Physics 56500 Assignment #2 - Due February 8th

1. The z-coordinates of two tracks are expressed in terms of their longitudinal impact parameters and their polar angles as follows:

$$z = z_0 + s\ \cot\theta$$
$$z' = z_0' + s'\cot\theta'$$

Suppose that the tracks are known to originate from a common point in the s-z plane for values of s and s' that have been determined from other information, for example, analysis of the transverse track parameters. Furthermore, suppose that the longitudinal track parameters have uncorrelated measured values $(\hat{z}_0, \cot\hat{\theta})$ and $(\hat{z}'_0, \cot\hat{\theta}')$ with covariance matrices

$$V = \begin{pmatrix} \sigma_{z_0}^2 & \rho\sigma_{z_0}\sigma_{\cot\theta} \\ \rho\sigma_{z_0}\sigma_{\cot\theta} & \sigma_{\cot}^2 \end{pmatrix}$$
$$V' = \begin{pmatrix} \sigma_{z'_0}^2 & \rho'\sigma_{z'_0}\sigma_{\cot\theta'} \\ \rho'\sigma_{z'_0}\sigma_{\cot\theta'} & \sigma_{\cot\theta'}^2 \end{pmatrix}$$

Determine expressions for the longitudinal track parameters that minimize the χ^2 statistic subject to the constraint $g = z_0 + s\ \cot\theta - z_0' - s'\cot\theta' = 0$.

2. Poisson processes have an unknown signal yield, S, and a predicted background yield, B. If the total number of signal and background events observed in an experiment is N, what is the upper 95% confidence limit on S?

3. The file **graph.root** contains a set of data points that are Gaussian distributed about the function

$$f(t) = \sin\omega t$$

where ω is a parameter to be determined by fitting the data to a function of this form. This is a one-parameter minimization problem because the amplitude and phase are assumed to be known precisely.

   (a) Following the example in the macro **GraphExample.C**, fit the data stored in the graph to a function of the form of $f(t)$ with ω as the only free parameter. Determine the value of ω that minimizes the χ^2 statistic and its (symmetric) 1-sigma confidence interval. Provide a graph of χ^2 as a function of ω.

   (b) The χ^2 statistic, with N degrees of freedom, has a probability density function given by

$$\frac{1}{\Gamma(N/2)}\ x^{N/2-1}e^{-x/2}$$

   which has a cumulative distribution function that can be calculated using the TMath::Gamma(N/2,x/2) method. Calculate the probability that the value of the χ^2 statistic would be greater than the minimal value obtained from the fit. In this case, N is the number of data points minus one, since this is a one parameter fit.