Physics 56500 Assignment #2 - Due February 20th

1. Write a program to generate pairs of correlated Gaussian random numbers with means $\mu_1 = 1$ and $\mu_2 = 2$ with covariance matrix V given by
$$V = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$
where $\sigma_1 = 0.1$, $\sigma_2 = 0.1$ and $\rho = 0.3$. Demonstrate that this program is correct by calculating the sample means and sample covariance matrix elements:
$\langle x_1 \rangle = \frac{1}{N}\sum_i x_1^{(i)}$, $\langle x_2 \rangle = \frac{1}{N}\sum_i x_2^{(i)}$, and $\langle (x_i - \langle x_i \rangle)(x_j - \langle x_j \rangle) \rangle$

2. In the second assignment you were asked to calculate improved estimates of track parameters z_0 and $\cot\theta$ subject to the constraint that a pair of tracks emerge from a common point in the s-z plane. You should have arrived at the expressions
$$z_0 = \hat{z}_0 - \frac{\sigma_{z_0}^2 + \rho s \sigma_{z_0}\sigma_{\cot\theta}}{AVA^T} g$$
$$\cot\theta = \cot\hat{\theta} - \frac{\rho\sigma_{z_0}\sigma_{\cot\theta} + s\sigma_{\cot\theta}^2}{AVA^T} g$$
where
$$g = \hat{z}_0 + s\cot\hat{\theta} - \hat{z}_0' - s'\cot\hat{\theta}'$$
and
$$AVA^T = \sigma_{z_0}^2 + 2\rho\sigma_{z_0}\sigma_{\cot\theta} + s^2\sigma_{\cot\theta}^2 + \sigma_{z'_0}^2 + 2\rho' s'\sigma_{z'_0}\sigma_{\cot\theta'} + s'^2\sigma_{\cot\theta'}^2$$
in which $\hat{z}_0$ and $\cot\hat{\theta}$ are the measured track parameters. But how can one check that these expressions are correct? The program **ToyModel.C** performs a Monte Carlo simulation of this process, and compares the distribution of the measured track parameters with the distributions of track parameters after imposing the constraint. It assumes that both tracks have the same covariance matrix elements but it does not take into account correlations between z_0 and $\cot\theta$.

   Modify the program so that it correctly generates correlated Gaussian random numbers with $\rho = 0.3$ and demonstrate that the constrained parameters have resolutions that are better than the unconstrained measurements.