Physics 564 - Fall 2005, Assignment #2

This set of exercises will convince you that the quark model can be very good at explaining the properties of hadrons. It will also give you some experience using the MINUIT minimization class, which is very widely used in the field of High Energy Physics.

Masses of constituent quarks

We proposed an emperical mass formula for the baryons based on the masses of their constituent quarks and the hyperfine splitting between their spin states:

$$M_{q_1 q_2 q_3} = m_{q_1} + m_{q_2} + m_{q_3} + \frac{\kappa \vec{s_1} \cdot \vec{s_2}}{m_{q_1} m_{q_2}} + \frac{\kappa \vec{s_2} \cdot \vec{s_3}}{m_{q_2} m_{q_3}} + \frac{\kappa \vec{s_1} \cdot \vec{s_3}}{m_{q_1} m_{q_3}} \qquad (1)$$

where κ is a constant with dimensions $[\text{mass}]^3$. Recall that the expectation values for the $\vec{s_i} \cdot \vec{s_j}$ operators was given by

$$\begin{aligned} \langle \vec{s_1} \cdot \vec{s_2} \rangle &= \frac{1}{2}[S(S+1) - s_1(s_1+1) - s_2(s_2+1)] \\ &= \begin{cases} 1/4 & \text{when S=1} \\ -3/4 & \text{when S=0} \end{cases} \end{aligned}$$

We will also need to use the expectation value of the operator

$$\begin{aligned} \langle (\vec{s_1} + \vec{s_2}) \cdot \vec{s_3} \rangle &= \frac{1}{2}[S(S+1) - s_{12}(s_{12}+1) - s_3(s_3+1)] \\ &= \begin{cases} 0 & \text{when } s_{12} = 0 \\ -1 & \text{when } s_{12} = 1 \text{ and } S = 1/2 \\ 1/2 & \text{when } s_{12} = 1 \text{ and } S = 3/2 \end{cases} \end{aligned}$$

Since the spins of the quarks in the spin 3/2 baryons are all aligned, every combination of $\vec{s_i} \cdot \vec{s_j}$ has an expectation value of 1/4. Thus, the masses of the spin 3/2 baryons are as follows:

$$\begin{aligned} m_\Delta &= 3m_u + \frac{3\kappa}{4m_u^2} \\ m_{\Sigma^*} &= 2m_u + m_s + \frac{\kappa}{4m_u^2} + \frac{\kappa}{2m_u m_s} \\ m_{\Xi^*} &= 2m_s + m_u + \frac{\kappa}{4m_s^2} + \frac{\kappa}{2m_u m_s} \\ m_\Omega &= 3m_s + \frac{3\kappa}{4m_s^2} \end{aligned}$$

where we assume that $m_u = m_d$.

The mass formulas for the spin 1/2 baryons must be constructed taking into account the fact that the overall spin× flavor wavefunction must be symmetric. Consider, for example, the proton (uud) which has spin 1/2. The two up quarks must be in the $J_z = 0$ state of a $J = 1$ multiplet since their total spin wavefunction must be symmetric:

$$|p\uparrow\rangle = \frac{1}{\sqrt{2}}(u\uparrow u\downarrow + u\downarrow u\uparrow)(\uparrow d) + \text{permutations} \tag{2}$$

Hence, the the proton has $S_{12} = 1$ and its mass formula is

$$\begin{aligned} m_p &= 3m_u + \frac{\kappa \vec{s_1}\cdot\vec{s_2}}{m_u^2} + \frac{\kappa(\vec{s_1}+\vec{s_2})\cdot\vec{s_3}}{m_u^2} \\ &= 3m_u + \frac{\kappa}{4m_u^2} - \frac{\kappa}{m_u^2} \\ &= 3m_u - \frac{3\kappa}{4m_u^2} \end{aligned}$$

Similarly, the Ξ mass has the formula

$$\begin{aligned} m_\Xi &= 2m_s + m_u + \frac{\kappa \vec{s_1}\cdot\vec{s_2}}{m_s^2} + \frac{\kappa(\vec{s_1}+\vec{s_2})\cdot\vec{s_3}}{m_u m_s} \\ &= 2m_s + m_u + \frac{\kappa}{4m_s^2} - \frac{\kappa}{m_u m_s} \end{aligned}$$

and the Σ mass has the formula

$$\begin{aligned} m_\Sigma &= 2m_u + m_s + \frac{\kappa \vec{s_1}\cdot\vec{s_2}}{m_u^2} + \frac{\kappa(\vec{s_1}+\vec{s_2})\cdot\vec{s_3}}{m_u m_s} \\ &= 2m_u + m_s + \frac{\kappa}{4m_u^2} - \frac{\kappa}{m_u m_s}. \end{aligned}$$

Finally, the Λ is an isospin singlet, so the u and d quarks are in an anti-symmetric combination. To render the overall wavefunction symmetric, the spins of the u and d quarks must also be in an anti-symmetric state, which will have $S_{12} = 0$. Thus, the mass formula for the Λ is

$$\begin{aligned} m_\Lambda &= 2m_u + m_s + \frac{\kappa \vec{s_1}\cdot\vec{s_2}}{m_u^2} + \frac{\kappa(\vec{s_1}+\vec{s_2})\cdot\vec{s_3}}{m_u m_s} \\ &= 2m_u + m_s - \frac{3\kappa}{4m_u^2}. \end{aligned}$$

1. Run the example program called "`baryons`" described on the web page

`http://www.physics.purdue.edu/~mjones/phys564/minuit_examples.html`

to fit for the masses of the u/d and the strange quark. What are the fitted values of the u/d and s quark masses? Comment on how well the model describes the mass spectrum of these baryons.

2. Like the Λ, the Λ_c is an isospin singlet state but it contains a charm quark instead of a strange quark. Similarly, the Σ_c, Σ_c^*, Ξ_c^* and Ω_c consist of (udc), (udc), (usc), (ssc) states, respectively. Modify the program so that it will fit for the mass of the charm quark by including the masses of the charm baryon states:

$$\begin{aligned} m_{\Lambda_c} &= 2284.9 \text{ MeV}/c^2 \\ m_{\Sigma_c} &= 2455.0 \text{ MeV}/c^2 \\ m_{\Sigma_c^*} &= 2517.5 \text{ MeV}/c^2 \\ m_{\Xi_c^*} &= 2644.5 \text{ MeV}/c^2 \\ m_{\Omega_c} &= 2697.5 \text{ MeV}/c^2. \end{aligned}$$

What value does the model determine for the charm quark mass? Compare the quality of the fit to the previous one.

3. The $D^0 = (c\overline{u})$ and $D^+ = (c\overline{d})$ are spin 0 members of an isospin doublet. The D^{*+} meson also a member of an isospin doublet but it has spin 1. The D^{*+} decays to $D^0\pi^+$ and to $D^+\pi^0$ via a strong transition. Calculate the branching fractions of these final states.

The $D_{s1}^+ = (c\overline{s})$ meson has spin 1 and since it contains no up or down quarks so it must be an isospin singlet. The D_{s1}^+ decays to the $D^{*0}K^+$ and $D^{*0}K^0$ final states. Calculate the branching fractions for these two final states.