Physics 564 - Assignment #4

1. From the expressions

$$
\begin{aligned}
\frac{d\sigma}{d\Omega}(e^+_L e^-_R \to f_L \overline{f}_R) &= f_{\rm color}\frac{\alpha^2}{4s}(1+\cos\theta)^2|Q_f - r c^f_L c^e_L|^2 \\
\frac{d\sigma}{d\Omega}(e^+_L e^-_R \to f_R \overline{f}_L) &= f_{\rm color}\frac{\alpha^2}{4s}(1-\cos\theta)^2|Q_f - r c^f_R c^e_L|^2 \\
\frac{d\sigma}{d\Omega}(e^+_R e^-_L \to f_L \overline{f}_R) &= f_{\rm color}\frac{\alpha^2}{4s}(1-\cos\theta)^2|Q_f - r c^f_L c^e_R|^2 \\
\frac{d\sigma}{d\Omega}(e^+_R e^-_L \to f_R \overline{f}_L) &= f_{\rm color}\frac{\alpha^2}{4s}(1+\cos\theta)^2|Q_f - r c^f_R c^e_R|^2
\end{aligned}
$$

where $c^f_L = c^f_V + c^f_A$ and $c^f_R = c^f_V - c^f_A$ and

$$
r = \frac{\sqrt{2} G M_Z^2}{s - M_Z^2 + i M_Z \Gamma_Z}\left(\frac{s}{4\pi\alpha}\right)
$$

calculate the forward-backward asymmetry for the $\mu^+\mu^-$ final state at $\sqrt{s} = M_Z$. The LEP experiments measured an average forward-backward asymmetry of $A^\mu_{FB} = 0.0141$ at $\sqrt{s} = 91.2$ GeV. Use this to calculate $\sin^2\theta_W$.

2. The SLC linear collider produced a polarized electrons and collided them with unpolarized positrons. Using the expressions given in question 1, show that

$$
\sigma(e^-_{L/R} e^+ \to f\overline{f}) = f_{\rm color}\frac{4\pi\alpha^2}{3s} B^f_{L/R}
$$

where

$$
B^f_{\rm L/R} = Q_f^2 - 2Q_f(c^e_V \pm c^e_A)c^f_V {\rm Re}(r) + (c^e_V \pm c^e_A)((c^f_V)^2 + (c^f_A)^2)|r|^2
$$

3. The Left-Right asymmetry is defined in terms of the cross sections for scattering left-handed and right-handed electrons from unpolarized positrons:

$$
A^f_{LR} = \frac{\sigma(e^-_L e^+ \to f\overline{f}) - \sigma(e^-_R e^+ \to f\overline{f})}{\sigma(e^-_L e^+ \to f\overline{f}) + \sigma(e^-_R e^+ \to f\overline{f})}.
$$

Show that when $\sqrt{s} = M_Z$, A^f_{LR} has the expression

$$
A^f_{LR}(s = M_Z^2) = \frac{2c^e_V c^e_A}{(c^e_V)^2 + (c^e_A)^2}
$$

4. In practice, the electrons produced at the SLC are not perfectly polarized. Instead, the polarization of a bunch of electrons is defined by

$$P = \frac{n_L - n_R}{n_L + n_R}$$

where n_L and n_R are the numbers of left- and right-handed electrons in each bunch. Many electroweak measurements carried out at the SLC rely on an accurate measurement of the electron beam polarization. For an electron beam with polarization P, show that the measured left-right asymmetry is given by

$$A^f_{LR}|_{\mathrm{measured}} = P A^f_{LR}.$$