Physics 564 - Assignment #2

1. Cross sections are expressed in terms of the reduced matrix element, $\mathcal{M}$, the Lorentz invariant phase space, dQ and the flux factor, F as follows:

$$d\sigma = |\mathcal{M}|^2 \frac{dQ}{F}. \tag{1}$$

For a reaction $A + B \to C + D$, the phase space and flux factors are defined:

$$\begin{aligned} dQ &= \frac{d^3p_C}{(2\pi)^3 2E_C}\frac{d^3p_D}{(2\pi)^3 2E_D} \cdot (2\pi)^4 \delta^4(p_C + p_D - p_A - p_B) \\ F &= 4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2} \end{aligned}$$

In the centre-of-mass frame, show that dQ and F can be expressed as follows:

$$\begin{aligned} dQ &= \frac{1}{4\pi^2}\frac{p_f}{4\sqrt{s}} \\ F &= 4p_i\sqrt{s} \end{aligned}$$

where p_i and p_f are the momenta of the initial and final state particles and $s = (E_A + E_B)^2$ is the total centre-of-mass energy squared.

(Recall that the Dirac delta function has the property that $\delta(f(x))dx = (\delta(u)/f'(x))du$.)

2. The Dirac spinors $u^{(s)}(p)$ satisfied the Dirac equation:

$$(\vec{\alpha}\cdot\vec{p}+\beta m)u^{(s)}(p) = Eu^{(s)}(p) \tag{2}$$

and are therefore eigenvectors of the Hamiltonian with energy, E. These were written as:

$$u^{(s)}(p) = \sqrt{E+m}\begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{pmatrix} \tag{3}$$

where $\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ corresponding to an electron with spin up or spin down when projected along the z-axis.

An alternative procedure is to define solutions of the form

$$u_{L/R}(p) = \sqrt{E+m}\begin{pmatrix} \chi_{L/R} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_{L/R} \end{pmatrix} \tag{4}$$

such that $u_{L/R}(p)$ are also eigenvectors of the matrix

$$\frac{1}{2}\vec{\Sigma}\cdot\hat{p} = \frac{1}{2}\begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}. \tag{5}$$

Show that the eigenvalues are $\pm 1/2$. Can you identify a coordinate system in which $\frac{1}{2}\vec{\Sigma}\cdot\hat{p}$ is diagonal?

3. Let $u_L(p)$ and $u_R(p)$ represent the two spinors that are eigenvectors of the matrix $\frac{1}{2}\vec{\Sigma}\cdot\hat{p}$ with eigenvalues $\lambda = +1/2$ and $-1/2$, respectively. Spin 1/2 particles with $\lambda = +1/2$ have their spin aligned along their momentum vector and are called "right-handed", while those with $\lambda = -1/2$ have their spin alligned opposite their momentum and are called "left-handed".

In the limit where $E \gg m$, show that the projection operators

$$P_L = \frac{1}{2}(1-\gamma^5) \tag{6}$$

and

$$P_R = \frac{1}{2}(1+\gamma^5) \tag{7}$$

satisfy

$$\begin{aligned} P_R u_R(p) &= u_R(p) \\ P_L u_L(p) &= u_L(p) \\ P_L u_R(p) &= 0 \\ P_R u_L(p) &= 0 \end{aligned}$$