Question #1

For a potential energy $\frac{1}{2} M \omega^2 x^2$, the number of physical solutions of the time independent Schrodinger equation (for random E), $E \psi(x) = -(\hbar^2/2M)d^2\psi/dx^2 + V(x) \psi(x)$, is

(a) 0

(b) 1

(c) 2

(d) 3

(e) depends on V in a complicated way.

Question #2

In the classically forbidden region (x where $E - V(x) < 0$) the solution of $E\,\psi(x) = -(\hbar^2/2M)d^2\psi/dx^2 + V(x)\,\psi(x)$

(a) oscillates with x.

(b) is 0

(c) must be imaginary.

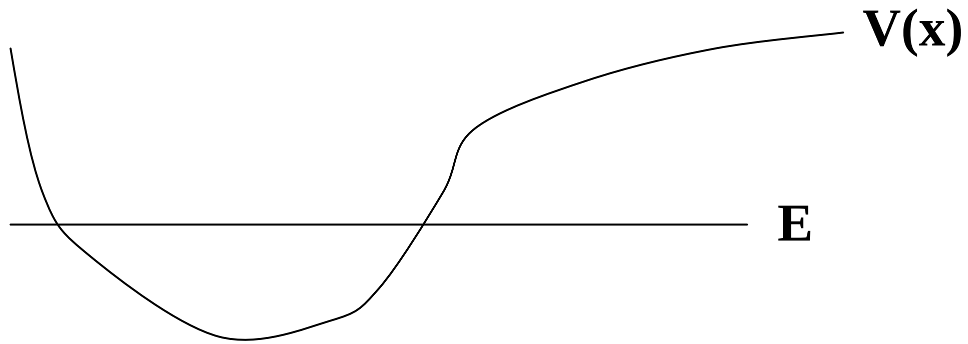
(d) exponentially diverges or converges with x

(e) linearly increases with x

Eigenstates & Eigenvalues of S.E.

The time independent Schrodinger equation is
$H_{op}\ \psi_n(x) = E_n\ \psi_n(x)$
What is the eigenvalue? eigenstate?

E can not take every possible value if classical motion restricted to finite range.



Properties of Eigenstates of S.E.

(1) E_n are all real and increase with n.

(2) The $\psi_n(x)$ are ortho-normal (orthogonal & normalized).

(3) The eigenstates can be chosen to be real at every x.

(4) Eigenstates are continuous.

(5) Derivative of eigenstate is continuous if V(x) is finite.

(6) $\langle H_{op} \rangle$ does not depend on t.

(7) $\psi_n(x) = \psi_n(-x)$ or $-\psi_n(-x)$ if $V(x) = V(-x)$

Ortho-normality properties

What is <1> = ?

Suppose you've found the eigen-states [standing waves, $\psi_n(x)$] and eigen-energies [E_n]:

$$E_n \psi_n(x) = -(\hbar^2/2M)d^2\psi_n/dx^2 + V(x)\,\psi_n(x)$$

You've set the size (normalized) $\int_{-\infty}^{\infty} |\psi_n(x)|^2\, dx = 1$

Is $\Psi(x,t) = A\,[\psi_n(x)\exp(-i\,E_n\,t/\hbar) + \psi_m(x)\exp(-i\,E_m\,t/\hbar)]$ a solution of Schrodinger Eq? Why/why not?

For this $\Psi(x,t)$, compute <1>. Use it to determine A & show that ψ_n & ψ_m must be orthogonal if E_n doesn't equal E_m.

General Behavior

$d^2\psi/dx^2 = -2\ M\ [E - V(x)]\ \psi/\hbar^2$

What sort of behavior when at positions where $E > V(x)$?

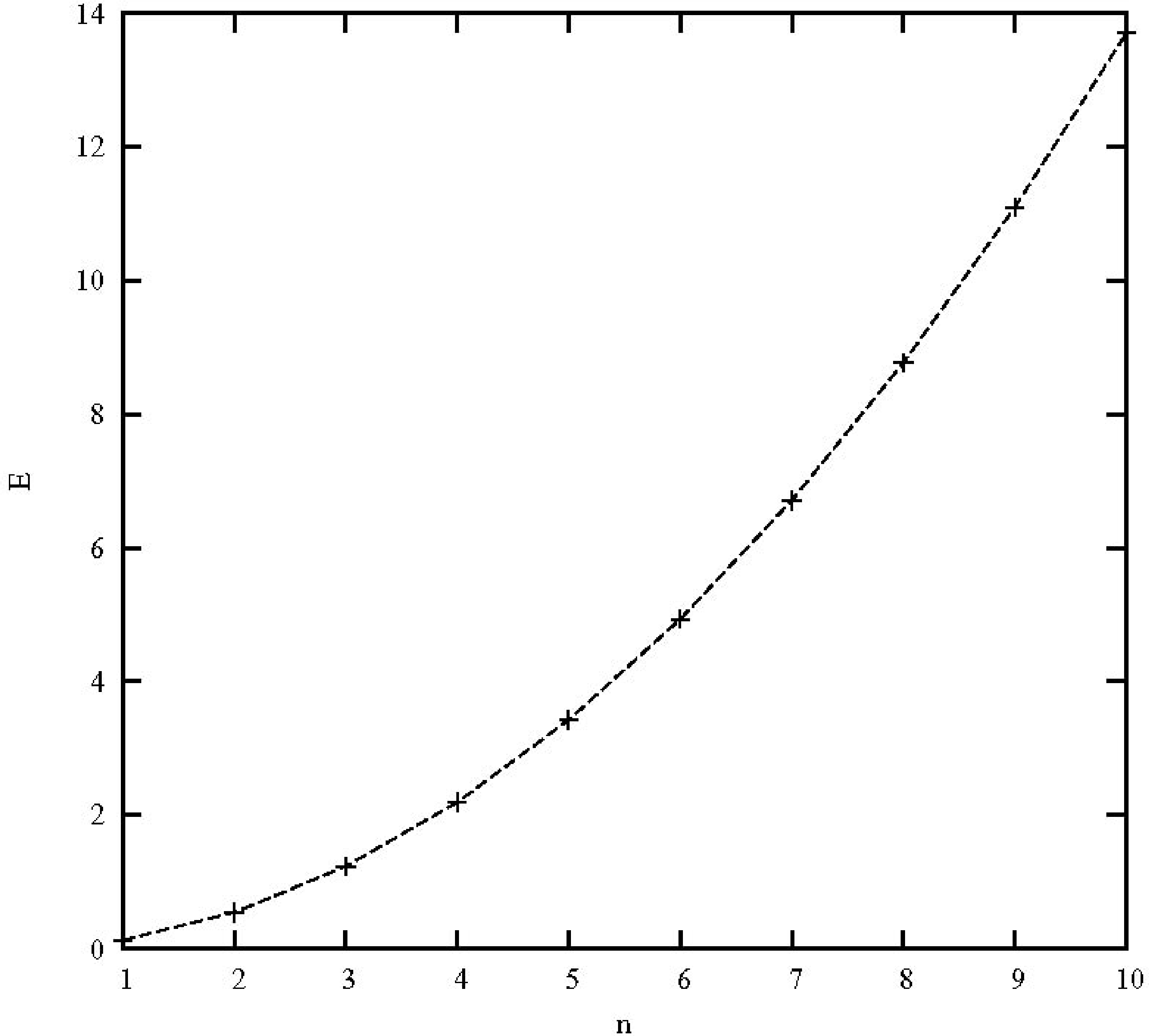
What sort of behavior when at positions where $E < V(x)$?

For a given potential where does ψ oscillate fastest w/ x?

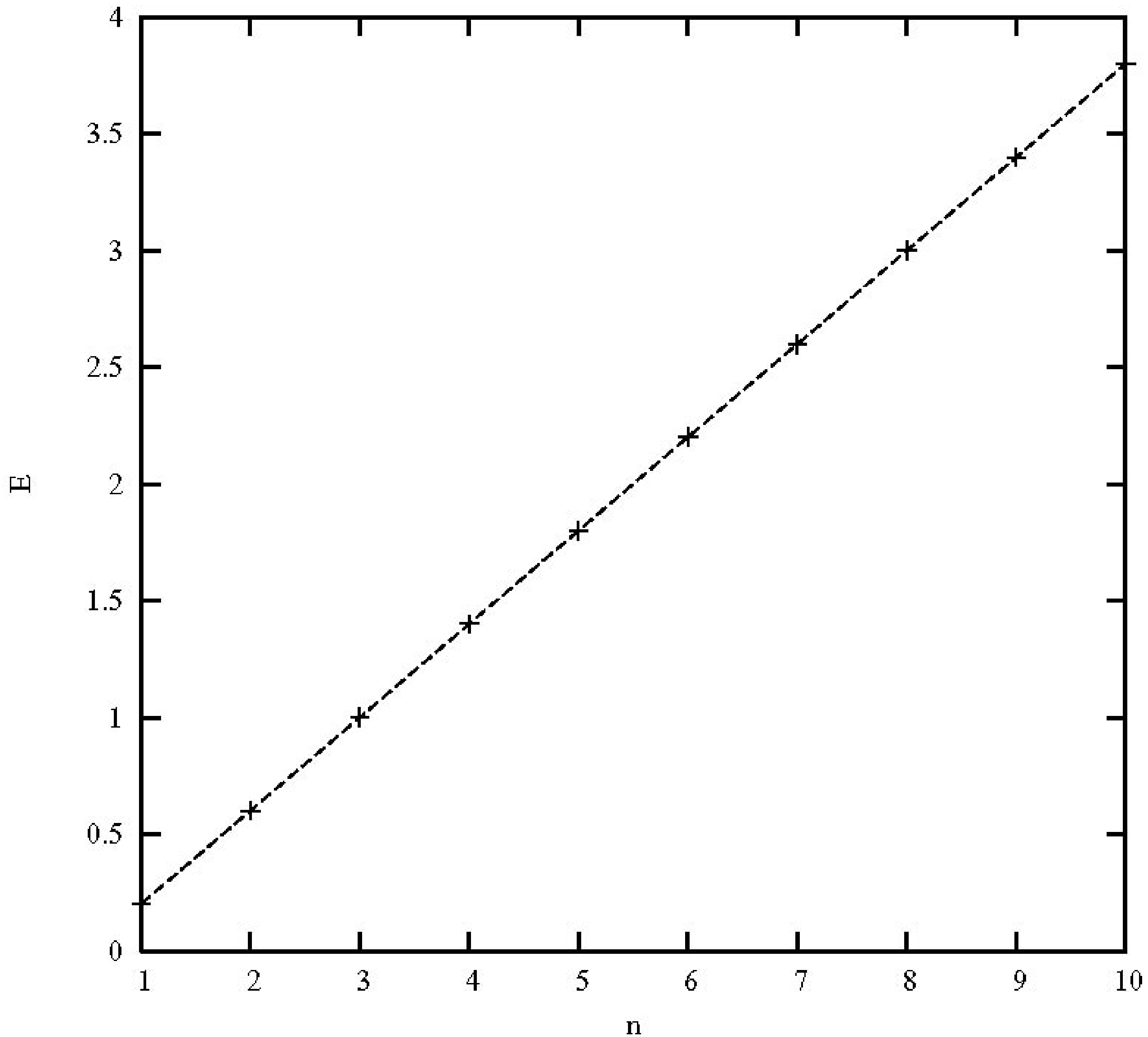
Where are the positions where curvature of ψ is 0?

Correspondence principle: period = $h/(E_{n+1} - E_n)$ If we plot E vs n, how should the curve look if classical period increases with E? decreases with E? is independent of E?

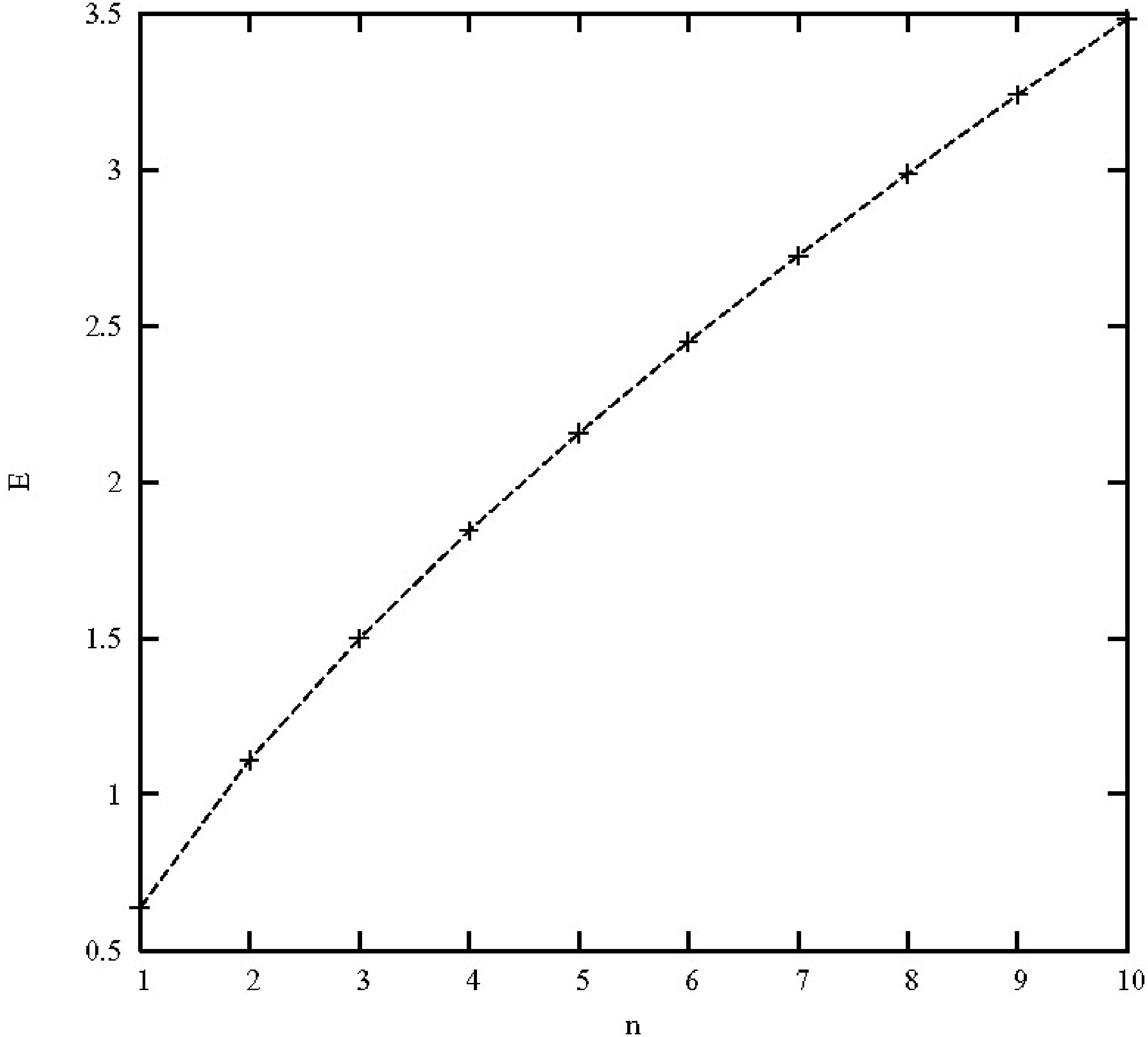
Infinite Square Well



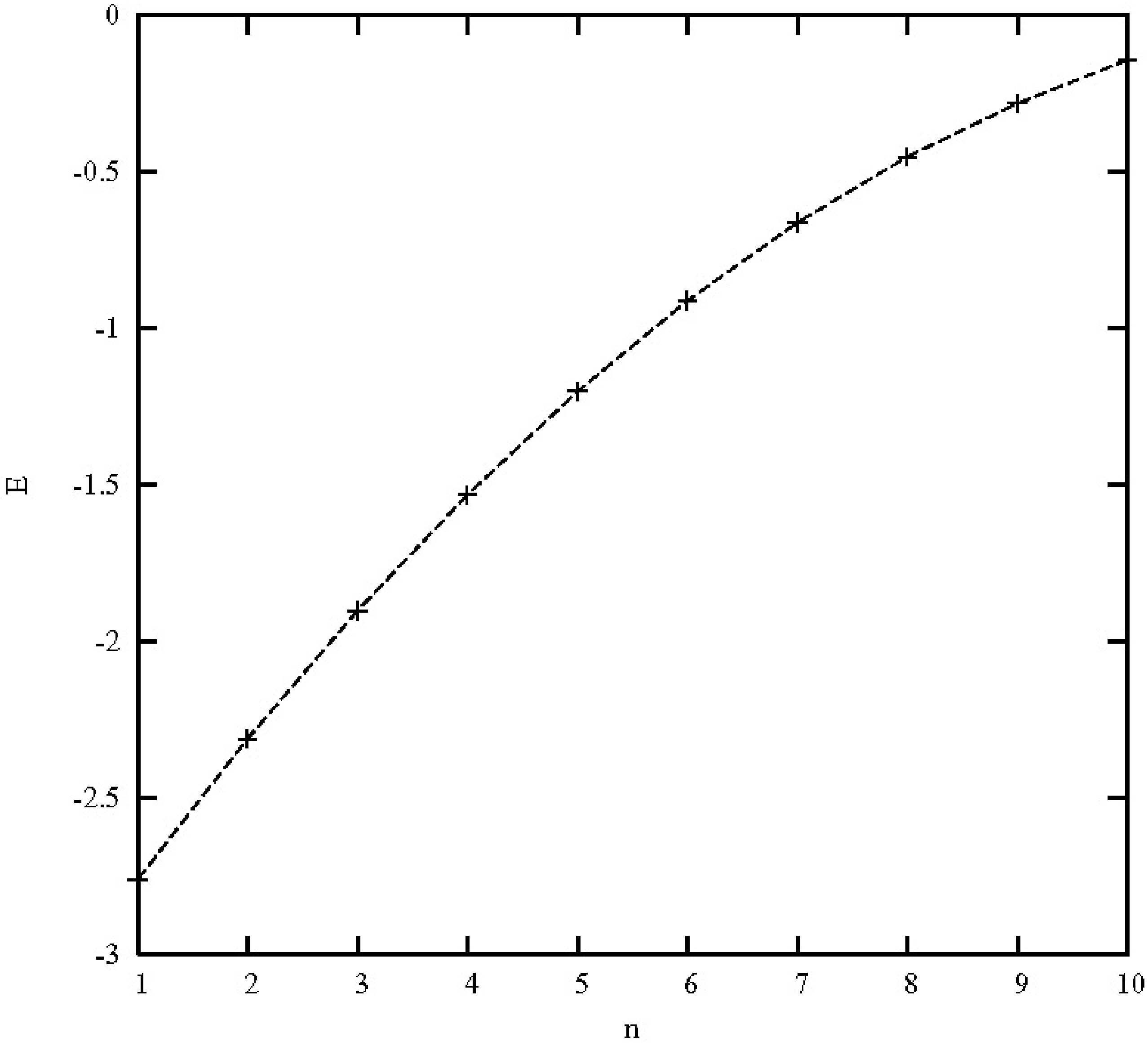
Harmonic Oscillator



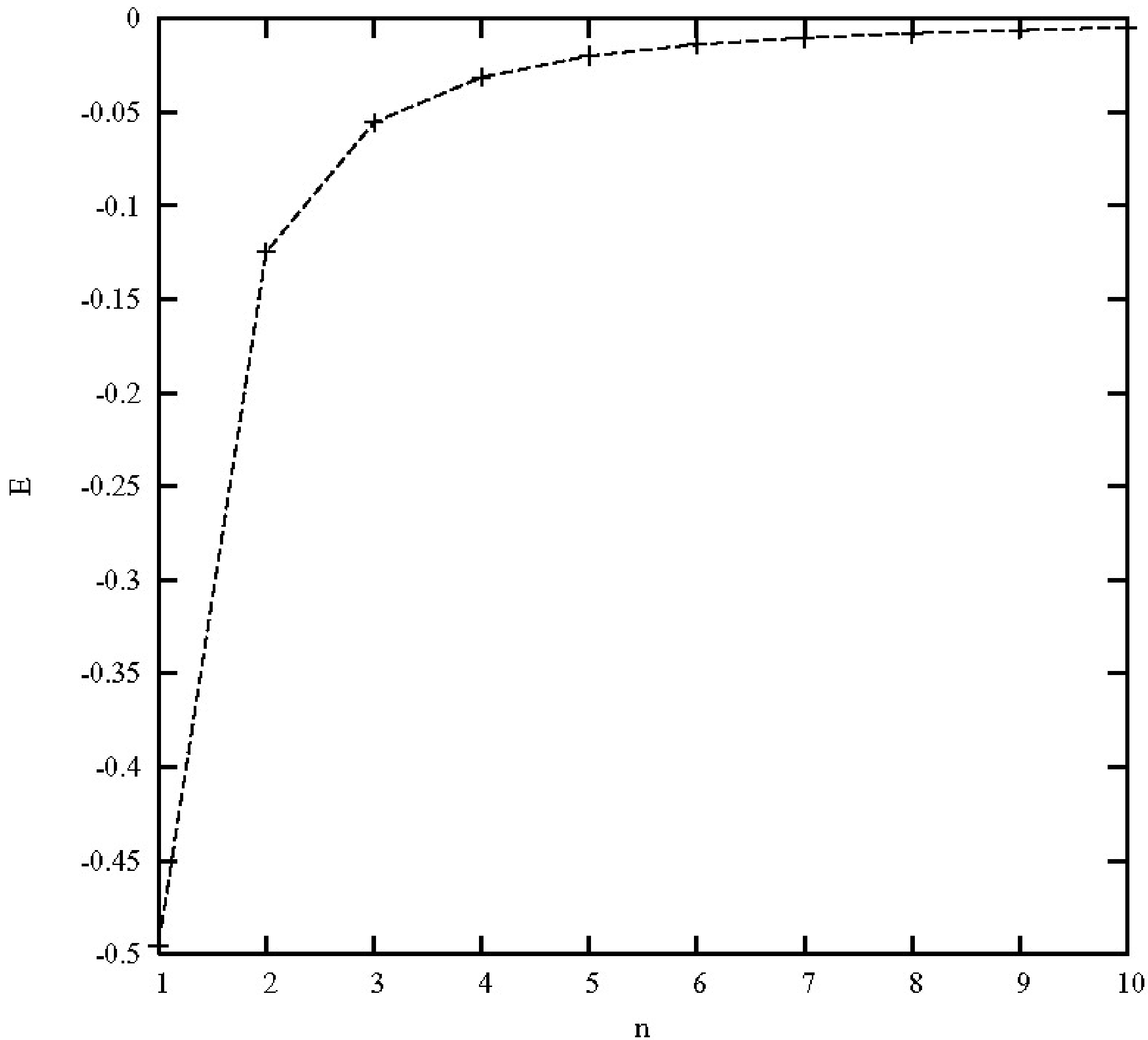
Linear Potential, Wall x=0



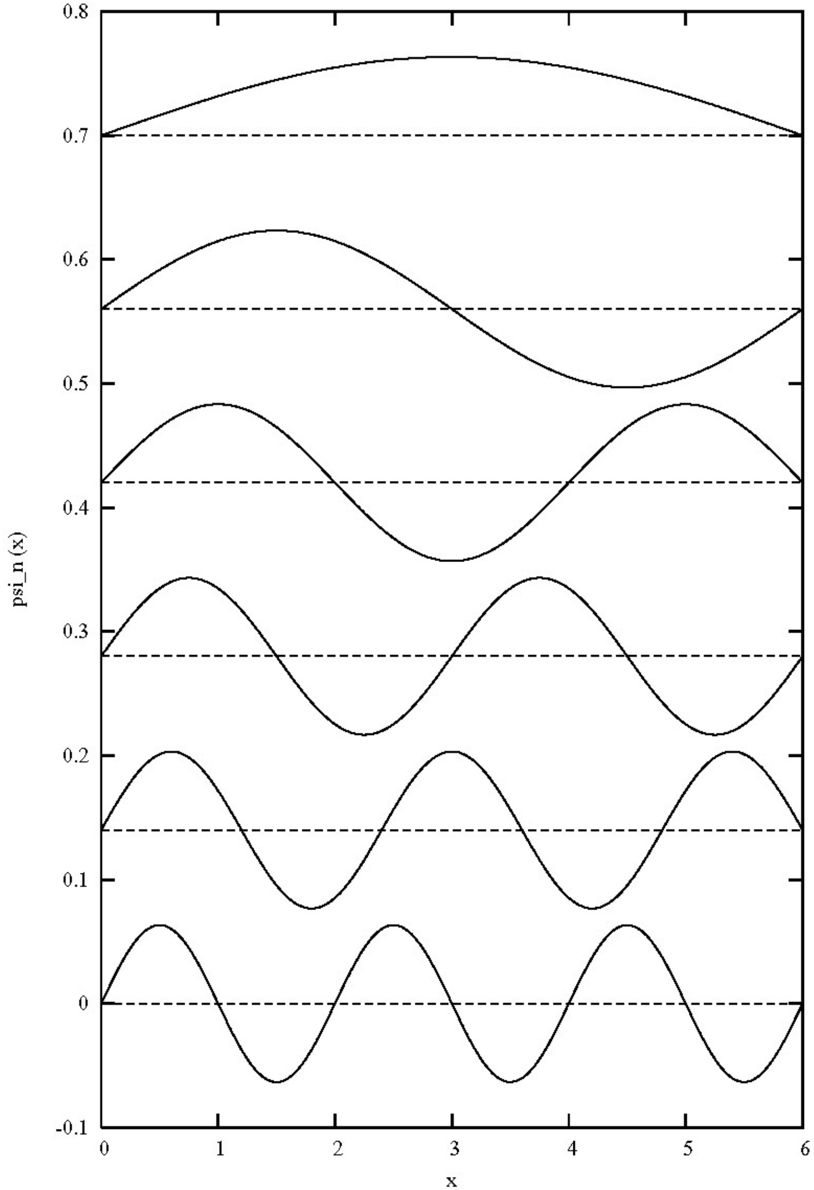
“Molecule” Vibration



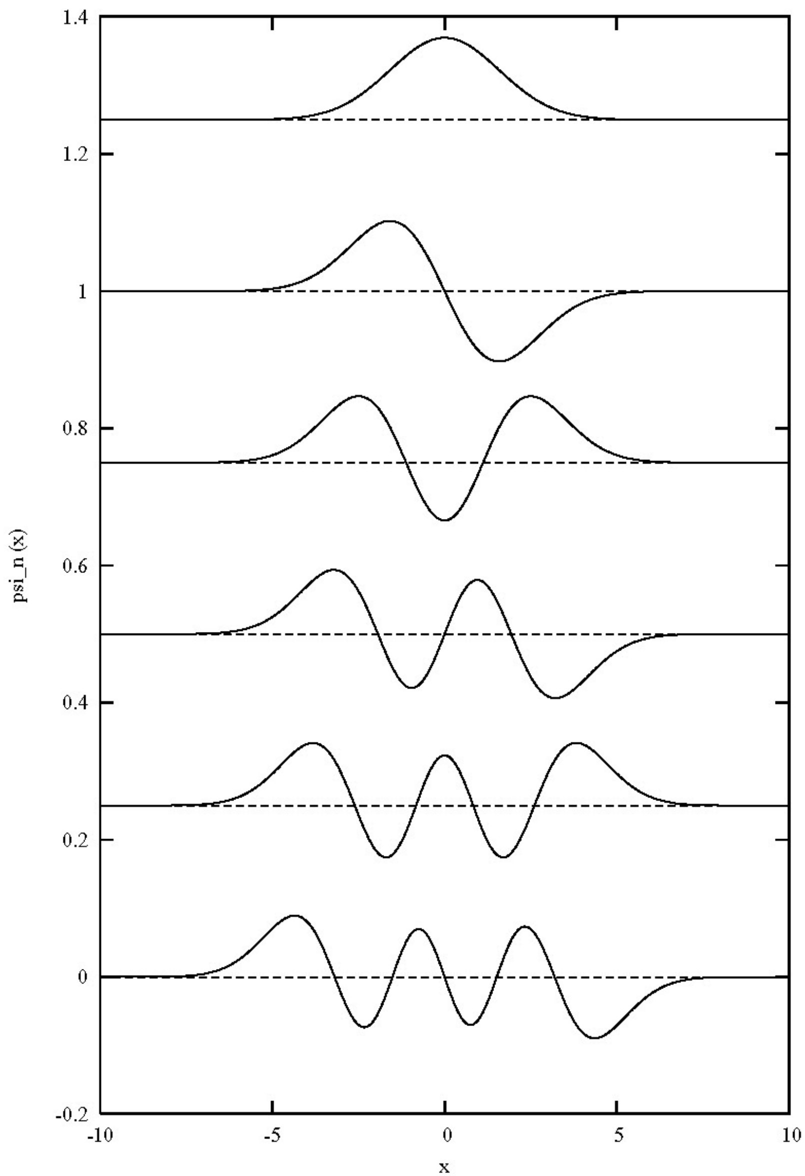
Coulomb Potential (-1/r)



Infinite Square Well



Harmonic Oscillator



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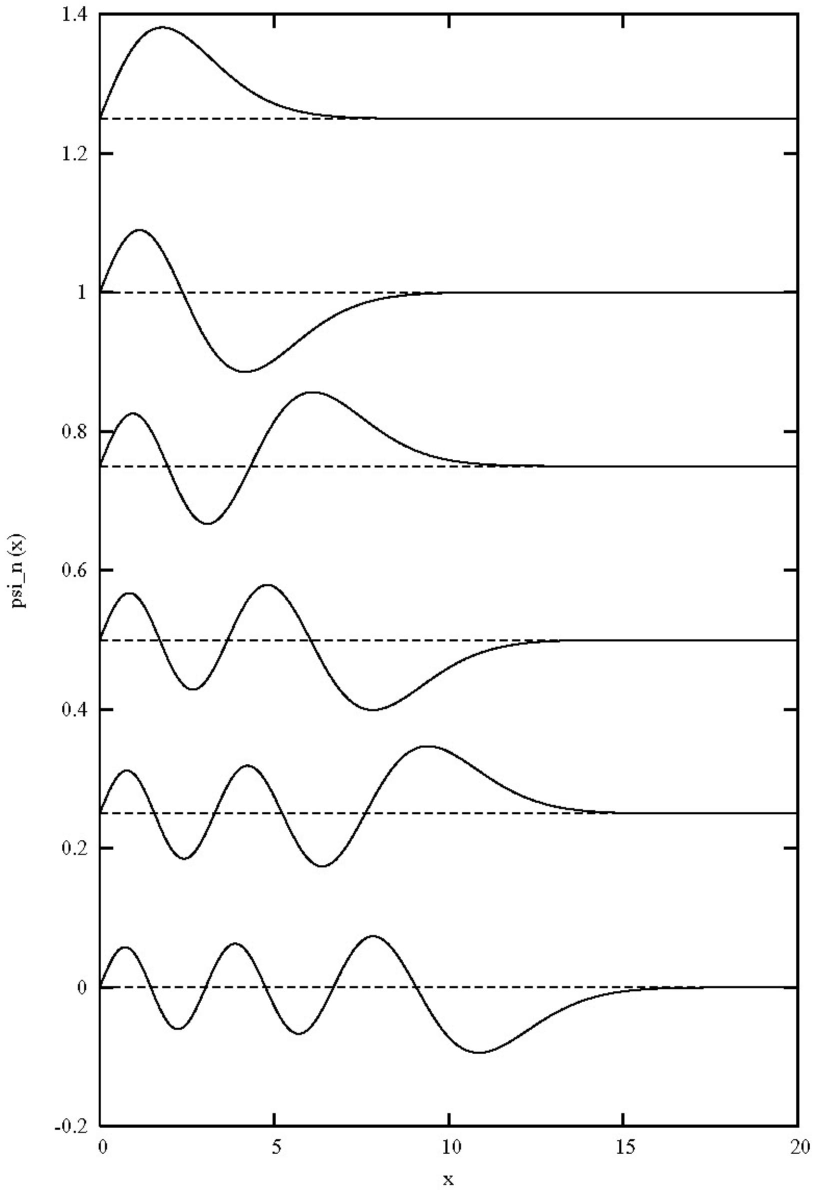

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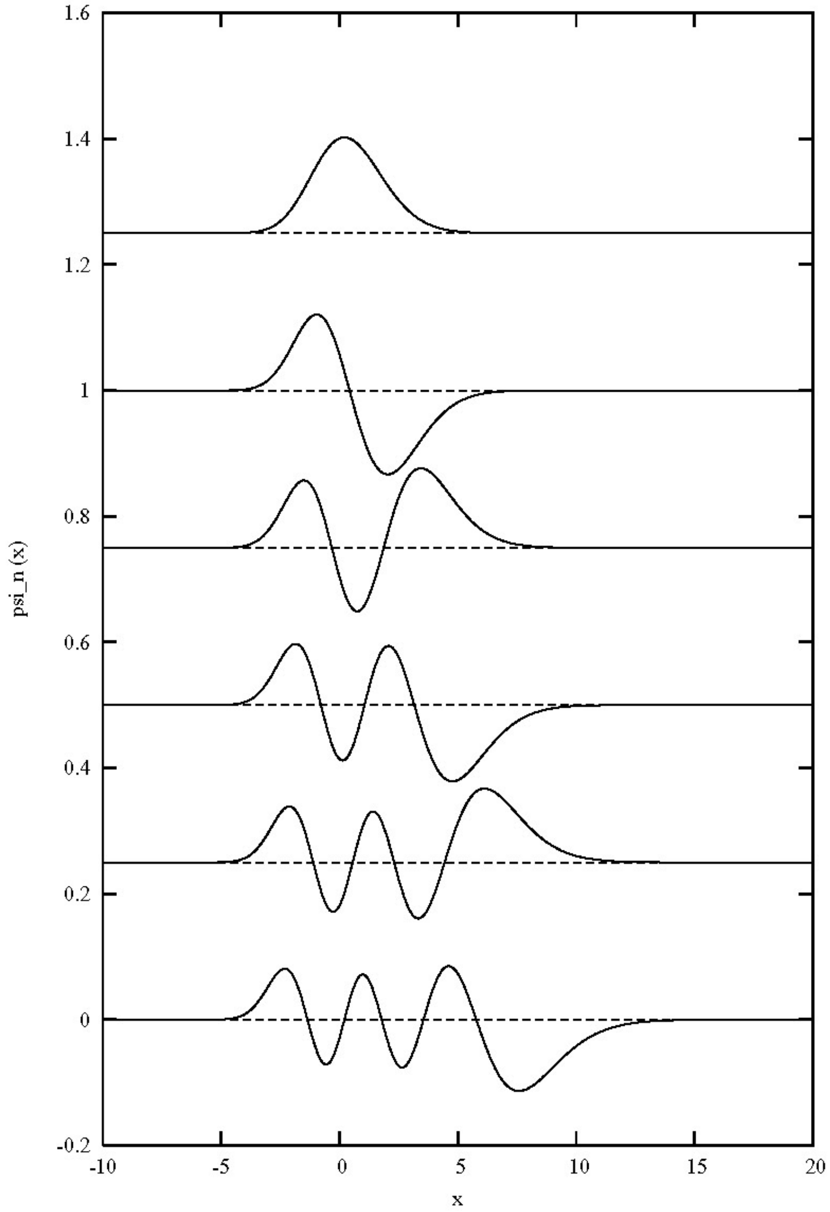

Coulomb Potential (-1/r)