Physics 56500 Assignment #4 - Due March 8th

1. The tau lepton is a fundamental particle with spin ½. One of its decay modes is $\tau^- \to \pi^- \nu_\tau$ and because the neutrino has left-handed helicity, the angular distribution of the pion in the tau rest frame is given by
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{1}{2}(1 + P_\tau \cos\theta^*)$$
where $P_\tau = \pm 1$ for taus with right- or left-handed helicity. This can be re-written in terms of the variable $x = E_\pi/E_\tau$ as
$$\frac{1}{\Gamma}\frac{d\Gamma}{dx} = 1 + P_\tau(2x - 1)$$
which implies that right-handed taus would emit high energy pions whereas left-handed taus would produce pions that are boosted to low energies. In any case, the average polarization of a sample of tau decays can be determined by fitting the distribution of x to a function of this form.
   (a) Write a program to randomly generate the distribution of x given an average polarization $\langle P_\tau \rangle$. **Produce histograms of the generated distributions with exactly 200 events for the cases $\langle P_\tau \rangle = 0.3$ and $\langle P_\tau \rangle = -0.8$.**
   (b) In practice, the number of observed events will be Poisson distributed about the expected number of events, Y. **Produce a histogram filled with Poisson distributed random numbers with the expected yield $Y = 200$. Comment on the observed RMS of the distribution and compare it with the expected behavior when Y is large.**
   (c) Simulate a large number of experiments in which the number of events is Poisson distributed about the mean $Y = 200$ for the two cases of polarization $\langle P_\tau \rangle = 0.3$ and $\langle P_\tau \rangle = -0.8$ considered above. For each simulated experiment, perform a χ^2 fit to the generated distribution of x with a function of the form
$$f(x) = N\,\Delta x\,(1 + \alpha(2x - 1))$$
where N is the actual number of events in the simulated experiment and Δx is the histogram bin width. **Produce a histogram showing the difference between the fitted value, α, and the true value, $\langle P_\tau \rangle$. What is the mean (with uncertainty) of these distributions?**
   (d) Repeat the experiments using a binned likelihood fit (option “L” in the TH1::Fit() method). **What is the mean (with uncertainty) of the fitted polarization performed using binned likelihood fits?**

2. An experimental complication with the analysis described in question (1) is the fact that low energy pions may not be reconstructed. The expected distributions for $P_\tau = +1$ and $P_\tau = -1$ are therefore sculpted by the detector acceptance, but can be used as templates for fitting the observed distribution of x.

   Download the macro **TauPolarizationTemplates.C** and examine the code. This generates two template distributions for each polarization state, but simulates the effect of a loss of acceptance at low x.

   (a) Produce a graph showing the generated acceptance as a function of x.
   (b) Write an expression for the likelihood of observing n_i events in each bin of x as a function of the number of events, n_+ and n_- with $P_\tau = +1$ and $P_\tau = -1$, and the contents of the corresponding template distributions. Assume Poisson statistics in each bin.
   (c) The TMinuit class is a general purpose minimization package. The macro is set up to maximize a likelihood function (equivalently minimize $-\log L$) which depends on multiple parameters. In this example, the likelihood is calculated in the fcn() function and $-2 * \log L$ is returned in the variable f. Implement the likelihood function described in part (b) in the fcn() function.
   (d) Show how the fitted polarization $\langle P_{fit} \rangle$ can be calculated in terms of the fitted parameters n_+ and n_-.
   (e) Provide histograms showing that the fitted polarization is unbiased for the cases where the generated polarization is $\langle P_\tau \rangle = 0.3$ and $\langle P_\tau \rangle = -0.8$.
   (f) For each case, provide an example of one toy experiment showing the generated distribution of dN/dx and the fitted templates.