

Transfer of a wavepacket between atoms

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Abstract

We calculate the properties of a two-atom system where an electronic wavepacket on one atom is transferred to a second atom. We focus on two situations that could be experimentally accessible. In both situations, the dipole–dipole interaction causes the wavepacket character to evolve between the two atoms. The effect will only be observable if the atoms preserve their coherence on the microsecond time scale.

1. Introduction

There has been recent progress in the experimental and computational study of the interaction between highly excited (Rydberg) atoms. Rydberg atoms have exaggerated properties that allow for pairs of atoms to interact even when separated by large distances. Interesting effects were seen in experiments on ‘frozen Rydberg gases’ [1, 2] where the states could be tuned in energy to give strong interactions. There was further impetus for investigating these systems when a theoretical treatment [3] showed the possibility for dipole blockade. Experiments have observed the suppressed excitation of atoms so that more than 1000 atoms can be suppressed by a single excitation. [4] On the experimental side, there has been the direct observation of dipole blockade between two Rydberg atoms [5, 6]. The hopping of an excitation between two atoms has also been observed. [7] The possibility for interesting effects when a single type of excited state coherently ‘hopped’ through a background consisting of a different type of excited state was simulated in [8]. On the theory side, there has been a plethora of suggestions for using the interaction between two or more Rydberg atoms to create interesting systems (for example, see [9]).

In all of these studies, the atoms are excited to single-atom *eigenstates* using narrow bandwidth, cw-lasers or a chirped laser. The Rydberg atoms start in a single state that later evolves due to the interaction between pairs (or more) of atoms. While experiments in the frequency domain give useful information about many aspects of the interaction between Rydberg atoms, studies in the time domain can also lead to interesting behaviour. In complex systems, it can be useful to initiate a wavepacket in a specific degree of freedom and study its evolution throughout the coupled system.

In this paper, we will investigate some of the properties of interacting Rydberg atoms when the Rydberg atoms are

excited to coherent superpositions of states (wavepackets). We will consider two of the possible geometries in detail. In both situations, the initial wavepacket will be on one atom and the other atom will start in a stationary state, or both atoms will start in a wavepacket whose evolution is changed due to the atom–atom interaction. In the first geometry, one atom is in a superposition of two states that have the same angular momentum (for example, 50s and 51s) which leads to a wavepacket with radial motion and the second atom is in a state different from the two states of the first atom and which interacts with the first through the dipole–dipole potential (for example, 50p). In the second geometry, one atom is in a superposition of states with different angular momentum (for example, 50s and 50p) which leads to a wavepacket with radial and angular motion and the second atom is in a stationary state one of which is the same as the first atom (for example, 50s) or both atoms start in the same wavepacket (for example, a coherent superposition of 50s and 50p).

We study how the wavepacket evolves. We find that it can jump from one atom to the other. We also discuss how the wavepacket disperses and revives. Observation of these features would give a clear signature of the coherence of the interaction. Atomic units are used unless stated otherwise.

2. Theory

Before deriving the behaviour of a wavepacket on two atoms, we will review the treatment of wavepackets for time-independent Hamiltonians. If the Hamiltonian has no time dependence, the wavefunction can be represented as the sum over eigenstates ψ_α , the eigenenergies E_α and the $t = 0$ coefficients C_α to give

$$\Psi(t) = \sum_{\alpha} C_{\alpha} \psi_{\alpha} e^{-iE_{\alpha}t}, \quad (1)$$

where the coefficients are related to the $t = 0$ projections:

$$C_\alpha = \langle \psi_\alpha | \Psi(t = 0) \rangle. \quad (2)$$

This decomposition is useful when the eigenstates and energies can be found. In the situations below, we will be able to determine these quantities analytically. However, the eigenstates and energies typically need to be obtained numerically.

Another important notation used below is that representing the electronic states of two Rydberg atoms. We will use the notation $\psi_\alpha(1)\psi_{\alpha'}(2)$ to represent that state where atom 1 has the Rydberg electron in the state ψ_α and atom 2 has its Rydberg electron in the state $\psi_{\alpha'}$. The atoms are assumed to be separated by a large distance compared to their physical size. In this situation, there is an interaction between the atoms that is well approximated by the dipole–dipole term of the $1/r_{12}$ operator:

$$V \simeq [\vec{r}_1 \cdot \vec{r}_2 - 3(\vec{r}_1 \cdot \hat{R})(\vec{r}_2 \cdot \hat{R})]/R^3, \quad (3)$$

where $\vec{r}_{1,2}$ is the vector dipole operator for atoms 1,2, R is the distance between the atoms and \hat{R} is the unit vector that points from atom 1 to atom 2. The accuracy of this approximation depends on the states and on the separation between atoms. We do not use this approximation below but it guides the choice of which types of states to use. It could also be useful when calculating the size of some of the matrix elements described below.

For the situations considered below, the dominant effect of this interaction potential causes a transition on atom 1 from a state α_1 to a state β_1 and a transition on atom 2 from a state α_2 to a state β_2 . If the states are such that $E_{\alpha_1} + E_{\alpha_2} = E_{\beta_1} + E_{\beta_2}$, then there is no energy cost for the transition. In this case, the total wavefunction oscillates with a superposition of α_1, α_2 states and β_1, β_2 states.

We do not give explicit expressions for the matrix elements of the dipole–dipole operator, equation (3), and specific states. The form of the matrix element has been given in many different papers (for example, see [10, 11]).

Finally, we assume that the atoms are stationary during the time scales of the simulation. This can be achieved by starting with cold atoms or holding the atoms in a trap.

2.1. Situation 1

The first geometry we will present is the case where there is a radial wavepacket on one atom and a stationary state of a different character on the other atom. In order for the interaction to be strong enough for interesting physics, the states should differ by 1 in orbital angular momentum. A possible situation would be to have a wavepacket consisting of 50s plus 51s states on one atom and the second atom being a 50p. For some alkali atoms, the dipole matrix element between the ns and np states is similar in size to that between the $(n+1)s$ and np states.

This situation already has complications that can mar the wavepacket motion due to different orientations of the states. One of the problems is that the different total angular momenta projected along the inter-atomic axes can interact with different strengths. As a concrete example, examine the case of 50s on

one atom and 50p_{3/2} on the other. There are 16 states in total. Taking the z -direction to be along the inter-atomic direction, there are two states with $J_z = 2$, four states with $J_z = 1$, four states with $J_z = 0$, four states with $J_z = -1$ and two states with $J_z = -2$.

In order for the wavepacket dynamics to be clearest, the orientation of the states needs to be chosen such that there are only two states available for the atom–atom interaction.

We will explore the details of the case where the single atom wavepacket is constructed from a superposition of states $n_1\ell$ and $n_2\ell$. The second atom is prepared in the state $n_{dd}, \ell+1$ and the angular momenta are such that J_z is a maximum. In this situation, the states are mixed due to the dipole–dipole interaction. The eigenstates are

$$\psi_{n_1, \pm} = \frac{1}{\sqrt{2}} [\psi_{n_1\ell}(1)\psi_{n_{dd}, \ell+1}(2) \pm \psi_{n_{dd}, \ell+1}(1)\psi_{n_1\ell}(2)] \quad (4)$$

with eigenenergies

$$E_{n_1, \pm} = E_{n_1\ell} + E_{n_{dd}, \ell+1} \pm \Delta_{n_1}, \quad (5)$$

where Δ_{n_1} is the matrix element of the dipole–dipole operator which is proportional to the square of the dipole matrix element between states $n_1\ell$ and $n_{dd}, \ell+1$ and is inversely proportional to the cube of the separation. Similarly, the other eigenstates are

$$\psi_{n_2, \pm} = \frac{1}{\sqrt{2}} [\psi_{n_2\ell}(1)\psi_{n_{dd}, \ell+1}(2) \pm \psi_{n_{dd}, \ell+1}(1)\psi_{n_2\ell}(2)] \quad (6)$$

with eigenenergies

$$E_{n_2, \pm} = E_{n_2\ell} + E_{n_{dd}, \ell+1} \pm \Delta_{n_2}. \quad (7)$$

Now that the eigenstates and energies are determined, we can make a wavepacket by superposing the four states with the appropriate coefficients. But to get the coefficients we need to know the wavefunction at $t = 0$. An interesting choice is to make a wavepacket localized only on one atom at $t = 0$. For example we can choose

$$\Psi(t = 0) = \frac{1}{\sqrt{2}} [\psi_{n_1\ell}(1) + \psi_{n_2\ell}(1)] \psi_{n_{dd}, \ell+1}(2) \quad (8)$$

which gives a radial wavepacket on atom 1. In the absence of the atom–atom interaction, the time-dependent wavefunction would be

$$\Psi(t, \Delta = 0) = \frac{1}{\sqrt{2}} [\psi_{n_1\ell}(1) e^{-iE_{n_1\ell}t} + \psi_{n_2\ell}(1) e^{-iE_{n_2\ell}t}] \times \psi_{n_{dd}, \ell+1}(2) e^{-iE_{n_{dd}, \ell+1}t} \quad (9)$$

which is the single atom radial wavepacket on atom 1 times the eigenstate on atom 2. The observables on atom 1 can have time-dependent behaviour but not those on atom 2. For example, the expectation value of the radius of atom 1 will oscillate with a frequency $\omega = (E_{n_1\ell} - E_{n_2\ell})$.

If the atom–atom interaction is present, then an interesting two-atom wavepacket ensues. We can get the coefficients of the eigenstates by projecting onto the $t = 0$ wavefunction. For the initial state of equation (8), all of the coefficients are 1/2. This means the time-dependent wavefunction can be written as

$$\Psi(t) = \frac{1}{2} [\psi_{n_1, +} e^{-iE_{n_1, +}t} + \psi_{n_1, -} e^{-iE_{n_1, -}t} + \psi_{n_2, +} e^{-iE_{n_2, +}t} + \psi_{n_2, -} e^{-iE_{n_2, -}t}] \quad (10)$$

which can be simplified using the expression for the eigenstates and eigenvalues to

$$\Psi(t) = [\Psi^{(c)}(1, t)\psi_{n_{dd}, \ell+1}(2) + \Psi^{(s)}(2, t)\psi_{n_{dd}, \ell+1}(1)] \times e^{-iE_{n_{dd}, \ell+1}t}, \quad (11)$$

where

$$\Psi^{(c)}(1, t) = \frac{1}{\sqrt{2}} [\cos(\Delta_{n_1}t) e^{-iE_{n_1}t} \psi_{n_1 \ell}(1) + \cos(\Delta_{n_2}t) e^{-iE_{n_2}t} \psi_{n_2 \ell}(1)] \quad (12)$$

is the wavepacket on atom 1 and

$$\Psi^{(s)}(2, t) = \frac{i}{\sqrt{2}} [\sin(\Delta_{n_1}t) e^{-iE_{n_1}t} \psi_{n_1 \ell}(2) + \sin(\Delta_{n_2}t) e^{-iE_{n_2}t} \psi_{n_2 \ell}(2)] \quad (13)$$

is the wavepacket on atom 2. Note that equation (11) reduces to equation (9) when the interaction between the atoms is 0. Note that in all cases of interest, the magnitude of the energy splitting $|E_{n_1} - E_{n_2}|$ is much larger than the atom-atom interaction Δ_{n_1} or Δ_{n_2} .

A special case occurs when the interaction strength is the same for both states: $\Delta_{n_1} = \Delta_{n_2}$. In this case, the wavepackets on the different atoms have exactly the same form as the non-interacting atom situation but the amplitude has an overall factor of $\cos(\Delta_{n_1}t)$ or $\sin(\Delta_{n_1}t)$. This gives the effect of having the wavepacket, without any change except for an overall phase, oscillate from one atom to the other.

There are a few classes of ways to probe the wavefunction. We will now examine what type of time dependence is expected for different kinds of probes. One of the important quantities is the ratio of the interaction strengths. To make the plots simple, we will choose $\Delta_{n_1} = 2\pi$ MHz and Δ_{n_2} to be a fraction of that value. The two interaction strengths can be almost the same; for example, we checked the ratio of the $ns-np$ and $(n+1)s-np$ dipole matrix elements between $n = 40$ and 60 and found it to be between $0.94-0.95$ for $np_{1/2}$ and $0.97-0.98$ for $np_{3/2}$.

2.1.1. Probe 1. We will first consider a probe that strongly detects either the probability for atom 1 to be in the state $n_{dd}, \ell + 1$ or for atom 2 to be in that state. The probability for atom 2 to be in the state $n_{dd}, \ell + 1$ is

$$P_{n_{dd}, \ell+1}(2, t) = \langle \Psi^{(c)}(1, t) | \Psi^{(c)}(1, t) \rangle = \frac{1}{2} [\cos^2(\Delta_{n_1}t) + \cos^2(\Delta_{n_2}t)] \quad (14)$$

and the probability for atom 1 to be in the state $n_{dd}, \ell + 1$ is

$$P_{n_{dd}, \ell+1}(1, t) = \langle \Psi^{(s)}(2, t) | \Psi^{(s)}(2, t) \rangle = 1 - P_{n_{dd}, \ell+1}(2, t). \quad (15)$$

The probability for atom 2 to be in the state $n_{dd}, \ell + 1$ is plotted in figure 1 for the cases where $\Delta_{n_2} = \Delta_{n_1}, 0.95\Delta_{n_1}, 0.90\Delta_{n_1}, 0.85\Delta_{n_1}$ and $0.80\Delta_{n_1}$.

This graph shows the typical features that arise from beats between the two different frequencies. At a time corresponding to $(\pi/2)/|\Delta_{n_1} - \Delta_{n_2}|$, the hopping of the two states between the atoms is exactly out of phase and gives a non-oscillating probability of $\sim 1/2$. The first revival is

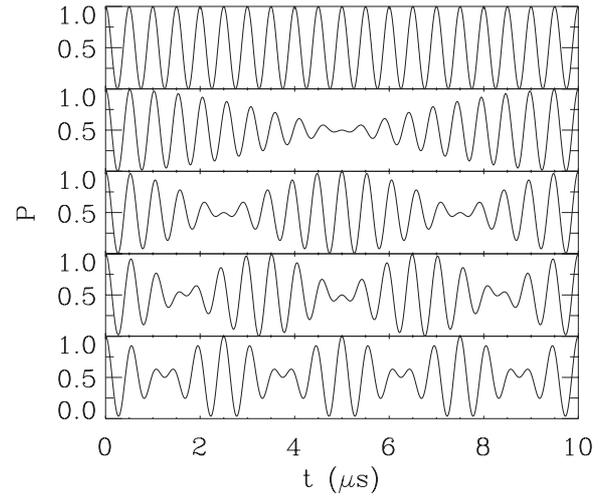


Figure 1. The probability for finding atom 2 with the state $n_{dd}, \ell + 1$ for five different ratios of the interaction strength. The ratio of strengths from top to bottom are 1, 0.95, 0.90, 0.85 and 0.80.

at a time of $\pi/|\Delta_{n_1} - \Delta_{n_2}|$. For atoms interacting through the dipole-dipole potential, the strength of the interaction depends strongly on the separation distance. However, the ratio of the interaction strengths, $\Delta_{n_1}/\Delta_{n_2}$, is independent of the separation so that the number of oscillations between revivals gives a direct measure of this ratio.

2.1.2. Probe 2. Now we will consider a probe that detects a coherence property of the wavepacket on atom 1. For example, the probe might detect the probability for finding the electron near the nucleus; in this case, the detector will measure the probability for the atom to be in the combination $\psi_{n_1 \ell} + \psi_{n_2 \ell}$. If the probe measures the probability to be near the outer turning point, the combination $\psi_{n_1 \ell} - \psi_{n_2 \ell}$ is more relevant. As an example, the probability for the wavepacket to be in the + combination on atom 1 is

$$P_{1c} = \left| \frac{1}{\sqrt{2}} (\psi_{n_1 \ell} + \psi_{n_2 \ell}) \langle \Psi^{(c)}(1, t) | \right|^2 = \frac{1}{4} |\cos(\Delta_{n_1}t) e^{-iE_{n_1}t} + \cos(\Delta_{n_2}t) e^{-iE_{n_2}t}|^2 \quad (16)$$

which can be simplified to

$$P_{1c}(t) = \frac{1}{4} [\cos^2(\Delta_{n_1}t) + \cos^2(\Delta_{n_2}t) + 2\cos(\Delta_{n_1}t)\cos(\Delta_{n_2}t)\cos([E_{n_1} - E_{n_2}]t)]. \quad (17)$$

This probability will now depend on the slow time variation due to Δ and the fast time variation due to the wavepacket oscillation on a single atom.

Calculations were performed using $\Delta_{n_2} = 0.90\Delta_{n_1}$ with $\Delta_{n_1} = 2\pi$ MHz and where $E_{n_2 \ell} - E_{n_1 \ell} = 2\pi/20$ ps; thus, the period of the wavepacket motion is 20 ps. Figure 2 shows the probability for the wavepacket to be in the + state on atom 1 for short time ranges starting at the times of 0, 2.5 μ s, 5.0 μ s, 7.5 μ s and 10.0 μ s. There are a few interesting features of this graph. The first is that at the times 2.5 μ s and 7.5 μ s there is no oscillation and the value of the probability is 1/4. In figure 1, the middle panel shows the times when there is no oscillation. But, more importantly, these are times where $\cos(\Delta_{n_1}t) = 0$;

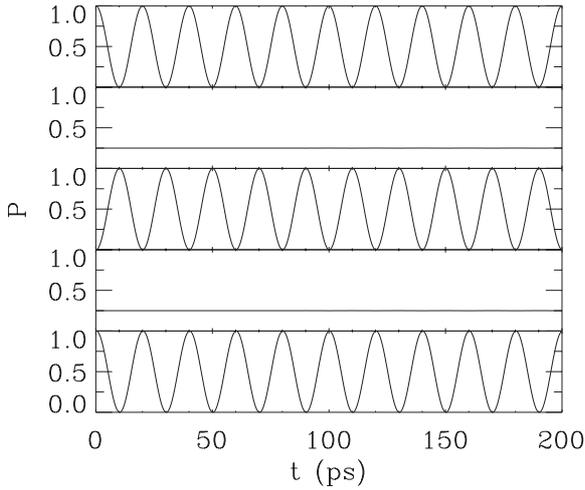


Figure 2. The probability for finding atom 1 with the + state of the wavepacket for the interaction strength ratio of 0.90. The time is measured from 0, 2.5 μ s, 5.0 μ s, 7.5 μ s and 10.0 μ s.

from equation (17) we find that there will be no fast wavepacket oscillation whenever $\cos(\Delta_{n_1}t) = 0$ or $\cos(\Delta_{n_2}t) = 0$. At these two times, the $\psi_{n_1\ell}(1)$ amplitude is 0 or the $\psi_{n_1\ell}(2)$ amplitude is 0 so there can be no fast beats between these states. Another interesting feature is that the middle panel shows a π phase shift compared to the bottom or top panels; this is because the dipole–dipole interaction has introduced a sign difference between the two parts of the wavefunction. This property probably cannot be measured because it would require knowing the experimental times relative to the dipole–dipole interaction strength to roughly one part in 10^5 . Finally, there is almost no variation of the range of oscillation because the time shown is short compared to the hop time between atoms.

These oscillations are extremely short compared to the time scales so it is not obvious whether coherence can be maintained for sufficiently long times. A coherent, Rydberg wavepacket for μ s time scales was maintained in [12], which is a promising result for our proposed studies of wavepackets and atom–atom interactions.

Equation (17) was for the + character to be on atom 1. The probability for the + character to be on atom 2 is the same expression but with all of the $\cos(\Delta t)$ terms replaced with $\sin(\Delta t)$:

$$P_{2c}(t) = \frac{1}{4} [\sin^2(\Delta_{n_1}t) + \sin^2(\Delta_{n_2}t) + 2 \sin(\Delta_{n_1}t) \sin(\Delta_{n_2}t) \cos([E_{n_1\ell} - E_{n_2\ell}]t)]. \quad (18)$$

This clearly shows that there will be a fast wavepacket motion on atom 2 except for the times where $\sin(\Delta_{n_1}t) = 0$ or $\sin(\Delta_{n_2}t) = 0$. If the two Δ are nearly the same, the earliest strong wavepacket character will be when $t = (\pi/2)/\Delta_{n_1}$. Measuring the wavepacket motion on atom 2 will be a clear signature of the coherence of the interaction.

Finally, if we measure the + character on either atom, we add equations (17) and (18). This gives

$$P_+(t) = \frac{1}{2} [1 + \cos([(\Delta_{n_1} - \Delta_{n_2})t] \cos([E_{n_1\ell} - E_{n_2\ell}]t))], \quad (19)$$

where a trigonometric identity was used for the last term. This means that whenever the time is near $(\pi/2)/|\Delta_{n_1} - \Delta_{n_2}|$, $(3\pi/2)/|\Delta_{n_1} - \Delta_{n_2}|$, etc there will be little wavepacket motion that is detectable. However, integer multiples of $\pi/|\Delta_{n_1} - \Delta_{n_2}|$ will give the largest wavepacket motion.

2.1.3. Other probes. There are other possible probes of the coherent properties of this wavefunction. Probably the most interesting is detecting linear combinations of $n_{dd}, \ell + 1$ and $n_{1\ell}$ or $n_{2\ell}$ states. For example, this could be accomplished by a microwave pulse that was resonant between one of the transitions. The basic idea is similar to that discussed in the previous two sections so we will not give explicit expressions for this case.

2.2. Situation 2

The second situation we will investigate is the case where there are only two possible states on each atom, instead of three as in the previous section. For example, a case might be where one atom has a linear combination of 50s and 50p and the second atom has a (perhaps) different linear combination of the same two states. As with the situation above, there can be complications if there are more than two states that are accessible so we will assume that the system is such that only two possible states are involved.

We will explore details of the case where the states are $n_{1\ell}$ and $n_2, \ell + 1$. There are four eigenstates and eigenvalues:

$$\psi_{n_1, n_1} = \psi_{n_1\ell}(1)\psi_{n_1\ell}(2) \quad (20)$$

with eigenenergy $E_{n_1, n_1} = 2E_{n_1\ell}$,

$$\psi_{n_2, n_2} = \psi_{n_2, \ell+1}(1)\psi_{n_2, \ell+1}(2) \quad (21)$$

with eigenenergy $E_{n_2, n_2} = 2E_{n_2, \ell+1}$ and

$$\psi_{\pm} = \frac{1}{\sqrt{2}} [\psi_{n_1\ell}(1)\psi_{n_2, \ell+1}(2) \pm \psi_{n_2, \ell+1}(1)\psi_{n_1\ell}(2)] \quad (22)$$

with eigenenergies $E_{\pm} = E_{n_1\ell} + E_{n_2, \ell+1} \pm \Delta$ where Δ is the matrix element of the dipole–dipole operator.

Now that the eigenstates and energies are determined we can make a wavepacket by superposing the four states with the appropriate coefficients. As with the derivation above, we need to know the wavefunction at $t = 0$. An interesting choice is to imagine that the wavepacket is made by starting with both atoms in the state n_1, ℓ . A microwave pulse then gives an equal admixture of n_1, ℓ and $n_2, \ell + 1$ on each atom. If this occurs by acting with an operator such as $\exp(\pm i\sigma_x \pi/4)$ (with σ_x the formal Pauli spinor in this two-state space), then the wavefunction would have the form

$$\Psi(t = 0) = \frac{1}{2} [\psi_{n_1\ell}(1) \pm i\psi_{n_2, \ell+1}(1)] [\psi_{n_1\ell}(2) \pm i\psi_{n_2, \ell+1}(2)]. \quad (23)$$

To achieve this form, the operation that causes the mixing $\psi_{n_1\ell} \pm i\psi_{n_2, \ell+1}$ needs to be fast compared to the time $\sim 1/\Delta$; it does not need to be fast compared to $\sim 1/|E_{n_1\ell} - E_{n_2, \ell+1}|$. In the absence of the atom–atom interaction, the time-dependent wavefunction would be

$$\Psi(t, \Delta = 0) = \frac{1}{2} [\psi_{n_1\ell}(1) e^{-iE_{n_1\ell}t} \pm i\psi_{n_2, \ell+1}(1) e^{-iE_{n_2, \ell+1}t}] \times [\psi_{n_1\ell}(2) e^{-iE_{n_1\ell}t} \pm i\psi_{n_2, \ell+1}(2) e^{-iE_{n_2, \ell+1}t}] \quad (24)$$

which is both atoms having a single-electron wavepacket.

If the atom–atom interaction is present, then a more interesting wavepacket evolves. For the initial condition of equation (23), there is no overlap with the ψ_- eigenstate. The time-dependent wavefunction can be written as

$$\Psi(t) = \frac{1}{2}[\psi_{n_1 n_1} e^{-iE_{n_1 n_1} t} - \psi_{n_2 n_2} e^{-iE_{n_2 n_2} t} \pm i\sqrt{2}\psi_+ e^{-iE_+ t}]. \quad (25)$$

Another possible initial state is to start with a wavepacket only on atom 1 and have atom 2 in the eigenstate $\psi_{n_1, \ell}(2)$. This initial state is

$$\Psi(t=0) = \frac{1}{\sqrt{2}}[\psi_{n_1 \ell}(1) \pm i\psi_{n_2, \ell+1}(1)]\psi_{n_1 \ell}(2). \quad (26)$$

We chose the state on atom 2 arbitrarily to be $\psi_{n_1, \ell}(2)$, but the other eigenstate could have been chosen and will give similar results. Projecting on the eigenstates gives the time-dependent wavefunction

$$\Psi(t) = \frac{1}{\sqrt{2}} \left[\psi_{n_1 n_1} e^{-iE_{n_1 n_1} t} \pm \frac{i}{\sqrt{2}} \psi_+ e^{-iE_+ t} \mp \frac{i}{\sqrt{2}} \psi_- e^{-iE_- t} \right]. \quad (27)$$

2.2.1. Probe 1. One possible probe of the system is to perform an operation that effectively projects the initial state on the time-dependent state. This can be accomplished by performing operations at time t that undo the formation of the initial state. For the situation of equation (25), this projection gives

$$P(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2 = \frac{1}{16} |e^{-iE_{n_1 n_1} t} + e^{-iE_{n_2 n_2} t} + 2e^{-iE_+ t}|^2 \quad (28)$$

which has both a fast wavepacket oscillation and a slower oscillation due to the atom–atom interaction. If we use the notation $\omega_{wp} = E_{n_1} - E_{n_2}$, this can be simplified to

$$P(t) = \frac{1}{4} |\cos(\omega_{wp} t) + e^{-i\Delta t}|^2 = \frac{1}{4} [1 + \cos^2(\omega_{wp} t) + 2 \cos(\omega_{wp} t) \cos(\Delta t)] \quad (29)$$

which clearly shows the time dependence from the fast wavepacket motion, ω_{wp} , and the slower oscillation from the atom–atom interaction, Δ .

Because of the fast wavepacket oscillation, it may not be clear how to use this measurement. One possibility is to take the difference between the maximum of $P_2(t)$ minus the minimum of $P_2(t)$ during one wavepacket oscillation (that is, during a time interval of $\sim 2\pi/\omega_{wp}$). When $\cos(\Delta t) = \pm 1$, the maximum is 1 and the minimum is 0 giving a difference of 1. When $\cos(\Delta t) = 0$, the maximum is 1/2 and the minimum is 1/4 giving a difference of 1/4. Thus, the difference oscillates with a period equal to π/Δ . This oscillation should be detectable and will only be present if the interaction is coherent. Figure 3 shows the difference, δP , between the maximum and minimum for the case where $\omega_{wp} = 2\pi/20$ ps and $\Delta = 2\pi$ MHz.

Figure 3 is a classic case of dispersion and revival of the wavepacket. If decohering interactions are present, then the oscillatory property of the wavepacket will be destroyed with time and will not revive. Only for the case of coherent interactions will the revivals occur.

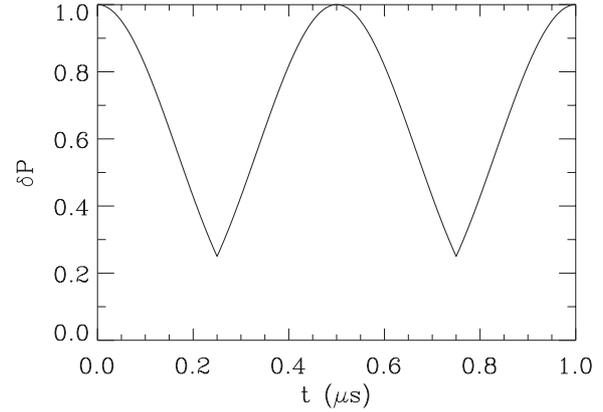


Figure 3. Shows the difference between the maximum of P_2 and the minimum of P_2 during one wavepacket period (20 ps) as a function of time. The atom–atom interaction strength is $\Delta = 2\pi$ MHz.

2.2.2. Probe 2. In this section, we will present results when the initial state is given by equation (26). Two interesting ways to probe the behaviour of this state is to either measure the wavepacket on atom 1 or the wavepacket on atom 2. A simple test of the wavepacket character is to project onto the initial wavepacket. For the wavepacket on atom 1, the projection is equivalent to projecting onto the initial state:

$$P_1(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2 = \frac{1}{4} |e^{-iE_{n_1 n_1} t} + \frac{1}{2} e^{-iE_+ t} + \frac{1}{2} e^{-iE_- t}|^2 \quad (30)$$

which can be simplified to

$$P_1(t) = \frac{1}{4} |1 + e^{i\omega_{wp} t} \cos(\Delta t)|^2 = \frac{1}{4} [1 + \cos^2(\Delta t) + 2 \cos(\omega_{wp} t) \cos(\Delta t)], \quad (31)$$

where $\omega_{wp} = E_{n_1} - E_{n_2}$. As above, there is both a fast oscillation from the wavepacket and a slow oscillation from the atom–atom interaction. Whenever $\cos(\Delta t) = 0$, the fast oscillation from the wavepacket has 0 amplitude.

For the wavepacket on atom 2, the projection is onto the state

$$\Psi_2 = \psi_{n_1 \ell}(1) \frac{1}{\sqrt{2}} [\psi_{n_1 \ell}(2) \pm i\psi_{n_2, \ell+1}(2)]. \quad (32)$$

For this case, the probability is given by

$$P_2(t) = |\langle \Psi_2 | \Psi(t) \rangle|^2 = \frac{1}{4} |e^{-iE_{n_1 n_1} t} + \frac{1}{2} e^{-iE_+ t} - \frac{1}{2} e^{-iE_- t}|^2 \quad (33)$$

which can be simplified to

$$P_2(t) = \frac{1}{4} |1 - i e^{i\omega_{wp} t} \sin(\Delta t)|^2 = \frac{1}{4} [1 + \sin^2(\Delta t) + 2 \sin(\omega_{wp} t) \sin(\Delta t)]. \quad (34)$$

This probability also shows a fast wavepacket oscillation and a slow oscillation from the atom–atom interactions. Thus, the wavepacket behaviour can hop from atom 1 to atom 2. For this atom, the fast wavepacket oscillation has 0 amplitude when $\sin(\Delta t) = 0$ and has a maximum in the oscillation when $\sin(\Delta t) = \pm 1$.

2.2.3. *Other probes.* There are other possible probes of the coherent properties of the wavefunction. Probably the most interesting would be simultaneously projecting both atoms onto the initial wavepacket state. This projection gives $(1/2) \cos^2[(\omega_{wp} - \Delta)t/2]$; except for a slow shifting of phase due to Δ , the atom–atom interaction does not have any effect on this measurement. This probe of the wavepacket does not show any effect of dispersion, revivals, etc.

3. Conclusion

We have presented four possible situations where time-dependent wavepackets and atom–atom interactions lead to interesting measurable effects. We presented results where the wavepacket character of the atom could hop to a different atom as well as cases where the atom–atom interaction caused dispersion and then revival of the wavepackets. The few cases discussed in this paper do not exhaust the possibility for combining wavepackets and atom–atom interactions. These studies seem to be within the range of current experiments.

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