

Physics 564 - Fall 2005, Assignment #1

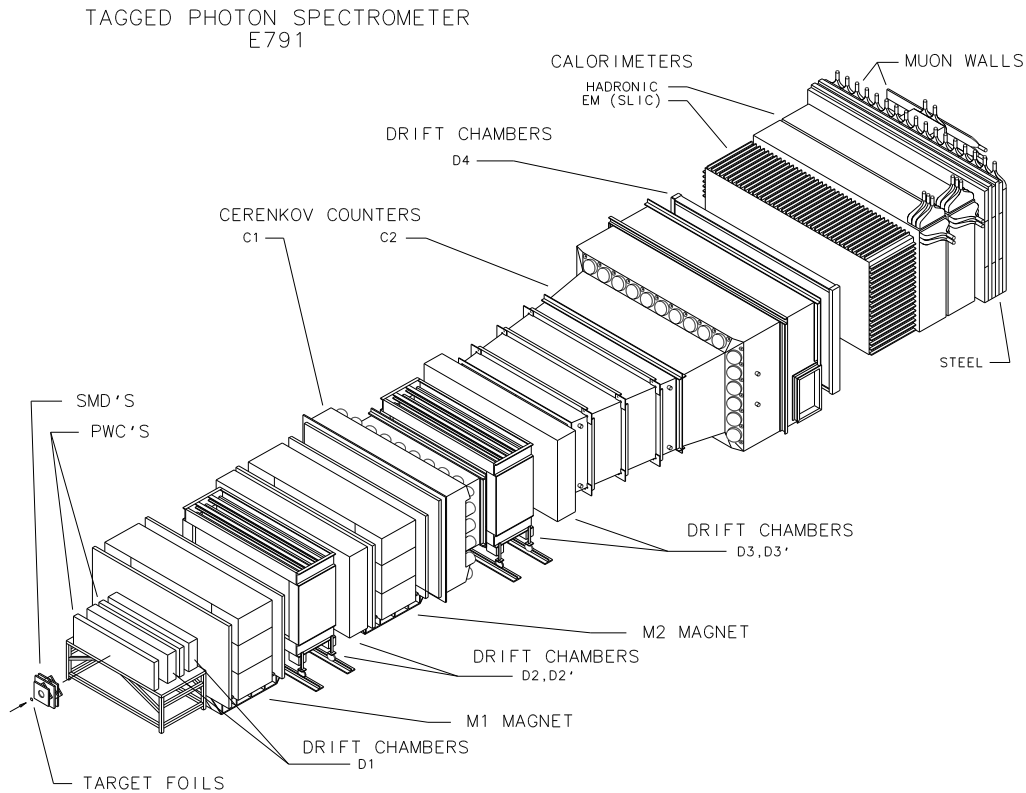
This set of exercises is intended to give you some experience with generating distributions of random numbers, creating, filling and displaying histograms, the simulation of 2-body particle decays and their analysis using relativistic kinematics in ways that are very frequently encountered in high energy physics research.

The information that describes how to log on to the PCN cluster, edit, compile and run your programs, can be accessed from the web page or directly at the URL's:

<http://www.physics.purdue.edu/~mjones/phys564/computing.html>
http://www.physics.purdue.edu/~mjones/phys564/root_examples.html

Background

Fermilab experiment E791 used a $500 \text{ GeV}/c \pi^-$ beam that hit a target made of carbon to produce D^0 mesons. Many particles are produced in these collisions and the D^0 's are produced with a range of energies. Their decays are recorded using a set of detector elements located downstream of the target as shown:



In this assignment, we will make a number of simplifying assumptions about the experiment. First, we will assume that the D^0 mesons are produced with no transverse momentum. That is, if the beam is in the $+z$ -direction, then when a D^0 meson is produced, its momentum will also be in

the $+z$ -direction. It is convenient to describe this momentum in terms of a variable, $x_F = p_z/p_z^{max}$ which represents the fraction of the momentum out of the maximum possible momentum with which the D^0 could have been produced, both evaluated in the center-of-mass frame. We will suppose that x_F is distributed according to a function that is of the form

$$\frac{1}{N} \frac{dN}{dx_F} = 6x_F(1 - x_F) \quad (1)$$

which is normalized so that

$$\int_0^1 \frac{1}{N} \frac{dN}{dx_F} dx_F = 1. \quad (2)$$

The momentum of charged particles is measured using two dipole magnets, M1 and M2. The resolution with which the momentum is measured can be parameterized by the formula:

$$\frac{\delta p}{p} = (0.02p)\% \quad (3)$$

where p is in GeV/ c . Hence, high momentum particles have less precise momentum measurements than do low momentum particles.

Kaons are identified using the Cerenkov counters, C1 and C2, but sometimes a pion is incorrectly identified as a kaon with a probability of approximately 10%.

Questions

1. What is the centre of mass energy, \sqrt{s} , in the collision between a π^+ in the beam and a stationary nucleon in the target?
2. What is the maximum longitudinal momentum, p_z^{max} , with which the D^0 can be produced, calculated in the centre-of-mass frame?
3. Using either the rejection or inversion method, produce a histogram of random numbers, x_F , distributed according to Equation 1. Fill another histogram with the distribution of $p_z = x_F p_z^{max}$, that is, the longitudinal momentum of the D^0 in the centre-of-mass frame.
4. Calculate p'_z , the longitudinal momentum of the D^0 in the lab frame and fill a histogram with its distribution.
5. Loop up the lifetime of the D^0 meson. Produce a histogram of the distribution of decay lengths in the lab. Estimate the probability that a D^0 could travel more than 1 cm in the detector before decaying.
6. The D^0 decays to $K^-\pi^+$ with a branching fraction of 3.8%. Calculate the energies of the K^- and π^+ decay products in the D^0 meson rest frame.
7. The decay $D^0 \rightarrow K^-\pi^+$ is isotropic in the D^0 rest frame. If θ^* represents the angle of the K^- with respect to the $+z$ -axis in the D^0 rest frame, fill histograms with the distributions of $\cos\theta^*$ and θ^* .
8. For a given decay angle, θ^* , calculate the 4-momenta of the K^- and π^+ in the D^0 rest frame. Then apply a Lorentz boost to calculate their 4-momenta in the lab frame. Fill histograms with $p_z(K^-)$ and $p_z(\pi^+)$, and the angles $\theta(K^-)$, $\theta(\pi^+)$, measured in the lab frame.

9. Using the experimental resolutions on the momentum measurements in the transverse and longitudinal directions to smear the calculated momenta of the K^- and π^+ . Then fill a histogram with the measured invariant mass of the $K^-\pi^+$ pair.

10. As described above, a pion is incorrectly identified as a kaon 10% of the time. Fill a histogram with the distribution of invariant mass calculated with the *wrong* mass hypothesis, that is, were the “kaon” energy is calculated from the pion momentum and a kaon mass and the “pion” energy is calculated from the kaon momentum and a pion mass. Compare the shapes of the distributions of these “right” and “wrong” mass assignments.