Numerical Issues

"Inherent Accuracy" - A numerical method is almost always an approximation which has an accuracy that depends on the parameters.

For example, ① show $\frac{f(x+\delta x)-f(x)}{\delta x} = f'(x) + O(\delta x)$

② Compute $\frac{\sin(1+\delta x)-\sin(1)}{\delta x} - \cos(1)$ versus δx
Should see this goes to zero proportional to δx

Another example, ③ Show $\frac{f(x+\delta x)-f(x-\delta x)}{2\delta x} = f'(x) + O(\delta x^2)$

④ Compute $\frac{\exp(2+\delta x)-\exp(2-\delta x)}{2\delta x} - e^2$ versus δx
Should see this goes to zero proportional to δx

⑤ Compute derivative of $\sqrt{x}$; what is $f'(x=0)$?

"Round off error" float numbers have 8 significant digits
double numbers have 16 significant digits

⑥ Make 2 numbers $a = 1.0 + \delta x$ and $b = 1.0$
Print out $a-b$ versus δx for $\delta x = 1, 1E\text{-}1, 1E\text{-}2, \ldots 1E\text{-}16$

⑦ Use $f(x) = x$, this means $f'(x) = 1$; compute $\frac{f(2+\delta x)-f(2)}{\delta x} - 1$ versus δx

⑧ On a log-log scale plot $\frac{\sin(1+\delta x)-\sin(1)}{\delta x} - \cos(1)$ versus δx

⑨ Repeat for $\frac{\sin(1+\delta x)-\sin(1-\delta x)}{2\delta x} - \cos(1)$ versus δx