The “origin of mass” in the Standard Model

Extension to Non-Abelian theories and the electroweak interaction

- In the SM, the Higgs mechanism generates masses for a *triplet* of weak gauge bosons (W_μ^+, W_μ^0, W_μ^-), which form a *weak isospin* triplet.
- Isospin, like spin, are eigenstates of the rotation (angular momentum) operators in SU(2), except in an abstract *internal space* rather than coordinate space (but each of the fields in the triplet are vector fields, spin-1 in coordinate space, hence the subscript $_\mu$).
- More precisely, the W^0 and another gauge boson field B^0 (an Abelian U(1) field) mix together to form the Z^0 and the A^0; the Z^0 gets a mass (91 GeV) while the A0 remains massless (the photon). In this sense, the weak and electromagnetic interactions are deeply related (unified).
- The angular momentum operators that generate weak isospin rotations do not commute; the theory is non-Abelian.
- This adds extra mathematical complexity to the description of gauge invariance, spontaneous symmetry breaking, and the Higgs mechanism.
- But conceptually, it’s a straightforward extension of the Abelian theory; you just have to extend your thinking to include abstract internal spaces, and multiplets of fields that transform in a well defined way under rotations in those internal spaces.
- So in this class we won’t bother developing the non-Abelian mathematics; leave that to a course in particle physics.
- For the Higgs mechanism, the most important consequence is that we will need (at a minimum) a weak-isospin-double of complex fields, for a total of four real fields.
- Three get “eaten” by the gauge bosons, giving them mass; the fourth is the physical, massive Higgs Boson. This is the simplest way to do it, not the only way!

Massive vector bosons

- The “massless” intermediate vector bosons interact with the Higgs VEV as they swim through it; this “slows them down”, so that they behave as if they have mass.
- The otherwise massless “chiral” fermions also couple to the Higgs ether and develop a mass.
- This is directly analogous to the “effective mass” gained by electrons (and holes) in a crystal, due to the more complex energy / dispersion relation E(k) as the electrons swim through the crystal and polarize it.
- Helicity = projection of spin along direction of motion.
  - Spin-½ particles can have two helicities: R (|m=-½>) and L (|m=+½>).
  - Vector (S=1) bosons can have three helicities (-1, 0, 1).
  - But massless vector bosons can only have two helicity states: R (|m=-1>) and L (|m=+1>).
  - This is a consequence of current conservation (for massless vector fields like the EM photon).
  - The helicity=0 state is the longitudinal state; it’s non-existence means that the electric field is always perpindicular to the direction of motion of the wave: E·p = 0, the E field is transverse, not longitudinal
  - But in a polarizable medium like a crystal, a longitudinal component can exist; light develops an effective mass and moves slower than the speed of light in vacuum.
  - The Higgs is like a polarizable medium (in the Standard model).

Massive vector bosons

- Massless particles travel at the speed of light; it is not possible to boost into a frame in which the particle is moving in the other direction. Helicity is a conserved quantum number. L and R-handed particles are forever distinct and uncoupled – chiral symmetry.
- Massive particles can't travel at the speed of light; it is possible to boost into a frame in which the particle is moving in the other direction. L and R-handed helicity are not good quantum numbers, the states are coupled – chiral symmetry is broken.
- In the Lagrangian, mass terms are of the form: $m \, \psi_L \, \psi_R$
- The intermediate vector bosons of the weak interaction "eat" the Higgs VEV, get fat (massive), and pick up a longitudinal helicity component (which *is* the Higgs).
- In this sense, the Higgs has already been discovered, even if it doesn't exist…
- Because they're massive, the W and Z particles can (very quickly) decay, and are very short-lived; this is the origin of the very short range, and feeble strength, of the weak interaction. The production and decay (and indeed, all the detailed properties) of these particles have been studied with high precision – the theory is right!

The left-over Higgs particle

- Because we have three massive vector mesons (W^+, W^-, Z^0), there must have been three Higgs fields that were “eaten”.
- But they can’t be three components of (eg) a vector field. They have to be (complex) scalar fields. Which come in pairs. So there must be a fourth, which is not eaten, and which is (in the simplest theory, the SM) is a spin-0 (scalar) neutral boson with mass.
- We are (inevitably?) stuck with The Higgs particle (AKA God), for which there is no (direct) evidence. and as we will see, this creates a host of problems! (Sound familiar?)
- NOTE that there’s nothing in the theory that requires the Higgs to be elementary; it could be a composite object, or be generated “dynamically” (Technicolor). This is how it happens in the BCS theory of superconductivity, where Cooper pairs of electrons play the same role.
- This underscores how little we know about the Higgs mechanism!

The left-over Higgs particle

- Is there any other way to retain these gauge particles and give them mass? Not that anyone has thought of, and (using arguments that I am not well versed in) it seems like something like the Higgs mechanism is the *only* way.
- The prediction of an otherwise-not-looked-for particle to solve a problem in the mathematical structure (symmetry) of a physical theory is not unprecedented: Dirac struggled with the negative-energy solutions of his Lorentz-invariant equation (from $E^2 = p^2 + m^2 \Rightarrow E = \pm (p^2 + m^2)^{1/2}$) until those solutions were re-interpreted as antimatter. This is a direct consequence of Lorentz invariance, and it doubled the number of particles. Here again, history repeats itself: supersymmetry does the same.

Summary of the origin of EW symmetry breaking

- When I was a student, I was told that the goal of elementary particle physics was to learn the basic laws of the strong and weak interactions.
- Today, this is a solved problem. These laws are explained by the “Standard Model”: The strong, weak, and electromagnetic interactions are described by a Yang-Mills gauge theory with the gauge group SU(3) x SU(2) x U(1) .
- But, in science, the solution to every problem leads to new questions at a deeper level.
- To describe the weak interactions in a Yang-Mills theory, the gauge symmetry must be spontaneously broken. We know that this happens, but we do not know why.
- The agent of electroweak symmetry breaking represents a novel fundamental interaction at an energy of a few hundred GeV. We do not know the nature of the new force.
- This is one of the great puzzles in modern science. It is the key to further progress in microscopic physics. You are likely to solve this problem during your careers, from discoveries at the next generation of accelerators.
- But just understanding the origin of the puzzle, the origin of mass, is enough challenge for this class.

If electroweak symmetry were not hidden . . .

- Quarks and leptons would remain massless
- QCD would confine them into color-singlet hadrons
- Nucleon mass would be little changed, but proton outweighs neutron
- QCD breaks EW symmetry, gives (1/2500×observed) masses to W, Z,
- so weak-isospin force doesn’t confine
- Rapid β-decay $\Rightarrow$ lightest nucleus is one neutron; no hydrogen atom
- Probably some light elements in BBN, but ∞ Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite
- structures like the solids and liquids we know

. . . the character of the physical world would be profoundly changed!

What is the Higgs?

What is the nature of the mysterious new force that hides electroweak symmetry?

A fundamental force of a new character, based on interactions of an elementary scalar field whose nature is presently unknown.

- A new gauge force, perhaps acting on undiscovered constituents?
- A residual force that emerges from strong dynamics among the weak gauge bosons?
- An echo of new symmetries or extra spacetime dimensions?

Which path has Nature taken?

The essential step toward understanding this new force that shapes our world:

- Find the Higgs boson and explore its properties.
- Is it there? How many Higgs bosons are there?
- Verify the spin and parity of the Higgs boson(s)
- Does the Higgs generate mass only for gauge bosons, or also for the fermions?
- How does the Higgs interact with itself?

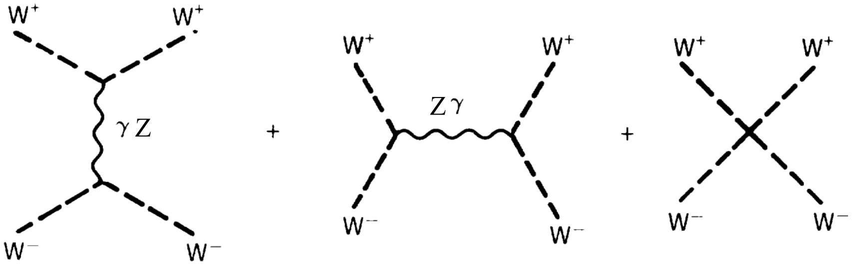
Finding the Higgs boson starts a new adventure!

The Higgs Boson

The standard model requires that at least one scalar particle exist.
This particle, known as the "Higgs" (after Peter Higgs) does two things:

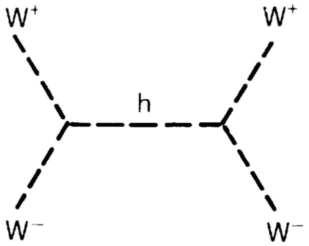
a) makes the theory renormalizable
b) "generates" the masses of the W, Z, and fermions

Renormalizable means that (e.g.) that scattering amplitudes and cross sections will be finite at high energy. Diagrams with the exchange of a virtual Higgs cancel other diagrams with virtual W's and Z's.



For example the cross section for $W^+W^- \rightarrow W^+W^-$ grows as E_{cm}^2! At a few TeV the cross section grows so large that is would violate unitarity (probability >1)!

The cross section can be made to be finite by adding diagrams (amplitudes) of the form:



Adding the Higgs amplitudes makes the total amplitude for $W^+W^- \rightarrow W^+W^-$ finite.

The Higgs Boson and Mass

In the minimal standard model the Higgs field is a scalar in an SU(2) doublet. Only one component of the doublet has to have mass. Thus there is only one massive Higgs particle in this model. The mass of this particle is given by:

$$M_H^2=2v^2\lambda$$

Both λ and v are constants.

But only one of them can be calculated from already measured quantities!

$$v = \frac{M_W \sin\theta_W}{\sqrt{\alpha\pi}} \approx 246\,\mathrm{GeV/c^2}$$

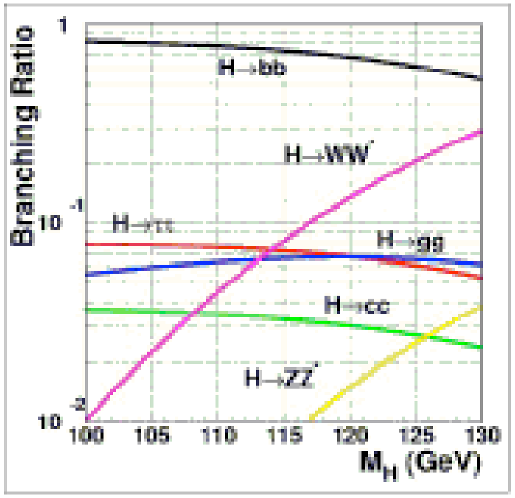
The mass of the fermions are related to the Higgs field.

The standard model Lagrangian contains terms of the form:

$$L_{\text{int}} = m_f f\bar{f} + \frac{m_f}{v} f\bar{f}H$$

The strength of the Higgs coupling to a fermion anti-fermion pair depends on the mass of the fermion. Thus we would expect Higgs to decay preferentially to the fermion with mass closest to $M_H/2$.

$$BR(H \to b\bar{b}) > BR(H \to \tau^+\tau^-) > BR(H \to c\bar{c}) \cdots > BR(H \to e^+e^-)$$

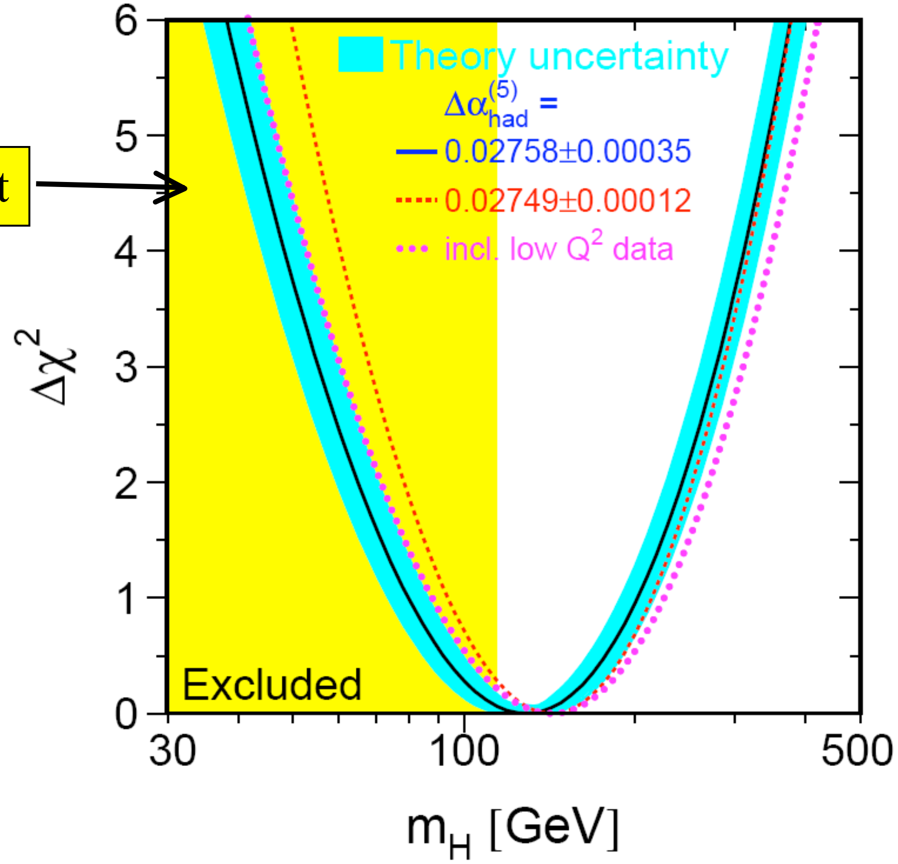


Where is the Higgs Boson?

Present experimental limits on the Higgs suggest M_H >114 GeV/c^2.
Constraints from theory predict a low mass Higgs (M_H ~ 130 GeV/c^2).

excluded by experiment

Mass of Higgs must be < 1TeV otherwise higher order corrections cause problems with the model.



Higgs may be discovered at Fermilab in next 1-3 years.
Will definitely be discovered (or ruled out) at LHC/CERN in 2-5 years.

Upper bound on Higgs mass

- Unitarity: the cross-section for as $E_{CM} \rightarrow \infty$ with $m_H = \infty$ violates the unitarity bound at CM energies ≥ 1.7 TeV unless in which case

$$\sigma_{M_H \to \infty}(e^+e^- \to W_L^+ W_L^-) = \frac{G_F^2 m_e^2}{16\pi}$$

$$\sigma_{S-wave}(unitary\ bound) = \frac{8\pi}{E_{cm}^2}$$

$$m_H < \sqrt{\frac{8\sqrt{2}\pi}{G_F}} = 1.7\ TeV$$

$$\sigma_{E_{cm} >> m_H}(e^+e^- \to W_L^+ W_L^-) = \frac{G_F^2 m_W^2 \sin^2\theta_W}{\pi}\left(\frac{m_W^2}{E_{cm}^2}\right)$$

- The Higgs quartic self-coupling runs; it becomes negative (driving the VEV to ∞) if m_H >> 200 GeV.

Quantum corrections

- quantum theory (quantization of a classical lagrangian) demands that there are higher order corrections to any physical process (collisional cross section, decay rate), beyond the “tree level” in the Lagrangian.
- There is a perturbation expansion, and higher order terms involving loops inevitably arise. Examples:
  - corrections to EM scattering -> renormalization of the electric charge, the running coupling constants in QED and QCD.
  - Real and imaginary parts of the mass propagator -> renormalized mass, absorption / decay -> exponential lifetimes or attenuation.
- These can be thought of as more complex configurations in a Feynman path integral from an initial state to a final state. They are important if they contribute to the action by an amount of order hbar or less.
- Many of these more complex configurations result in nothing more than changes to the (a priori unknown) parameters in the original Lagrangian, renormalizing them into physical (measurable) parameters, which now become a function of the distance or energy probed. This is renormalization. Renormalizable theories carry the guarantee that all complex configurations can be reduced in this way. Gauge invariance can guarantee renormalizability.

Dimensional regularization

- Some couplings (terms in the Lagrangian) are dimensionful, such as the ones in the Lagrangian for the Higgs sector. Higher order configurations give infinite results which cannot be absorbed into the parameters of the original Lagrangian.
- Such terms are non-renormalizable, infinite, non-predictive. Unless … they are effective terms in a deeper, more accurate theory at shorter distances.
- This was already seen in Fermi's theory of the weak interaction, in which quarks and leptons (Dirac fields with dimension (mass)$^{3/2}$) coupled in a "4-fermi" interaction. To get the dimensionality of the Lagrangian (mass)^4 right, the coupling constant G_F needs to be of dimension (mass)$^{-2}$.

$$L = \cdots + G_F \overline{\psi}_1 \psi_2 \overline{\psi}_3 \psi_4$$

- Immediately, this led to higher order diagrams which diverge quadratically, leading to meaningless corrections – ie, not a quantum theory (well, not a renormalizable one).
- It is immediately manifest in, eg, nue-e cross-section, which goes like $G_F^2 E_\nu$
- In the SM, this problem is fixed by finding a deeper (shorter-distance) theory involving the exchange of weak gauge bosons (W,Z), "explaining" G_F in terms of a dimensionless coupling g and a massive boson propagator.
- But the problem recurs in the Higgs sector, and recurs with a vengeance in gravity.

Divergence of the 4-fermi theory

$$\sigma(\nu_e e) = \frac{2G_F^2 m_e E_\nu}{\pi} \quad \times \quad \left| \frac{M_W^2}{2m_e E_\nu - M_W^2 + iM_W \Gamma_W} \right|^2$$

$$G_F/(\hbar c)^3 = \left(1.16639 \pm 0.00001\right) \times 10^{-5}\, GeV^{-2}$$

$$G_F/(\hbar c)^3 = \frac{g^2}{4\sqrt{2} M_W^2}$$

And because of the unification (mixing) of weak and EM interactions, g^2 can be related to the electromagnetic fine structure constant α_{EM} via the Weinberg mixing angle: $g^2 = \alpha / \sin^2 \theta_W$.
Since G_F , α_{EM}, and M_W are all very well measured, we can determine $\sin^2 \theta_W$. But we can also measure this a half-dozen other ways, all of which give excellent agreement!
Now that we know that G_F is only an effective coupling, and that the true coupling changes with energy, falling when $E > M_W$, there is no problem with a rising cross section at high energies.

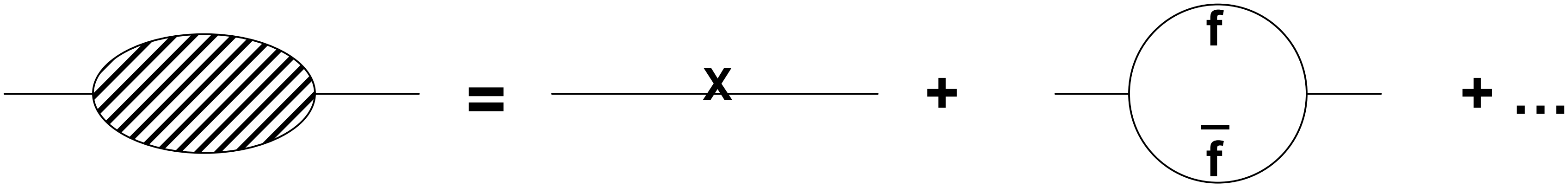
The Hierarchy problem in the SM, and beyond

- The EW symmetry is broken, via the Higgs mechanism, due to a VEV = $< \varphi >_0 = (G_F/\mathrm{sqrt}2)^{-1/2}$ = 246 GeV
- The mass of the Higgs is not strongly constrained, but should be of order ~100 GeV
- QFT demands that the mass of the Higgs (or any other particle) is modified from the “bare” value in the Lagrangian, by higher order corrections in a perturbative expansion.
- Mass corrections to all the fermions and gauge bosons in the SM are “manageable”, they can be *renormalized,* and the corrections do not introduce problems.
- Not so for the Higgs, or any other fundamental scalar. Due to its coupling to itself and to all the other particles in the theory, the mass corrections to the Higgs diverge strongly.

Higher-order corrections to the Higgs mass diverge quadratically

$$m_H^2 = m_0^2 + \delta m_H^2$$

$$\delta m_H^2 = -2\lambda_f^2 \int_0^\infty \frac{d^4k}{(2\pi)^4}\left[\frac{k^2 + m_f^2}{\left(k^2 - m_f^2\right)^2}\right] \quad + \quad \dots$$



Cutting off the loop integral

$$m_H^2 = m_0^2 + \delta m_H^2$$

$$\delta m_H^2 = -2\lambda_f^2 \int_0^\Lambda \frac{d^4k}{(2\pi)^4}\left[\frac{k^2+m_f^2}{\left(k^2-m_f^2\right)^2}\right] \quad + \quad \ldots \quad \propto \quad \Lambda^2$$

- These loop integrals can’t go to infinity; they will be cut off at a high energy (short distance) scale where new particles and laws of physics come into play.
- Beyond the SM, we only know of two new higher-energy scales: the GUT scale (~10^{16} GeV) and the Planck scale (~10^{19} GeV) .
- Either way, corrections to the Higgs mass now depend on physics at much higher energy scales. This breakdown of *decoupling*, so very important in physics, leads to very *unnatural* fine-tuning of parameters:

$$m_H^2 = 120GeV = m_0^2 + C\Lambda^2$$

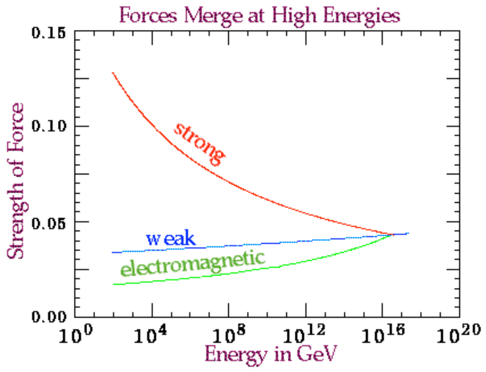
The Hierarchy problem

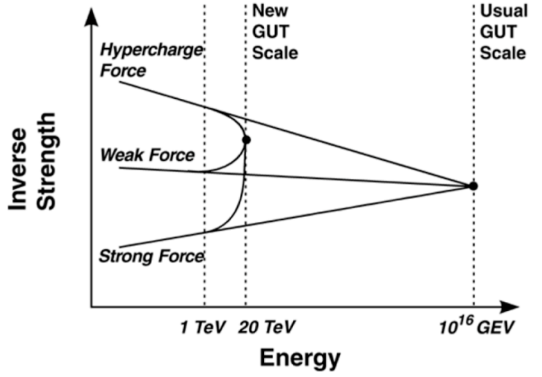
- This fine-tuning of parameters, this strong dependence of physics at the weak scale on the physics at (presumably) some much higher scale, is the hierarchy problem.
- If the loops are cut off at the scale of gravity, why is the scale of EW SSB so very different from the scale of gravity? Why is $M_W << M_{Pl}$?
- Equivalently, why is gravity so weak?

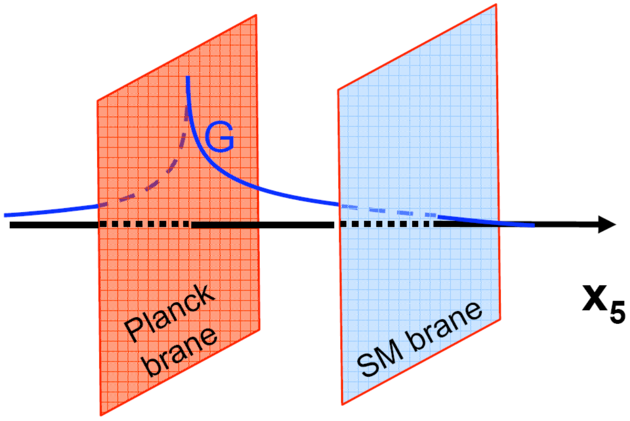
$$G_F = \frac{g^2}{4\sqrt{2}M_W^2} \quad >> \quad G_N = \frac{1}{M_{Pl}^2}$$

Possible solutions to the Hierarchy problem

- Some new physics appears at energy scales not far higher than the EW scale, to cut off (or otherwise “protect” against) the quadratic divergences. The “desert” between the EW and GUT/Planck scales is not empty!
- New physics changes the running of the couplings, bringing the GUT scale closer to the EW scale.
- Gravity is not as weak as we think, it’s only diluted in our 4D world but it’s EW-strong in, eg, 5 or more dimensions; $M_W \sim M_{Pl}^{5D}$.
- Fine-tuning is required; the theory is not natural. Theorists don’t accept this solution!







SUSY and the Hierarchy problem

- In SUSY, the loop diagrams that are quadratically divergent *cancel*, term by term, against equivalent diagrams involving superpartners.
- The cancellation is perfect if the particles and superpartners have the same mass.
- Else, the cancellation leaves residual contributions of order

$$\delta m_H^2 \approx m_{\tilde{f}}^2 - m_f^2$$

- If m_H is of order 100 GeV, then the masses of the superpartners must be only a little larger (any smaller and we would have detected them already), and definitely less than 1000 GeV.
- With these masses, some of them will be detected at the next accelerator, the LHC!