PHYS550, Test 2, Fall 2018

You must show work to get credit. You should read the important math info at the bottom of the page before working any problem; it might save you steps.

(1) (5 pts) T. Li, et al, Phys. Rev. Lett. **109**, 163001 (2012) proposed an experiment that would consist of a circular ring of 100 identical $^9Be^+$ ions. Treat the ions as being equally spaced in a ring of radius 50 nm allowed to move around the circle but otherwise constrained. See diagram on blackboard. Find the lowest 3 energy levels in Joules and Kelvins.

(2) (5 pts) An operator $\hat{M} = \exp(i\hat{p}a/\hbar)$ where $i, a, \hbar$ are constants and $\hat{p}$ is the momentum operator. (a) What are the units of a? (b) Find $g(x)$ where $g(x) = \hat{M}f(x)$.

(3) (5 pts) The two states $|\psi_1(t)\rangle$ and $|\psi_2(t)\rangle$ are the solution of the same time dependent Schrödinger equation with a time dependent $\hat{H}(t)$. Derive the equation for the time derivative of $\langle\psi_1(t)|\psi_2(t)\rangle$. Solve this equation to obtain the time dependent projection.

(4) (5 pts) The lowest order relativistic correction to the kinetic energy gives $KE \simeq p^2/(2m) - p^4/(8m^3c^2)$. (a) Write down the representation of the time independent 3D Schrödinger in terms of spatial derivatives that includes this correction. (b) For the case $V(\vec{r}) = V(r)$, *show* whether or not the eigenstates are still separable as $R_{n\ell}(r)Y_{\ell m}(\theta, \varphi)$?

(5) (10 pts) (a) Determine the matrix elements $H_{0,n'}$ and H_{n0} of the relativistically corrected Hamiltonian (see previous problem) for a 1D harmonic oscillator. (b) *Show* whether or not the eigenstates of the *corrected* Hamiltonian still have the property $\psi_n(x) = (\pm 1)\psi_n(-x)$. (c) Is the ground state energy of the corrected Hamiltonian larger or smaller than that of the non-relativistic Hamiltonian? You need to prove your answer.

(6) (10 pts) In 2D, an eigenstate has the wave function $\psi(\rho, \varphi) = A\rho e^{i\varphi}e^{-\beta\rho^2}$. (a) Determine the constant A. (b) Convert the wave function into a function of x, y. (c) Compute the probability current. (d) Compute the integral of the probability current over all space. (e) Compute the integral of $M\vec{\rho} \times \vec{j}$ over all space.

(7) (10 pts) Two identical quarks with mass M interact with each other through a radial potential $V(r) = Fr$ with $F > 0$. Ignoring Fermi/Bose statistics, find the $\ell = 0$ eigenstates and eigenenergies using properties of the Airy functions described below.

<u>Math info:</u>

The solution of $y''(x) - xy(x) = 0$ are the Airy functions: $Ai(x)$ and $Bi(x)$. The $Bi(x) \to \infty$ as $x \to \infty$ and $Ai(x) \to 0$ as $x \to \infty$. The zeros of both the $Ai(x)$ and $Bi(x)$ are at $x < 0$. The n-th zero (n starts at 1) are denoted $Ai(\alpha_n) = 0$ and $Bi(\beta_n) = 0$.

Possibly useful integral $\int_{-\infty}^{\infty} e^{-ax^2+bx}dx = \sqrt{\frac{\pi}{a}}e^{b^2/[4a]}$

Another possibly useful integral $\int_0^{\infty} x^n e^{-x}dx = n!$

In cylindrical coordinates the differential element is $\rho d\rho d\phi$

In spherical coordinates $-(\hbar^2/2\mu)d^2u_{n\ell}(r)/dr^2 + V_{eff}(r)u_{n\ell}(r) = E_{n\ell}u_{n\ell}(r)$

Harmonic oscillator eigenstates are

$$u_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right) e^{-m\omega x^2/(2\hbar)}$$