Physics 56500 Assignment #1 – Due January 23rd

1. First, to verify that you have access to ROOT, download the macro called "ErrorFunction.C" from the Phys56500 web site, and verify that it yields the following output:

```
jones105@planck:~/public_html/phys56500$ root -b ErrorFunction.C
  *******************************************
  *                                         *
  *        W E L C O M E  to  R O O T       *
  *                                         *
  *   Version   5.34/30     23 April 2015   *
  *                                         *
  *  You are welcome to visit our Web site  *
  *          http://root.cern.ch            *
  *                                         *
  *******************************************

ROOT 5.34/30 (v5-34-30@v5-34-30, Apr 23 2015, 18:31:46 on linuxx8664gcc)

CINT/ROOT C/C++ Interpreter version 5.18.00, July 2, 2010
Type ? for help. Commands must be C++ statements.
Enclose multiple statements between { }.
root [0]
Processing ErrorFunction.C...
ErrorFunction.C run by jones105 at Sun Jan 14 11:05:16 2018
         x     Erf(x)
---------- ----------
  -5.00000   -1.00000
  -2.00000   -0.99532
  -1.00000   -0.84270
   0.00000    0.00000
   1.00000    0.84270
   2.00000    0.99532
   5.00000    1.00000
root [1]
```

2. Show that the cumulative probability distribution for a Gaussian p.d.f.

$$g(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

can be written,

$$G(x;\mu,\sigma) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right).$$

3. If the random variable X is drawn from a Gaussian p.d.f., calculate the probability that
(a) $x > \mu - 5\sigma,\ x > \mu - 2\sigma, x > \mu - \sigma, x > \mu, x > \mu + \sigma, x > \mu + 2\sigma, x > \mu + 5\sigma$
(b) $|x-\mu| < 5\sigma, |x-\mu| < 2\sigma, |x-\mu| < \sigma$

4. The χ^2 statistic for two correlated measurements, $y_1 \pm \sigma_1$ and $y_2 \pm \sigma_2$, with correlation coefficient, ρ, is written:

$$\chi^2 = \begin{pmatrix} m - y_1 & m - y_2 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}^{-1} \begin{pmatrix} m - y_1 \\ m - y_2 \end{pmatrix}.$$

(a) Show that the value of m that minimizes χ^2 is

$$\widehat{m} = \frac{y_1(\sigma_2^2 - \rho\sigma_1\sigma_2) + y_2(\sigma_1^2 - \rho\sigma_1\sigma_2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$$

(b) Show that the variance in $\widehat{m}$ is

$$\sigma_{\widehat{m}}^2 = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}.$$