optomechanics of a Levitated Nonspherical Nanoparticle

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An optically levitated nanoparticle in vacuum is a paradigm optomechanical system for sensing and studying macroscopic quantum mechanics. While its center-of-mass motion has been investigated intensively, its torsional vibration has only been studied theoretically in limited cases. Here we report the first experimental observation of the torsional vibration of an optically levitated nonspherical nanoparticle in vacuum. We achieve this by utilizing the coupling between the spin angular momentum of photons and the torsional vibration of a nonspherical nanoparticle whose polarizability is a tensor. The torsional vibration frequency can be 1 order of magnitude higher than its center-of-mass motion frequency, which is promising for ground state cooling. We propose a simple yet novel scheme to achieve ground state cooling of its torsional vibration with a linearly polarized Gaussian cavity mode. A levitated nonspherical nanoparticle in vacuum will also be an ultrasensitive nanoscale torsion balance with a torque detection sensitivity on the order of $10^{-28} \text{ N m/s}^{1/2}\text{Hz}$ under realistic conditions.

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An optically levitated dielectric particle in vacuum [1–3] is an ultrasensitive detector for force sensing [4,5], milli-charge searching [6], and other applications [7,8]. It will provide a great platform to test fundamental theories such as objective collapse models [9,10] and quantum gravity [11] when its mechanical motion can be cooled to the quantum regime [12,13]. Recently, feedback cooling of the center-of-mass (c.m.) motion of a levitated nanoparticle to about 450 μK (about 63 phonons at 150 kHz) [14] and cavity cooling of the c.m. motion of a nanoparticle to a few mK [15] were demonstrated. The vibration mode would have already been in ground state at 450 μK [14] if its frequency is above 10 MHz. Increasing the vibration frequency of the nanoparticle can be a key to achieve ground state cooling. However, this cannot be achieved by simply increasing the intensity of the trapping laser, which induces heating and subsequently causes the loss of the nanoparticle [4,16]. Besides c.m. motion, a pioneering work has proposed to use multiple Laguerre-Gaussian (LG) cavity modes to achieve angular trapping of a dielectric rod and cool its torsional vibration (TOR) to the ground state [12]. This was later generalized to microwindmills [17], which have better overlap with LG cavity modes. These intriguing proposals of torsional optomechanics, however, have not yet been realized experimentally.

In this work, we report the first experimental observation of the torsional vibration of an optically levitated nonspherical nanoparticle in vacuum, and show that the torsional frequency can be 1 order of magnitude higher than the c.m. frequency at the same laser intensity. We explain our observation using a model of an ellipsoidal nanoparticle levitated by a linearly polarized Gaussian beam. For an ellipsoid much smaller than the wavelength of the trapping laser, its polarizability is a tensor due to its geometry [18]. In a linearly polarized Gaussian beam, the long axis of an ellipsoid tends to align with the polarization direction of the trapping laser to minimize the potential energy. When its long axis deviates from the polarization direction of the trapping laser, the ellipsoid will experience a torque pushing its long axis back to the equilibrium orientation. As a result, the ellipsoid will experience angular trapping and exhibit torsional vibration. Both the frequency and the quality factor of the torsional vibration can be 1 order of magnitude higher than those of the c.m. motion. Inspired by the experimental observation, we propose a simple yet novel scheme to achieve TOR ground state cooling with a cavity driven by a linearly polarized Gaussian beam. While we use nanodiamonds as examples in this Letter, our proposals will also work for other transparent nonspherical nanoparticles [19,20].

Besides being a platform for investigating fundamental physics, a levitated nonspherical nanoparticle in vacuum will also be a nanoscale torsion balance [21,22]. It can have a torque detection sensitivity on the order of $10^{-28} \text{ N m/s}^{1/2}\text{Hz}$ under realistic conditions, which will open up many new applications. Angular trapping and torsional vibration are
laser. As we consider only one torsional mode, we assume a 1550 nm laser beam [Fig. 1(a)]. The laser beam is tightly focused on the nanoparticles. The nanoparticles’ motion is monitored by the exiting trapping laser. The exiting beam is split by a beam splitter (BS) to a c.m. detector and a TOR detector. A 4/2 wave plate balances the power of the beams after the polarizing beam splitter (PBS) for the TOR detector. (b) A SEM image of irregular nanodiamonds. (c) A proposed scheme to cool the torsional vibration of a levitated ellipsoidal nanoparticle with an optical cavity driven by a linearly polarized Gaussian beam. (d) The relation between Cartesian coordinate systems of the nanoparticle (xN, yN, zN), the trapping laser (xT, yT, zT), and the cavity mode (xC, yC, zC). The xN axis aligns with the longest axis of the nanoparticle. xT and xC axes align with the polarization directions of the trapping laser and the cavity mode, respectively. The angle between xN and xT is θ, and the angle between xT and xC is β. The yC axis is the optical axis of the cavity. The zT axis is the propagation direction of the trapping laser. As we consider only one torsional mode, we assume zC and zN are parallel to zT for simplicity.

FIG. 1. (a) Experimental diagram for detecting torsional (TOR) vibration and center-of-mass (c.m.) motion of a levitated nonspherical nanoparticle (NP). A nanodiamond (represented by an ellipsoid) is levitated by a tightly focused linearly polarized 1550 nm laser beam. The nanoparticle’s motion is monitored by the exiting trapping laser. The exiting beam is split by a beam splitter (BS) to a c.m. detector and a TOR detector. A 4/2 wave plate balances the power of the beams after the polarizing beam splitter (PBS) for the TOR detector. (b) A SEM image of irregular nanodiamonds. (c) A proposed scheme to cool the torsional vibration of a levitated ellipsoidal nanoparticle with an optical cavity driven by a linearly polarized Gaussian beam. (d) The relation between Cartesian coordinate systems of the nanoparticle (xN, yN, zN), the trapping laser (xT, yT, zT), and the cavity mode (xC, yC, zC). The xN axis aligns with the longest axis of the nanoparticle. xT and xC axes align with the polarization directions of the trapping laser and the cavity mode, respectively. The angle between xN and xT is θ, and the angle between xT and xC is β. The yC axis is the optical axis of the cavity. The zT axis is the propagation direction of the trapping laser. As we consider only one torsional mode, we assume zC and zN are parallel to zT for simplicity.

also important for spin optomechanics of levitated nanodiamonds with nitrogen-vacancy centers [23–27], for which the orientations of nitrogen-vacancy centers are important.

Observation of the torsional vibration of levitated nanoparticles.—In the experiment, nanodiamonds are levitated using an optical tweezer formed by a linearly polarized 1550 nm laser beam [Fig. 1(a)]. The laser beam is tightly focused with a N.A. = 0.85 objective lens [27]. The nanodiamonds have broad distributions around their manufacturer size 100 nm. Some nanodiamonds have large aspect ratios, as shown in Fig. 1(b). The torsional vibration of the nanodiamond will change the polarization of the laser beam, which can be detected with a polarizing beam splitter (PBS) and a balanced detector [Fig. 1(a)]. Similar detection schemes have been used to detect the rotation of birefringent particles driven by circularly polarized lasers [28–30]. The c.m. motion of the nanodiamond changes the direction of the laser beam and can be detected with a balanced detector [31,32].

A sample of the power spectrum density (PSD) of the c.m. motion and the TOR motion of a levitated nanodiamond is shown in Fig. 2(a). Over 1/3 of our trapped nanodiamonds exhibit torsional signals. For this nanodiamond, the TOR frequency (Ω0/2π = 1.0 MHz) is about 6 times higher than the transverse c.m. frequency (Ωc/2π = 0.16 MHz and Ωc/2π = 0.18 MHz), which is promising for ground state cooling. For comparison, a factor of 6 increase in the c.m. frequency would require the laser power to be increased by a factor of 36, which can induce significant heating of the nanoparticle.

We investigated the motions of many different nanodiamonds as a function of trapping powers [Figs. 2(b) and 2(d)] and air pressures [Fig. 2(c)]. For each nanodiamond exhibiting torsional vibration, the TOR frequency is proportional to the...
square root of the trapping power, $\Omega_\theta \propto \sqrt{P}$, as shown in Fig. 2(b). This observation agrees with the prediction of the ellipsoidal model discussed below. Since the c.m. frequency is also proportional to the square root of the trapping power, $\Omega_x \propto \sqrt{P}$, the ratio $\Omega_\theta/\Omega_x$ is independent of the trapping power, as shown in Fig. 2(d). The TOR frequency is independent of the air pressure, as shown in Fig. 2(c), when the pressure is reduced from atmospheric pressure to a few Torr. The summary of 13 nanodiamonds yields $\Omega_\theta/\Omega_x = 1 \pm 0.03$ across the whole pressure range. Here $\Omega_\theta$ is the averaged torsional frequency over different pressures for each particle. For comparison, the rotational frequency of a birefringent microsphere driven by a circularly polarized laser is linearly proportional to the laser power, and increases when the air pressure decreases [30], which is very different from our results of the torsional vibration.

Further experimental evidence that supports the interpretation of torsional vibration instead of free rotation is from the measured damping factors which are anisotropic. The measured ratios $\Gamma_x/\Gamma_y$ for all the nanoparticles in Fig. 2(c) at different pressures and trapping powers yield a mean value of 0.8. A nonspherical nanoparticle rotating in the $xy$ plane should yield $\Gamma_x/\Gamma_y = 1$ on average [33]. $\Gamma_x/\Gamma_y < 1$ means the long axes of the nanoparticles align with the polarization direction ($x$ axis) of the trapping laser [34].

**Ellipsoidal model.**—As a minimal model to describe the torsional vibration of an irregular nanodiamond [Fig. 1(b)], we consider an ellipsoid with semiaxes $r_x > r_y = r_z$ in a linearly polarized optical tweezer. When the size of the ellipsoid is much smaller than the wavelength of the laser, we can use the Rayleigh approximation. The induced dipole will be $\mathbf{p} = \alpha_x E_x \hat{x} + \alpha_y E_y \hat{y} + \alpha_z E_z \hat{z}$, where the instantaneous electric field of the laser beam $\mathbf{E}$ is decomposed into components along the principle axes of the ellipsoid [18]. Here, $\alpha_x, \alpha_y, \alpha_z$ are polarizabilities along the principle axes. The force and torque on the ellipsoid are $\mathbf{F} = \langle \nabla (\mathbf{p} \cdot \mathbf{E}) \rangle / 2$ and $\mathbf{M} = (\mathbf{p} \times \mathbf{E})$, respectively [18,35]. To capture the essential properties of the system, we consider only the c.m. motion of the ellipsoid along the $y_T$ axis and the torsional vibration of the ellipsoid around the $z_T$ axis [Fig. 1(d)]. The potential energy of the ellipsoid in the optical tweezer is

$$U(y, \theta) = -\frac{V}{2c} |\chi_x - (\chi_x - \chi_y) \sin^2 \theta| I_L(y), \tag{1}$$

where $V = 4\pi r_x r_y r_z / 3$ is the volume of the ellipsoid, $c$ is the speed of light, $\chi_x = \alpha_x / (\epsilon_0 V)$ and $\chi_y = \alpha_y / (\epsilon_0 V)$ are the effective susceptibility of the ellipsoid, $\epsilon_0$ is the vacuum permittivity, $\theta$ is the angle between the longest axis of the ellipsoid and the electric field of the laser beam, and $I_L(y)$ is the laser intensity at the location of the ellipsoid. As an example, $\chi_x = 2.05, \chi_y = 1.74$ for an ellipsoidal nanodiamond with $r_x/r_z = 0.8$.

![FIG. 3. (a) Trapping potentials in the transverse ($U_x$) and angular ($U_\theta$) directions. (b) Frequencies of the torsional vibration and c.m. vibration as a function of the size of the ellipsoid when its aspect ratio is $r_x/r_z = 0.8$. (c) Quality factors of the torsional vibration and c.m. motion of a levitated nanoparticle as a function of the pressure. (d) Enhancement ratio $\Omega_\theta/\Omega_x$ as a function of the size of the ellipsoid with different aspect ratios. In the calculations, we assume the waist of the Gaussian optical tweezer to be 600 nm, the laser wavelength to be 1550 nm, and the laser power to be 100 mW. To calculate results shown in (a) and (c), we assume the semiaxes of the ellipsoid to be $r_x = 50$ nm and $r_y = r_z = 40$ nm. The corresponding vibration frequencies are $\Omega_\theta/2\pi = 1.26$ MHz and $\Omega_x/2\pi = 220$ kHz.](image)
While the frequency of the c.m. motion is independent of the size of the ellipsoid, the frequency of the torsional vibration increases when the size of the ellipsoid decreases [Fig. 3(b)]. Both experimental (Fig. 2) and theoretical results [Figs. 3(b) and 3(d)] demonstrate that the torsional frequency can be 1 order of magnitude higher than the c.m. frequency at the same laser intensity. The ratio $\Omega_\varphi/\Omega_y$ can be increased by decreasing the ratio $r_y/r_z$ or the particle size. Because of a higher frequency, the quality factor of the torsional vibration ($Q_\varphi$) [36] can also be 1 order of magnitude higher than that of the c.m. motion ($Q_y$) [34], which is another advantage for torsional ground state cooling [Fig. 3(c)]. In the calculation, we assume the collisions between air molecules and the ellipsoidal nanoparticle are mainly inelastic, with a momentum accommodation coefficient of 0.9 [34,36].

Torque sensing.—An optically levitated ellipsoid in vacuum will be an ultrasensitive nanoscale torsion balance [37] using the laser as a “string” to provide the restoring torque. Torsion balances have played historic roles in the development of modern physics. They were used in Coulomb’s experiment that discovered the law of electrostatic force, the Cavendish experiment that measured the gravitational constant [37], and many other important experiments [38–40]. The minimum torque that can be detected with a torsion balance is $M_{\text{min}} = \sqrt{4k_B T/\Omega_\varphi/(Q_\varphi \Delta t)}$ [41]. Here $T$ is the environmental temperature, $I$ is the moment of inertia, and $\Delta t$ is the measurement time. For a levitated $r_x = 50 \text{ nm}$, $r_y = r_z = 40 \text{ nm}$ ellipsoid at $10^{-8}$ Torr with a torsional frequency of $\Omega_\varphi/2\pi = 1.26 \text{ MHz}$, the torque sensitivity is about $2 \times 10^{-24} \text{ Nm}/\sqrt{\text{Hz}}$ at 300 K. This is several orders more sensitive than tethered nanoscale torsion sensors, which typically have sensitivities on the order of $10^{-21} \text{ Nm}/\sqrt{\text{Hz}}$ [21,22]. This system can be used to measure the torque on a single electron spin [42] or even a single nuclear spin. A proton in a 0.1 T magnetic field would experience a torque on the order of $10^{-27} \text{ Nm}$.

Torsional ground state cooling.—Inspired by the experimental observation and the ellipsoidal model, we propose to use a linearly polarized Gaussian beam to drive a cavity to cool the torsional vibration to the ground state [12,13,43]. We assume the nonspherical nanoparticle is levitated using another linearly polarized Gaussian beam [Figs. 1(c) and 1(d)]. Because the TOR vibration and c.m. motion can have very different frequencies, we can neglect their coupling. For simplicity, we consider only the torsional mode $\Omega_\varphi$ around the $z_T$ axis. The linear Hamiltonian of the system in the frame rotating at the cooling laser frequency can be approximated as [12,43]

$$\hat{H} = -\hbar \Delta_\varphi \hat{a}^\dagger \hat{a} + \hbar \Omega_\varphi \hat{b}^\dagger \hat{b} + \hbar |\alpha| (g_\varphi \hat{b}^\dagger + \hat{b}) (\hat{a}^\dagger + \hat{a}).$$

Here $2\pi\hbar$ is the Planck constant, $\Delta_\varphi = \omega_\varphi - \omega_\varphi + 2\Omega_\varphi^2 |\alpha|^2/\Omega_\varphi$ is the effective detuning, $\omega_\varphi$ is the laser frequency, $\omega_C$ is the cavity resonant frequency, $\hat{a}^\dagger (\hat{a})$ and $\hat{b}^\dagger (\hat{b})$ are the creation (annihilation) operators for the cavity field and the mechanical field, respectively, $|\alpha| = \sqrt{\langle n_p \rangle}$ is the steady amplitude of the cavity mode, $n_p$ is the number of photons in the cavity, and $g_\varphi$ is the coupling strength between a single torsional vibration phonon and a single cavity photon [43]. The coupling strength $g_\varphi$ will be maximized when the center of the nanoparticle is at the antinode of the cavity mode and the angle between the polarization directions of the trapping laser and the cavity beam is $\beta = 45^\circ$ [Fig. 1(d)]. We obtain the maximum coupling constant as [12,43]

$$g_\varphi = \sqrt{\frac{10\hbar \pi \rho r_x^2}{3\rho (r_x^2 + r_z^2)^2} (\chi_y - \chi_x^2)} \frac{64\pi c}{\hbar^2 L^2}. \quad (4)$$

Here $\lambda_C$ is the wavelength of cavity mode and $L$ is the length of the cavity. The waist of the cavity mode is $W_C = \sqrt{\lambda_C L/2\pi}$ for a confocal cavity.

The single phonon–single photon coupling strength $g_\varphi$ as a function of the cavity length is shown in Fig. 4(a). For comparison, we also plot the maximum coupling strength between the c.m. motion and the cavity mode, $g_c = \sqrt{2\hbar \pi \rho r_x^2 / (3\rho \lambda_C)} (16\pi^2 c / \lambda_C L^2)$, which happens when $\beta = 0^\circ$. For a $r_x = 50 \text{ nm}$ and $r_y = r_z = 25 \text{ nm}$ nanodiamond, $g_\varphi$ and $g_c$ have similar magnitudes and both increase when the length of the cavity decreases. Using a similar procedure as in Refs. [43,44], we can calculate the steady state phonon number of the levitated nanoparticle.

We assume the finesse of the cavity is $\mathcal{F} = 10^5$ and the detuning is $\Delta_L = -\sqrt{(\kappa/4) + \Omega_\varphi^2}$, with $\kappa$ being the decay...
rate of the cavity. As shown in Fig. 4(b), only the torsional mode can be cooled to the ground state with a $L = 0.5 \text{ mm}$ cavity. For a $L = 5 \text{ mm}$ cavity, both the torsional mode and the c.m. mode can be cooled to the ground states. The torsional vibration mode will have smaller final phonon numbers because of its higher vibration frequency. Thus, the torsional mode can be cooled to the ground state with a cavity with a broader range of cavity length and driving field strength than those of the c.m. mode. Comparing to former proposals with high-order LG cavity modes [12,17,45], our proposal requires only linearly polarized Gaussian beams. The observed motion is similar to the torsional vibrations (“pendular states”) of molecules [46,47] and spins [48] in an external field, and can be used to study torsional decoherence [49].

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