Quantum gravity and string theory

Primary sources:

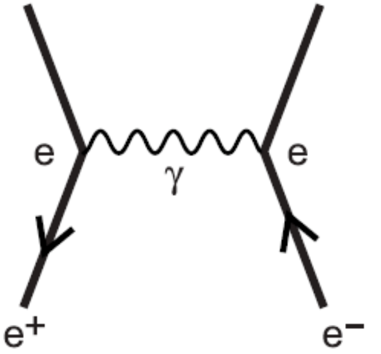
Steven B. Giddings, *SLAC Summer Institute (SSI04),*
http://www.slac.stanford.edu/econf/C040802/papers/L014.PDF

Joseph Polchinski (SSI98), arXiv:hep-th/9812104 v1 ()

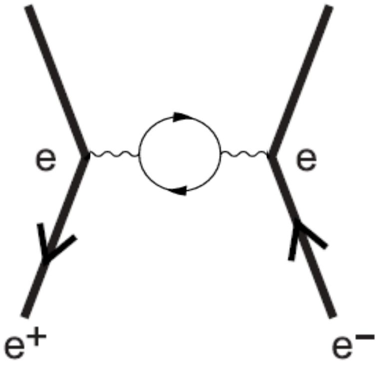
Barton Zweibach, A First Course in String Theory, Cambridge U Press (2004)

Quantum corrections, divergences, and renormalization

- A typical amplitude contributing to a scattering process, eg in QED, is $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering).
- Higher order quantum corrections will diverge, but all the infinite terms can be absorbed into the lowest order coupling e, resulting in finite (and energy-dependent, or running) couplings
- This means that QED (and the strong and electroweak SM theories) are renormalizable. Gauge invariance assures that the only higher order terms are the ones that only renormalize the couplings.
- This doesn’t work in quantum theories of gravity.

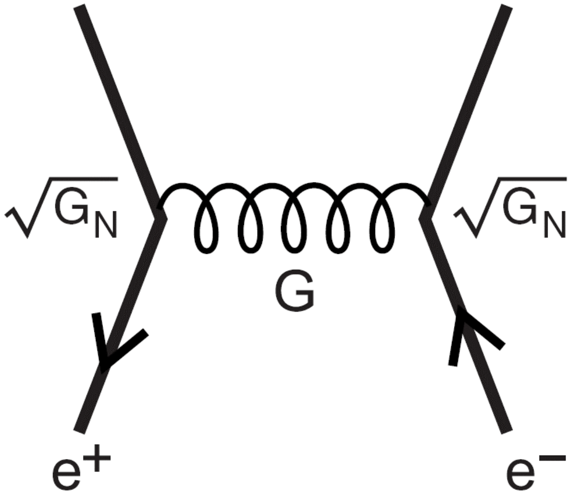


$$\mathcal{A}^{EM}_{e^+e^-} \propto e^2 = \alpha$$



$$\mathcal{A}^{EM}_{e^+e^-} \propto \alpha(E^2)$$

Quantum gravity



- Gravity contributes to any scattering process.
- GR has a prescription for incorporating gravity into the classical (or quantum, SM) Lagrangian $\mathcal{L}(\psi_e)$.
- The Lagrangian has terms that are written as dot products of various 4-vectors, like $g\, J_1^{\mu} J_{2\mu} = g\, J_1^{\mu} J_2^{\nu}\, \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the flat spacetime Minkowski metric
- To incorporate gravity, the GR prescription is to replace the Minkowski metric by a generalized metric that describes the curvature of spacetime; and then add a “kinetic energy” term for gravity, in the curvature scalar $\mathcal{R}$.

$$S_{\text{grav}} = \int d^4x \sqrt{-g}\left[\frac{\mathcal{R}}{G_N} + \mathcal{L}(\psi_e, g)\right]$$

- Since gravity is so weak, we can expand g about the Minkowski metric

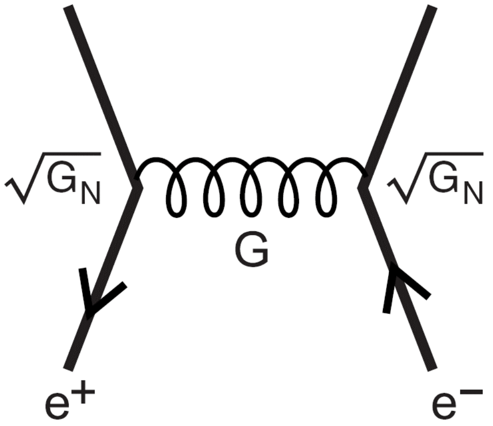
$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_N}\, h_{\mu\nu}$$

- Then, the gravity part of the Lagrangian linear in $h_{\mu\nu}$ looks like:
  where $\mathrm{T}_{\mu\nu}$ is the stress tensor
  for the electron field

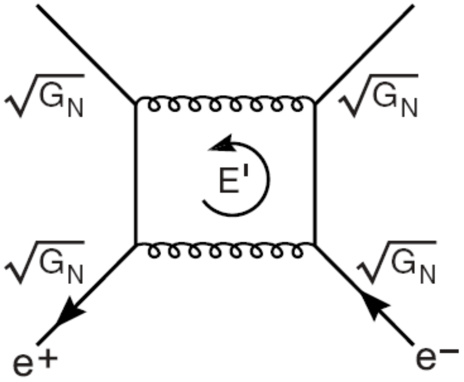
$$\sim \int d^4x \left[h^{\mu\nu} \Box_K h_{\mu\nu} + \sqrt{G_N}\, h^{\mu\nu} T_{\mu\nu} + \mathcal{O}(h^3)\right]$$

Divergent amplitudes

- The "tree level" amplitude for e^+e^- scattering via graviton exchange must be proportional to $G_N \sim 1/M_{Pl}^2$
  but it must also be dimensionless, which means it must also contain a factor involving the energy scale of the scattering process, E.
- Because E is so small compared with $M_{Pl} \sim 10^{19}$ GeV in any scattering process we have studied so far, gravity is very weak. We live in a low energy world!
- At sufficiently high energy, higher order corrections will become important. These will have more powers of G_N and will have integrals of the energy circulating in loops.
- These higher order corrections are badly divergent.
- At higher order, we have more powers of G_N, more powers of loop energy, and worse and worse divergences.
- There will be an infinite number of divergences, too many to be absorbed into a small number of parameters in the Lagrangian.



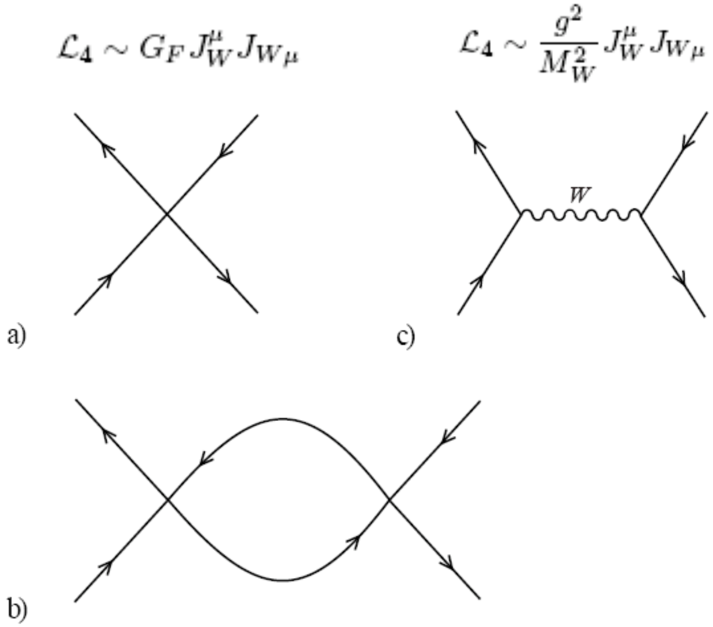
$$\mathcal{A}^{\text{grav}}_{e^+e^-} \propto G_N E^2 = (E/M_P)^2$$



$$\mathcal{A}^{\text{grav},1}_{e^+e^-} \propto G_N^2 E^2 \int dE' E'$$

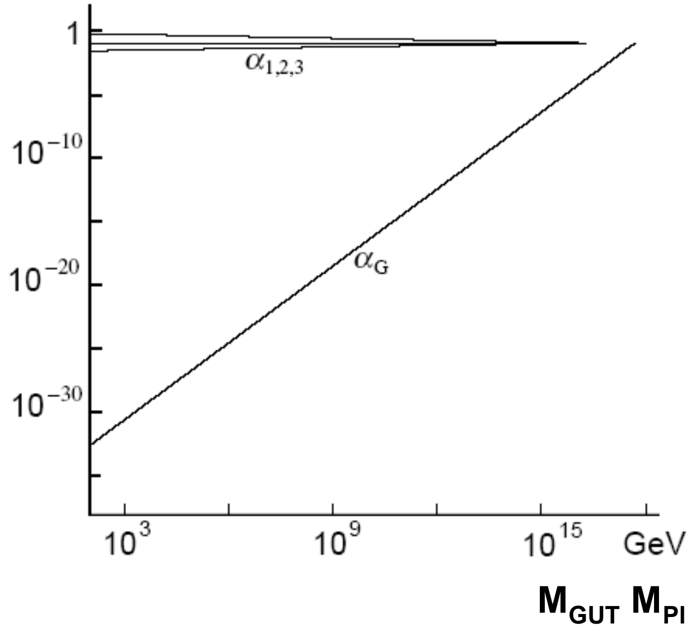
Effective theories

- The same thing happens in the Fermi theory of the weak interactions, with amplitudes like $G_F E^2$.
- But that is a low energy effective theory; the underlying theory contains a massive gauge boson, with $G_F \sim g^2/M_W^2$.
- At energies $E >> M_W$, the amplitude falls again. $\sim 1/(p^2 + M_W^2)$.
- The “new physics” (the W boson) acts to spread out the coupling in spacetime, softening the high-energy (short-distance) behavior.
- This is a unique solution, and it only works consistently because of gauge invariance and spontaneous symmetry breaking.
- We know of no analogous process for gravity, but it is clear that our low energy version of quantum gravity is only an effective theory, and new physics must come into play before we reach the Planck scale.



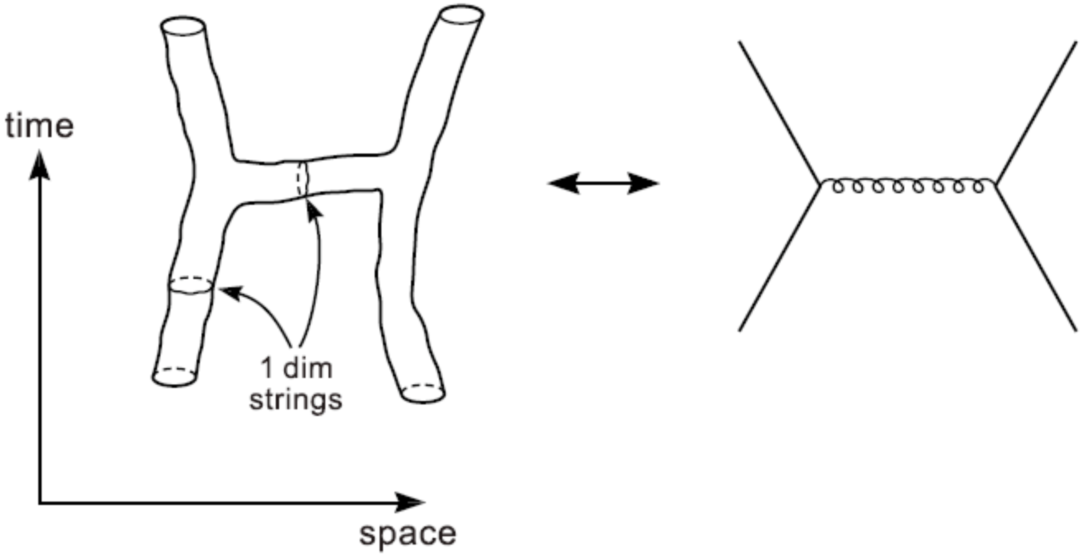
Why worry about gravity in the regime where $E > M_{Pl}$?

- Eventually, we can imagine studying e^+e^-scattering at energies $E > M_{Pl}$. The cross sections cannot grow without eventually violating unitarity.
- If models of extra dimensions are correct, we may not even be too far from this regime.
- In the early moments of the big bang, energy densities were arbitrarily high. We need a theory to describe this regime that does not contain infinities.
- In and near black holes, gravity is arbitrarily strong. We need a theory to describe this regime that does not contain infinities.
- Perhaps a complete quantum theory of gravity will help us understand the observed value of the cosmological constant.



The string theory solution

- The infinities that plague the higher order corrections to processes described by quantum field theory (except in special cases where the theories are “protected” by symmetries, especially gauge symmetry) have their origin in the fact that the fields couple at a single space-time location (ie, are local couplings).
- String theory avoids this by describing particles as excitations of strings. The string couplings occur over an extended spacetime region, providing a natural cut-off at short distance (high energy) characterized by the size of the string.
- We think of strings as infinitely thin filaments of energy, which can (in its various excitations) represent (at distances / resolution much greater then the string length, or energies much less than the string energy scale) electrons, photons, gravitons, etc.

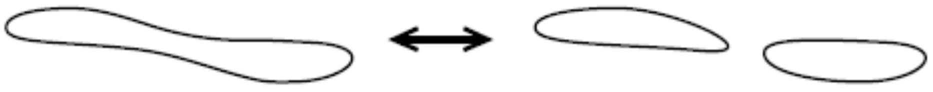


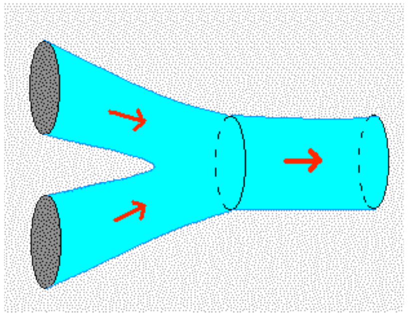
The rules of string theory

- The string length scale $L_S \sim 1/M_S$ is presumably too small to be observed so far; M_S is higher than ~ 1 TeV.
- Strings are 1-D extended objects. As they move through spacetime they sweep out world-sheets of some area.
- Amplitudes describing a scattering or decay process between initial and final states are given by extending Feynman’s sum-over-histories (all possible configurations of fields connecting |i> to |f>) to a sum over all possible string worldsheets interpolating between the string configurations in |i> to |f>. The area of the worldsheet takes the role of the action.

$$\mathcal{A}_{i\to f} \sim \int \mathcal{D}(worldsheets)\, e^{i\,Area}$$

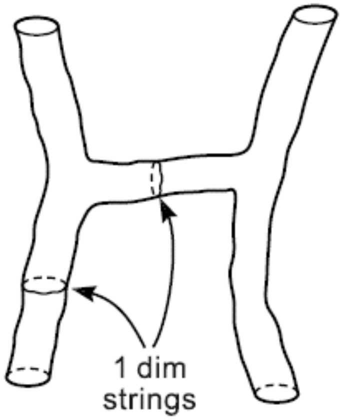
- The basic interaction, for a closed string, is a joining or splitting. This one interaction, depending on the states of the strings involved, can look like any of the interactions in nature: gauge, gravitational, Yukawa.

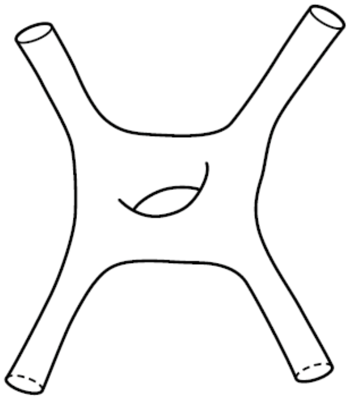


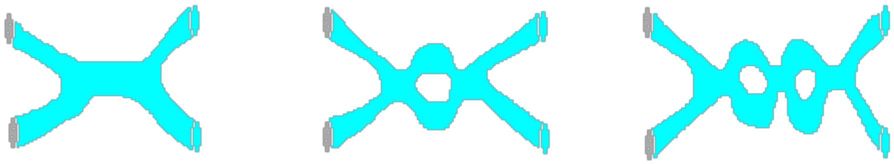


Amplitudes for scattering processes

- Applying this rule to the scattering diagram shown yields a result which is consistent with the exchange of a massless spin-2 boson. In other words, string theory automatically includes (weak field) gravity.
- Higher-loop diagrams are finite; no divergences. there are no singular space-time points where fields couple; the interaction is smoothed-out at the string scale.
- This is related to certain duality symmetries, which relate potential UV divergences to finite IR behavior.
- This continues to higher loops. String theory is UV finite order-by-order in perturbation theory. A renormalizable theory of gravity!

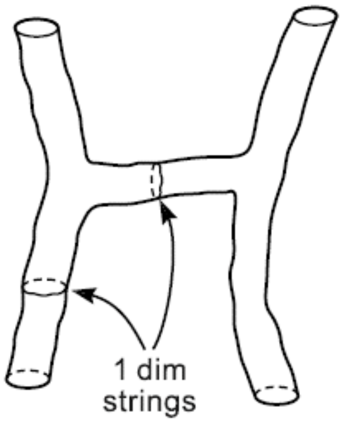


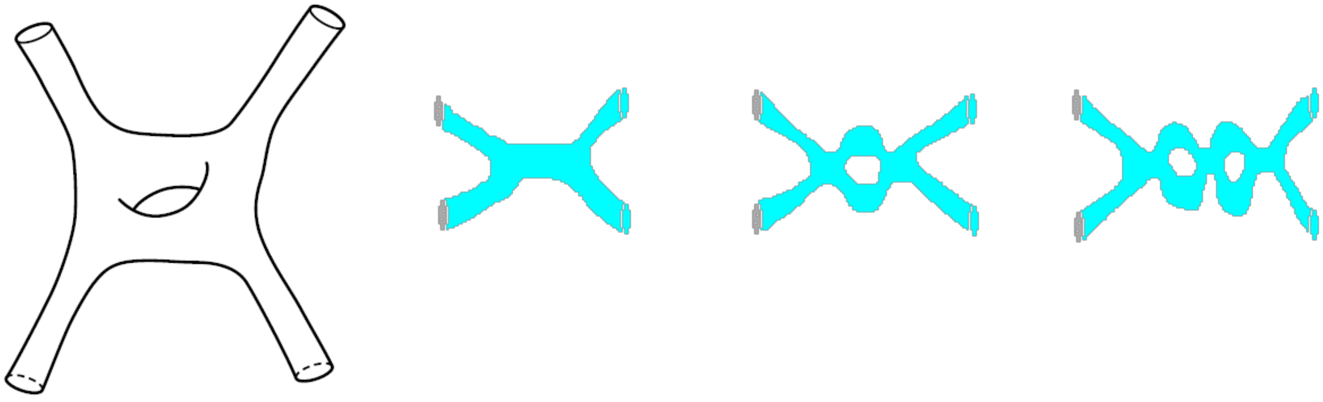




String theory in 26, um, 10 dimensions

- Well… some loop diagrams (anomalies) still diverge. This also happens in the SM, but in our 4-D spacetime, the “triangle” anomalies cancel when *all* the quarks and leptons are included in the loops (generation by generation). This is the primary evidence in the SM that quarks and leptons are related.
- In string theory, the divergent anomalous diagrams cancel only when the theory is expressed in 26 dimensions.
- In the supersymmetric extension, the cancellation occurs in 10 dimensional theories.



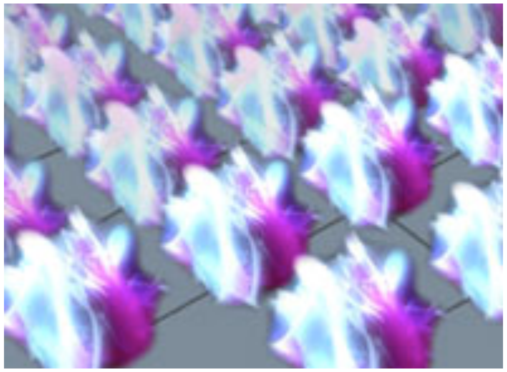


How about gauge bosons, fermions, etc?

- Closed strings describe gravitons. Open strings, with endpoints, have charges at the ends of the strings. Those charges obey symmetry rules.
- Evaluation of scattering amplitudes containing open strings results in states that are consistent with non-Abelian gauge bosons that couple to the charges at the endpoints.
- String theory has a tight mathematical structure, and only a handful of theories are mathematically consistent, free (in some number of dimensions) of divergent quantum anomalies.
- For now, this does not lead to predictive power because the theory has many vacuum states, with different physics.
- Consistent string theories are all supersymmetric; they contain both fermions and bosons, and symmetry operators that transform the one into the other. String theory *predicts* supersymmetry.
- So, the simple assumption that matter is composed of strings, not particles, gives us:
  - a renormalizable theory of gravity (but only in 10 spacetime dimensions)
  - gauge symmetry and gauge bosons
  - supersymmetry, and fermions (but, in general, too many of them, and no explanation of their patterns within and between generations)

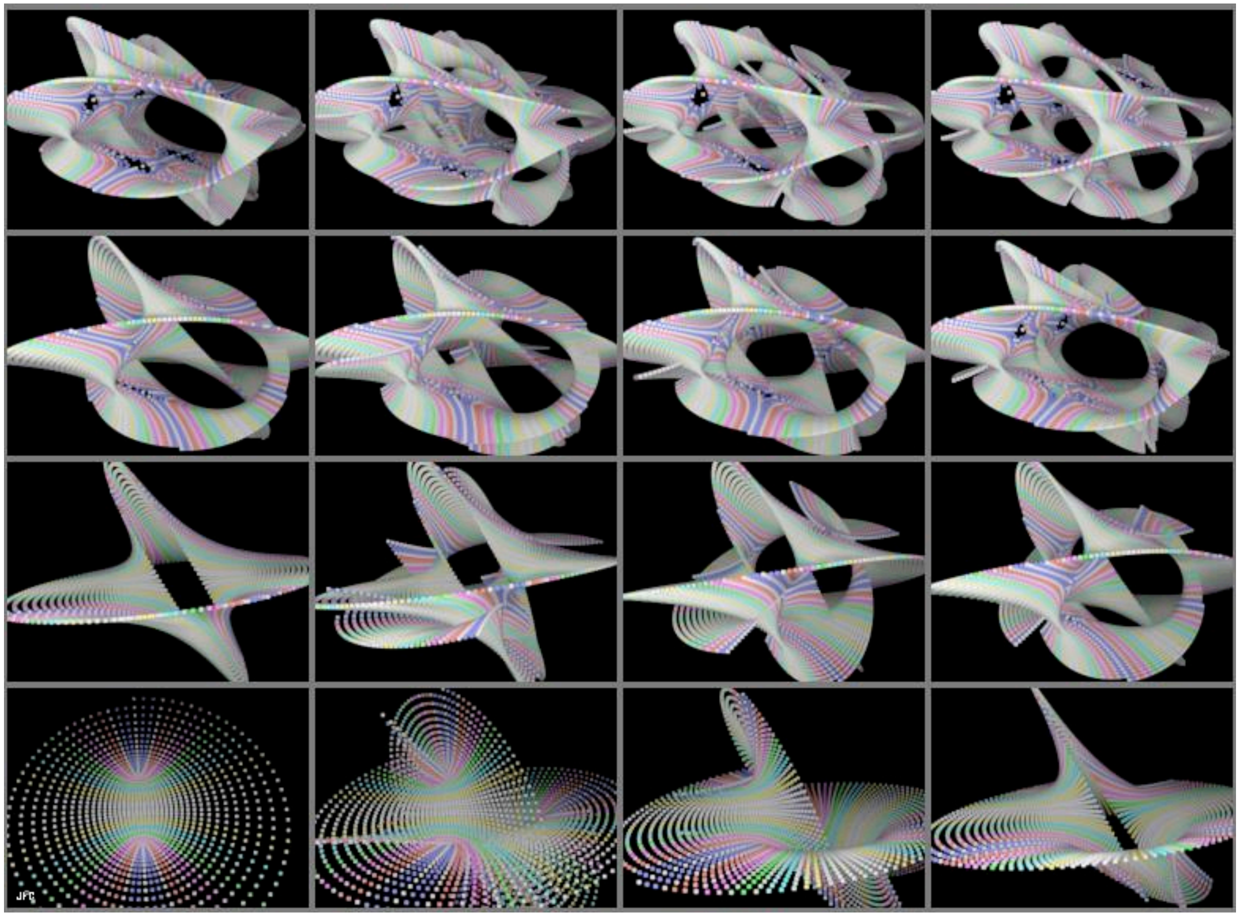
Compactifying the extra dimensions

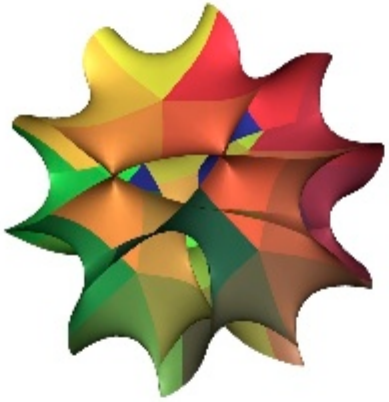
- We assume that 6 of string theory's 10 dimensions are compactified with characteristic size $R_S << 1/\text{TeV}$, so that we don't (yet) have sufficiently energetic probes to see them.
- This breaks some of the symmetries in the theory, resulting in far fewer fermions and gauge bosons in the low-energy 4-dimensional world. The pattern of symmetry breaking is highly constrained.
- Remarkably, it produces symmetry groups that are known to be viable candidates for grand unified theories (such as SO(10)), which contain all the quarks and leptons (and more, but not a lot more).
- The curled-up 6-dim compactified spaces, known as Calabi-Yau manifolds, can have KK-like excitations; in some models, these can lead to replicated multiplets like the SM generations.
- There are also scalar fields that can play the role of the Higgs.
- Starting to look like a Theory of Everything!

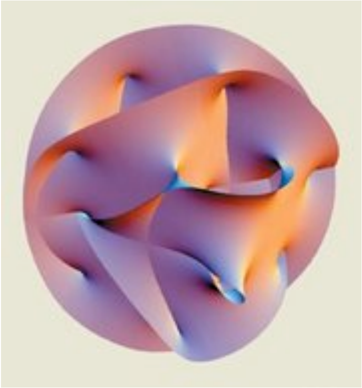


Calabi-Yau manifolds

Compact, complex super-spaces with nontrivial topologies







Entropy of black holes

- Black holes emit Hawking radiation, as if they were a black body with temperature $k_B T = \hbar c^3/(8\pi G M_{BH})$ and entropy $S = k_B A/(4L_{Pl}{}^2)$, where $A = 2\pi(GM_{BH}/c^2)^2$ is the surface area of a (non-spinning, uncharged) black hole (at its horizon) of mass M_{BH}, and L_{Pl} is the Planck length.
- $T = 6\times10^{-8}$ K $\cdot$ $(M_\odot / M_{BH})$, lifetime = 10^{71} seconds $\cdot$ $(M_{BH} / M_\odot)^3$
- Temperature and entropy are purely thermodynamic (macroscopic) properties, but in quantum statistical mechanics, they can be calculated by considering the microstructure. The entropy is k_B ln(number of microstates).
- In 1995, Strominger and Vafa calculated the right Bekenstein-Hawking entropy of a supersymmetric black hole in string theory, using methods based on D-branes.
- Their calculation was limited to extremal charged BHs; but it was followed by many similar computations of entropy of large classes of other extremal and near-extremal black holes, and the result always agreed with the Bekenstein-Hawking formula.

Problems with realistic string theory

- There are many (topologically distinct) ways to compactify the 6 dimensions, producing many different low-energy effective theories and mass spectra.
- Compactifications can vary continuously, with no potential barrier; there could be different compactifications at different 4-D spacetime locations.
- The different compactifications are connected by moduli fields, and threre are no constraints on them.
- They will act like massless Goldstone bosons interacting gravitationally, giving “fifth forces”, time-dependent values of G_N, time-dependent low-energy theories, and other phenomena that are not observed.
- There are new ideas about how to fix the moduli fields and the problems they create. An active field of research!

The cosmological constant

- String theory has many approximately stable vacua, corresponding to different shapes and sizes for the rolled-up dimensions. The physics that we see depends on which of these vacua we are in.
- As the universe expands and cools from the Planck phase, and the d-3 spatial dimensions compactify, various mechanisms can cause energy to be trapped into the compact dimensions, and play the role of a cosmological constant.
- Typical values of the cosmological constant Λ are of order 10^{120} (in units of present closure density), but the energy trapped during the Planck transition can be anything from 0 to 10^{120}.
- There are a huge number of ways in which this can happen… lots more than 10^{120}. The space of possible initial conditions (vacua) predicted in string theory is called the *landscape*; at each point in that space, there is a different vacuum in a different universe. Ours is one.
- Of course, in universes where the cosmological constant is close to 1 or higher, the universe expands too fast for galaxies to form, so life presumably wouldn't evolve.
- Since the distribution of cosmological constants is presumably dominated by the largest allowed value, this could serve as an explanation for the observed value of Λ, arising from the *anthropic* principle.
- Evocation of the anthropic principle applied to this landscape of possible universes can therefore solve one "fine-tuning" problem. Maybe it can also be applied to other fine tuning problems in high energy physics!