Physics 565 - Fall 2010, Assignment #4, Due Feburary 24$^{\text{th}}$

1. Consider the Lagrangian density, constructed from a Yang-Mills field with SU(2) symmetry:

$$\mathcal{L} = (D^\mu\vec{\phi})\cdot(D_\mu\vec{\phi}) - m^2\vec{\phi}\cdot\vec{\phi} - \frac{1}{4}\vec{F}^{\mu\nu}\cdot\vec{F}_{\mu\nu} \tag{1}$$

in which

$$D_\mu\vec{\phi} = \partial_\mu\vec{\phi} + g\vec{W}_\mu\times\vec{\phi} \tag{2}$$

$$\vec{F}_{\mu\nu} = \partial_\mu\vec{W}_\nu - \partial_\nu\vec{W}_\mu + g\vec{W}_\mu\times\vec{W}_\nu \tag{3}$$

(a) If the fields change under a gauge transformation as follows

$$\vec{W}_\mu \to \vec{W}'_\mu = \vec{W}_\mu - \vec{\lambda}(x)\times\vec{W}_\mu + \frac{1}{g}\partial_\mu\vec{\lambda}(x) \tag{4}$$

show that the term $-\frac{1}{4}\vec{F}^{\mu\nu}\cdot\vec{F}_{\mu\nu}$ is gauge invariant, to first order in $\vec{\lambda}$.

(b) Expand the Lagrangian 1 as a power series in g, explicitly listing the distinct terms at each power in g.

(c) Show that a mass term for the vector field of the form

$$\mathcal{L}_m = m^2\vec{W}_\mu\cdot\vec{W}^\mu \tag{5}$$

is *not* gauge invariant.