PHYS460, Test 2, Fall 2014

You must show work to get credit!!!!!!

(1) (5 pts) Sketch the $P_\ell^m(x)$ function in the relevant range of x for the case of $\ell = 7$, $m = 4$. Point out all of the relevant features for that ℓ, m.

(2) (5 pts) In T. Li, *et al*, Phys. Rev. Lett. **109**, 163001 (2012), they proposed an experiment that would consist of a ring of 100 trapped $^9Be^+$ ions. As a first step, they will trap one ion. You can approximate the ion motion as confined to a ring of radius 50 nm in the xy-plane. What are the lowest 3 energy levels in Joules and in Kelvins?

(3) (5 pts) You have a 1D potential with the form $V(x) = 0$ for $|x| > a$ and $V(x) = -(1/10)\hbar^2\pi^2/(2M[2a]^2)$ for $|x| < a$. There is one bound state. Give the bound state energy in the form $E = -f\hbar^2\pi^2/(2M[2a]^2)$ with your value of f good to 2 significant digits. Make sure to clearly write down your algorithm.

(4) (5 pts) Laser cooling and trapping techniques have progressed to the point where a quantum hamster with mass M_h is in the ground state of an infinite square well potential, $V(x) = 0$ for $0 < x < a$ and $V(x) = \infty$ elsewhere. (a) What is the probability to measure the hamster's momentum between p and $p + dp$? (b) Is it ethical to expose an innocent hamster to laser cooling and trapping techniques?

(5) (10 pts) The 3D potential energy for a quark can be (crudely) approximated as linearly increasing with distance from the origin. For a specified energy $E > 0$, give the first 4 nonzero terms in the power series expansion (in r) of the radial part of the wave function for $\ell = 2, m = -1$. Do not worry about normalization or whether E is an eigenenergy.

(6) (10 pts) For classical particles, the equations for the angular momenta are $d\vec{L}/dt = \vec{N}$ where the torque $\vec{N} = \vec{r} \times \vec{F}(\vec{r})$. (a) For a quantum particle, find $d\langle\vec{L}\rangle(t)/dt = \langle???\rangle$. (b) Evaluate the right hand side when the potential energy is spherically symmetric.

(7) (10 pts) You have a 2×2 Hamiltonian with elements $H_{11} = 3V$, $H_{22} = -3V$, and $H_{12} = 4V$. (1 pt) (a) What is the matrix element H_{21}? Give the reason for your answer. (3 pt) (b) Determine the two eigenenergies. (3 pt) (c) Determine the two eigenstates. (3 pt) (d) At time $t = 0$, the state is $|\Psi(0)>= |1>$. Determine $|\Psi(t)>$.