PHYS460, Test 1, Fall 2015

You must show work to get credit. Possible integrals you might need are in the back cover of the book.

(1) (5 pts) A particle is in the potential well that has the form $V(x) = Cx^4$ where $C > 0$. Describe the features of the eigenstates and/or eigenvalues. You will get 1 pt for each *independent* feature that is correct, but you will be deducted 1 pt for each feature that is wrong.

(2) (5 pts) For this problem, you are scientific advisor for the science fiction movie *Shelby's Conundrum* where the premise is $\hbar = 1$ J s. The hero, Shelby McShelby III, measures the position and velocity of a pack of killer gerbils ($M_{gerbil} = 1000$ kg) that are all in the same (angry) quantum state (don't ask why; I didn't say it was a good movie). He measured: 8.1 m 4 times, 8.3 m 10 times, 8.4 m 4 times, and 8.5 m 2 times for the position and -0.50 m/s 4 times, -0.51 m/s 12 times, and -0.52 m/s 4 times for the velocity. (a) Are these measurements consistent with the quantum mechanics of Shelby's world? (b) Would you use your real name in the movie credits?

(3) (5 pts) A particle experiences a constant force in the $-x$ direction. There is an infinite wall at $x = 0$ so that the particle is only measurable at $x > 0$. Give the wave function at some random energy $E > 0$ as a power series in x through the term proportional to x^5.

(4) (5 pts) The wave function $\Psi(x, 0) = C\exp(-\alpha|x| + ibx)$ where α, b, and C are positive real constants and $-\infty < x < \infty$. Compute the expectation value of $\hat{x}$ and the expectation value of $\hat{p}$.

(5) (10 pts) You have a potential $V(x) = -\alpha[\delta(x - a) + \delta(x + a)]$. Determine the ground state wave function and the transendental equation for the ground state energy.

(6) (10 pts) You have an electron in an infinite square well with $0 < x < a$. The wave function $\Psi(x, 0) = Bx^2(a - x)$. Compute the average energy that you would measure. Compare your result to the ground state energy.

(7) (10 pts) You have a mass that experiences the potential $V(x) = (1/2)M\omega^2x^2$. The wave function at $t = 0$ is $\Psi(x, 0) = Dx(1 - 4\sqrt{M\omega/\hbar}x)\exp(-M\omega x^2/[2\hbar])$. (a) Compute the average energy you would measure. (b) Compute $\Psi(x, t)$.