

PHYS461, Test 1, Spring 2018

You must show work to get credit. There are integrals at the back of the book that might be useful.

In doing variational problems, you might want to re-use trial wave functions (if possible) so you won't have to redo many of the integrals.

(1) (5 pts) Estimate the speed of the electron for the ground state of the hydrogen atom and what will be the fractional change of the ground state energy due to relativity. Clearly give your reasoning. I want numerical values.

(2) (5 pts) Approximate the energy of the ground state of the infinite square well using the trial wave function $\psi_{tr}(x) = Ax(a-x)$. Compare to the exact value and give the percentage your value is above or below the exact value.

(3) (5 pts) An electron is confined between two infinite walls at 0 and a . In addition, there is an extra potential $V(x) = \alpha\delta(x-L)$ where $0 < L < a$. (a) What are the units of alpha? (b) Obtain the eigen-energies, E_n , to order α^2 .

(4) (5 pts) As a simple model for a confined atom, treat the case of an electron that interacts with a proton fixed at $r = 0$ and an infinite wall at $r = b$. As accurately as possible find the b where the ground state energy is 0. (Hint: you should first figure out whether perturbation theory or the variational principle is more appropriate.)

(5) (10 pts) A proton is in a potential which is infinite for $x < 0$ and a uniform electric field of $E = 10^5$ V/m pushing it in the negative x direction for $x > 0$. (a) Give the potential energy for the proton. (b) Sketch the potential. (c) Get the ground state energy as accurately as possible. (Make sure to give your answer in J.) (d) Roughly, what is the extent of the wave function. (Make sure to give your answer in meters.)

(6) (10 pts) An atom is in a nearly harmonic well: $V(x) = (1/2)M\omega^2x^2 + Cx^3$. (a) What are the units of C? (b) Obtain the eigen-energies E_n to order C^2 .

(7) (10 pts) Two identical particles are confined in an infinite square well $V(x) = 0$ for $0 < x < a$ and is otherwise infinite. There is a small interaction between the two particles which can be approximated as $V(x_1, x_2) = \alpha\delta(x_1 - x_2)$. (a) For $\alpha = 0$, list the energies through $10E_1$ where E_1 is the energy of 1 particle in the infinite square well. (b) Obtain the first 6 eigenenergies to order α^1 and specify any other symmetry that each state has. Write your answer in terms of the symbol $I(n_1, n_2) = \int_0^\pi \sin^2(n_1x) \sin^2(n_2x) dx$.