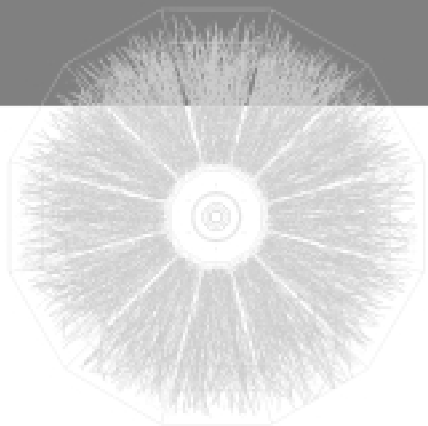

More
Interaction of Particles and Matter

In this section, we will discuss briefly the main interactions of charged particles, neutrons, and photons with matter. Understanding these interactions is important in the development of nuclear detectors, the design of radiation shielding, and the study of effects of radiation on living organisms. For the important latter case, we will examine the principal factors involved in stopping or attenuating a beam of particles.

Charged Particles

When a charged particle traverses matter, it loses energy mainly through collisions with electrons. This often leads to the ionization of the atoms in the matter, in which case the particle leaves a trail of ionized atoms in its path. If the energy of the particles is large compared with the ionization energies of the atoms, the energy loss in each encounter with an electron will be only a small fraction of the particle's energy. (A heavy particle cannot lose a large fraction of its energy to a free electron because of conservation of momentum, as we saw in Section 4-2. For example, when a billiard ball collides with a marble, only a very small fraction of the energy of the billiard ball can be lost.) Since the number of electrons in matter is so large, we can treat the loss of energy as continuous. After a fairly well-defined distance, called the *range,* the particle will have lost all its kinetic energy and will come to a stop. Near the end of the range, the view of energy loss as continuous is not valid because the kinetic energy is then small and individual encounters are important. For electrons, this can lead to a significant statistical variation in path length, but for protons and other heavy particles with energies of several MeV or more, the path lengths vary by only a few percent or less, for identical monoenergetic particles. The statistical variation of the path lengths is called *straggling.*

We can get an idea of the important factors in the stopping of a heavy charged particle by considering a simple model. Let ze be the charge and M the mass of a particle moving with speed v past an electron of mass m_e and charge e. Let b be the impact parameter. We can estimate the momentum imparted to the electron by assuming that the force has the constant value $F = kze^2/b^2$ for the time it takes the particles to pass the electron, which is of the order of $t \approx 2b/v$ (see Figure 12-18). The momentum given to the electron is equal to the impulse, which is of the order of magnitude

$$p \approx Ft = \frac{kze^2}{b^2}\frac{2b}{v} = \frac{2kze^2}{bv} \qquad \textbf{12-16}$$

(*Continued*)

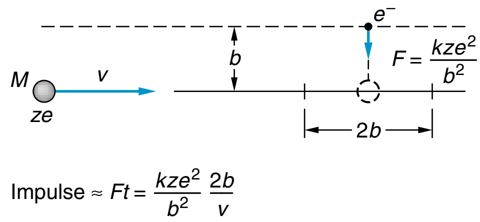

Fig. 12-18 Model for calculating the energy lost by a charged particle in a collision with an electron. The impulse given to the electron is of the order Ft, where $F = kze^2/b^2$ is the maximum force and $t = 2b/v$ is the time for the particle to pass the electron.

where k is the Coulomb constant. (The same result is obtained by integration of the variable impulse, assuming the particle moves in a straight line and the electron remains at rest.) The energy given to the electron is then

$$K_e = \frac{p^2}{2m_e} = \frac{2k^2z^2e^4}{m_e v^2 b^2} \qquad \textbf{12-17}$$

This is the kinetic energy lost by the particle in one encounter.

To find how many such encounters there are, consider a cylindrical shell of thickness db and length dx (see Figure 12-19). There are $Z\,(N_A/A)\rho 2\pi b\; db\; dx$ electrons in the shell, where Z is the atomic number, A the atomic weight, N_A Avogadro's number, and ρ the mass density. The energy lost to these electrons is then

$$-dK = \frac{2k^2z^2e^4}{m_e v^2 b^2} Z \frac{N_A}{A} 2\pi b\rho\; db\; dx$$

If we integrate from some minimum b to some maximum b, we obtain

$$-\frac{dK}{dx} = \frac{4\pi k^2 z^2 e^4 (Z/A) N_A \rho}{m_e v^2} L \qquad \textbf{12-18}$$

where

$$L = \ln\left(\frac{b_{\max}}{b_{\min}}\right) \qquad \textbf{12-19}$$

The range of the values of b can be estimated from general considerations. For example, this model is certainly not valid if the collision time is longer than the period the electron is in orbit. The requirement that $2b/v$ be less than this time sets an upper limit on b. The lower limit on b can be obtained from the requirement that the maximum velocity the electron can receive from a collision is $2v$ (obtained from the

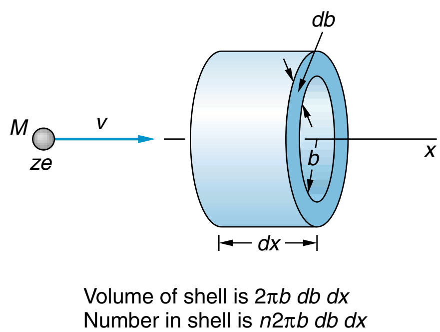

Fig. 12-19 In path length dx, the charged particle collides with $n2\pi b\; db\; dx$ electrons with impact parameters in db, where $n = Z(N_A/A)\rho$ is the number of electrons per unit volume in the material.

(Continued)

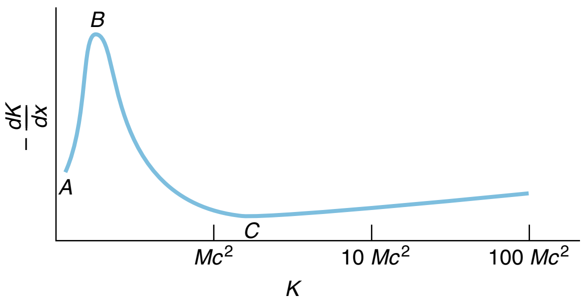

Fig. 12-20 Energy loss $-dK/dx$ vs. energy for a charged particle. The energy loss is approximately proportional to $1/v^2$, where v is the speed of the particle. Thus, in the nonrelativistic region B to C, $-dK/dx$ is proportional to K^{-1}, and in the relativistic region above C, $-dK/dx$ is roughly independent of K. At low energies, in the region A to B, the theory is complicated because the charge of the particle varies due to capture and loss of electrons.

classical mechanics of collisions of a heavy particle with a light particle). In any case, L is a slowly varying function of the energy, and the main dependence of the energy loss per unit length is given by factors other than L in Equation 12-18. We see that $-dK/dx$ varies inversely with the square of the velocity of the particle and is proportional to the square of the charge of the particle. Since $Z/A \approx 1/2$ for all matter, the energy loss is roughly proportional to the density of the material.

Figure 12-20 shows the experimentally measured rate of energy loss per unit path length $-dK/dx$ versus the energy of the ionizing particle. We can see from this figure that the rate of energy loss $-dK/dx$ is maximum at low energies and that at high energies it is approximately independent of the energy. Between points B and C on the curve, the energy loss is proportional to $1/v^2$, as suggested by Equation 12-18. For relativistic particles, those with energies beyond point C, the speed does not vary much with energy and the curve varies only because of the slow change of L. The low-energy portion of the curve, between A and B, is not given by our simple model. At very low energies the energy loss function is more complicated. Particles with kinetic energies greater than their rest energies mc^2 are called *minimum ionizing particles.* Their energy loss per unit path length is approximately constant, and their range is roughly proportional to their energy. Figure 12-21 shows the range versus energy curve for protons in air at very low energies, i.e., to the left of point A in Figure 12-20.

Since a charged particle loses energy through collisions with the electrons in a material, the greater the number of electrons, the greater the rate of energy loss. The

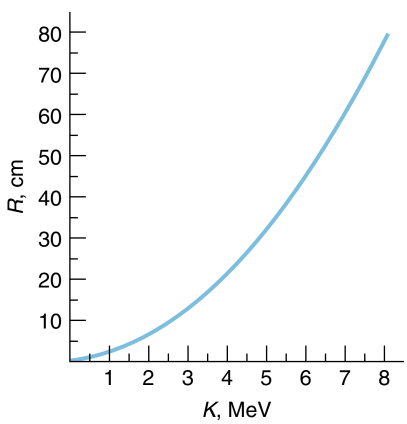

Fig. 12-21 Range vs. kinetic energy for protons in dry air. Except at low energies, the relationship between range and energy is approximately linear.

(*Continued*)

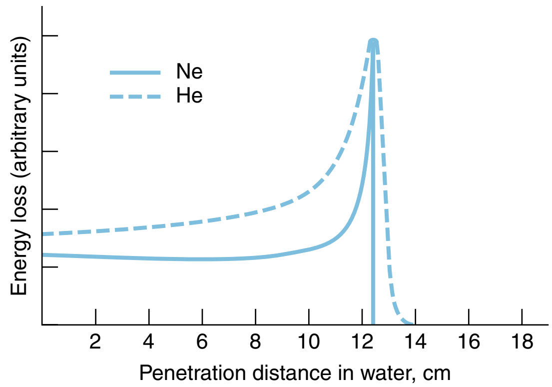

Fig. 12-22 Energy loss per unit length of helium ions and neon ions in water vs. depth of penetration. Most of the energy loss occurs near the end of the path in the Bragg peak. In general, the heavier the ion, the narrower the peak.

energy loss rate $-dK/dx$ is approximately proportional to the density of the material. For example, the range of a 6-MeV proton is about 40 cm in air; but in water, which is about 800 times more dense than air, its range is only 0.5 mm.

It is often convenient to divide out the density dependence in Equation 12-18 by defining thickness parameter:

$$l = \rho x \qquad \text{g/cm}^2 \tag{12-20}$$

If we then express the energy loss as dK/dl, it does not vary much from one material to another.

If the energy of the charged particle is large compared with its rest energy, the energy loss due to radiation as the particle slows down is important. This radiation is called *bremsstrahlung*. The ratio of the energy loss by radiation and that lost through ionization is proportional to the energy of the particle and to the atomic number Z of the stopping material. This ratio equals 1 for electrons of about 10 MeV in lead.

The fact that the rate of energy loss for heavy charged particles is very great at low energies (as seen from the low-energy peak in Figure 12-20) has important applications in nuclear radiation therapy. Figure 12-22 shows the energy loss per unit length versus penetration distance of charged particles in water. Most of the energy is deposited near the end of the range. The peak in this curve is called the *Bragg peak*. A beam of heavy charged particles can be used to destroy cancer cells at a given depth in the body without destroying other, healthy cells if the energy is carefully chosen so that most of the energy loss occurs at the proper depth.

Neutrons

Since neutrons are uncharged, their interaction with electrons in matter is via the magnetic moments of the two particles, rather than the Coulomb force. This interaction is used extensively to investigate the magnetism of bulk matter; however, it does not result in the transfer of kinetic energy to the electrons. Neutrons are removed from a beam by scattering from the nuclear potential or by capture. For kinetic energies that are large compared with thermal energies (kT), the most important processes are elastic and inelastic scattering. If we have a collimated neutron beam, any scattering or absorption will remove neutrons from the beam. This is very different from the case of a heavy charged particle, which undergoes many collisions that decrease the energy of the particle but do not remove it from the beam until its energy is essentially zero. A neutron is removed from the beam at its first collision.

(*Continued*)

The chance of a neutron's being removed from a beam within a given path distance is proportional to the number of neutrons in the beam and to the path distance. Let σ be the total cross section for the scattering plus the absorption of a neutron. If I is the incident intensity of the neutron beam (the number of particles per unit time per unit area), the number of neutrons removed from the beam per unit time will be $R = \sigma I$ per nucleus (Equation 12-5). If n is the number density of the nuclei (the nuclei per unit volume) and A is the area of the incident beam, the number of nuclei encountered in a distance dx is $nA\ dx$. The number of neutrons removed from the beam in a distance dx is thus

$$-dN = \sigma I(nA\ dx) = \sigma nN\,dx \qquad \textbf{12-21}$$

where $N = IA$ is the total number of neutrons per unit time in the beam. Solving Equation 12-21 for N, we obtain

$$N = N_0 e^{-\sigma n x} \qquad \textbf{12-22}$$

If we divide by each side of Equation 12-22 by the area of the beam, we obtain a similar equation for the intensity of the beam:

$$I = I_0 e^{-\sigma n x} \qquad \textbf{12-23}$$

We thus have an exponential decrease in the neutron intensity with penetration. After a certain characteristic distance $x_{1/2}$, half the neutrons in a beam are removed. After a second equal distance, half of the remaining neutrons are removed, and so on. Thus, there is no well-defined range.

At the half-penetration distance $x_{1/2}$, the number of neutrons will be $(1/2)N_0$. From Equation 12-22,

$$\begin{aligned} \frac{1}{2}N_0 &= N_0 e^{-\sigma n x_{1/2}} \\ e^{\sigma n x_{1/2}} &= 2 \qquad \textbf{12-24} \\ x_{1/2} &= \frac{\ln 2}{\sigma n} \end{aligned}$$

The main source of energy loss for a neutron is usually elastic scattering. (In materials of intermediate weight, such as iron and silicon, inelastic scattering is also important. We shall neglect inelastic scattering here.) The maximum energy loss possible in one elastic collision occurs when the collision is head-on. This can be calculated by considering a neutron of mass m with speed v_L making a head-on collision with a nucleus of mass M at rest in the laboratory frame (see Problem 12-24). The result is that the fractional energy lost by a neutron in one such collision is

$$\frac{-\Delta E}{E} = \frac{4mM}{(M + m)^2} = \frac{4(m/M)}{[1 + (m/M)]^2} \qquad \textbf{12-25}$$

This fraction has a maximum value of 1 when $M = m$ and approaches $4(m/M)$ for $M >> m$.

EXAMPLE 12-10 **Penetration of Neutrons in Copper** The total cross section for the scattering and absorption of neutrons of a certain energy is 0.3 barns for copper. (*a*) Find the fraction of neutrons of that energy that penetrates 10 cm in

(*Continued*)

copper. (*b*) At what distance will the neutron intensity drop to one-half its initial value?

Solution

(*a*) Using $n = 8.47 \times 10^{28}$ nuclei/m^3 for copper, we have

$$\sigma n x = (3.0 \times 10^{-28}\ \text{m}^2)(8.47 \times 10^{28}/\text{m}^3)(0.10\ \text{m}) = 0.254$$

According to Equation 12-22, if we have N_0 neutrons at $x = 0$, the number at $x =$ 0.10 m is

$$N = N_0 e^{-\sigma n x} = N_0 e^{-0.254} = 0.776 N_0 \qquad \textbf{12-26}$$

The fraction that penetrates 10 cm is thus 0.776, or 77.6 percent.
(*b*) For $n = 8.47 \times 10^{28}$ nuclei/m^3 and $\sigma = 0.3 \times 10^{-28}$ m^2, we have from Equation 12-24

$$x_{1/2} = \frac{\ln 2}{(0.3 \times 10^{-28}\ \text{m}^2)(8.47 \times 10^{28}/\text{m}^3)} = \frac{0.693}{2.54}\ \text{m} = 0.273\ \text{m} = 27.3\ \text{cm}$$

Photons

The intensity of a photon beam, like that of a neutron beam, decreases exponentially with distance in an absorbing material. The intensity versus penetration is given by Equation 12-23, where σ is the total cross section for absorption and scattering. The important processes that remove photons from a beam are the photoelectric effect,

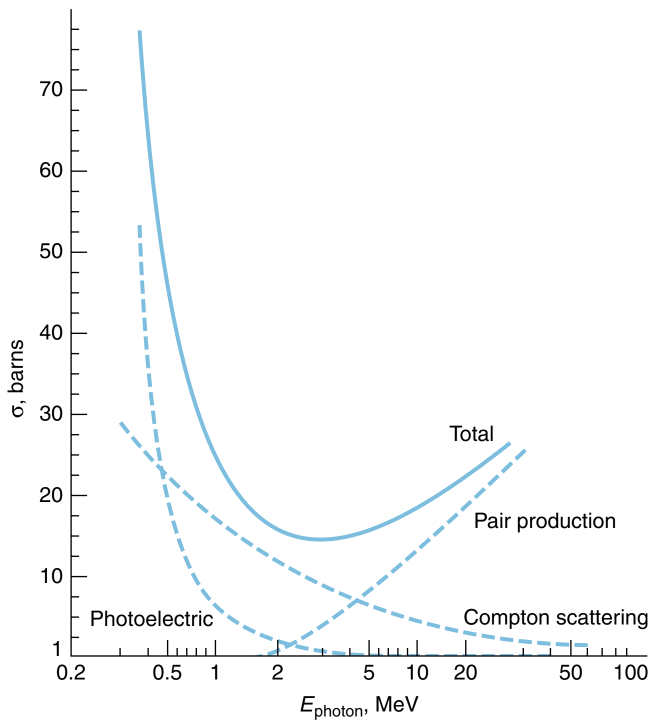

Fig. 12-23 Photon interaction cross sections vs. energy for lead. The total cross section is the sum of the cross sections for the photoelectric effect, Compton scattering, and pair production.

(*Continued*)

Compton scattering, and pair production. The total cross section for absorption and scattering is the sum of the partial cross sections for these three processes: σ_{pe}, σ_{cs}, σ_{pp}. These partial cross sections and the total cross section are shown as functions of energy in Figure 12-23. The cross section for the photoelectric effect dominates at very low energies, but it decreases rapidly with increasing energy. It is proportional to Z^4 or Z^5, depending on the energy region. If the photon energy is large compared with the binding energy of the electrons (a few keV), the electrons can be considered to be free, and Compton scattering is the principal mechanism for the removal of photons from the beam. The cross section for Compton scattering is proportional to Z. If the photon energy is greater than $2m_ec^2 = 1.02$ MeV, the photon can disappear, with the creation of an electron-positron pair. This process, called *pair production,* was described in Section 2-4. The cross section for pair production increases rapidly with the photon energy and is the dominant component of the total cross section at high energies. As was discussed in Section 2-4, pair production cannot occur in free space. If we consider the reaction $\gamma \rightarrow e^+ + e^-$, there is some reference frame in which the total momentum of the electron-positron pair is zero; however, there is no reference frame in which the photon's momentum is zero. Thus, momentum conservation requires that a nucleus be nearby to absorb momentum by recoil. The cross section for pair production is proportional to Z^2 of the absorbing material.