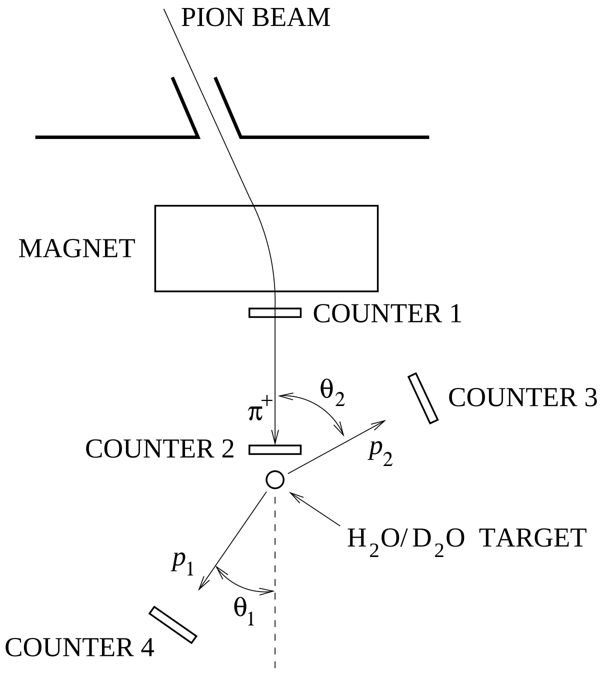
Physics 564 - Assignment #1

1. In 1951, the Berkeley Cyclotron was used to produce a beam of pions that hit a heavy water target to study the reaction $\pi^+ + d \rightarrow p + p$ as shown:

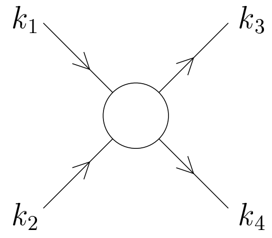


The differential cross section, $d\sigma/d\Omega$, was determined from the difference in counting rates measured with D_2O and H_2O targets. The magnet selected pions with a narrow range of kinetic energies centered at 70 MeV. The protons are emitted with an angle θ^* in their centre-of-mass frame, measured with respect to the incident pion beam direction.

(a) Calculate the angles θ_1 and θ_2 in the lab frame.

(b) The reaction is identified by simultaneous hits in the four scintillation counters. At what angles should counters 3 and 4 be placed to select events for which $\theta^* = 30°$?

2. The kinematics of a generic $2 \to 2$ scattering experiment can be described by the 4-momenta of the incomming and outgoing particles as shown:



The *Mandlestam variables*, s, t, and u, are defined

$$\begin{aligned} s &= (k_1 + k_2)^2 \\ t &= (k_1 - k_3)^2 = (k_2 - k_4)^2 \\ u &= (k_1 - k_4)^2 = (k_2 - k_3)^2. \end{aligned}$$

(Note that this uses a different sign convention than the one used in Perkins.)

(a) Show that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$.

The advantage of expressing quantities in terms of the Mandlestam variables is that they are Lorentz invariant. However, experiments are often performed in either the center-of-mass frame of particles 1 and 2, or in a fixed targed mode, where particle 2 is initially at rest.

(b) In the case where the masses of the particles is small and can be ignored, show that

$$\begin{aligned} s &= 4p^2 \\ t &= -4p^2 \sin^2(\theta/2) \\ u &= -4p^2 \cos^2(\theta/2) \end{aligned}$$

where p is the momentum of particle 1 and θ is the angle between particles 1 and 3, in the center-of-mass frame.

3. The D^{*+} meson is a $c\overline{d}$ pair with the spins of both quarks aligned. It decays almost exclusively to either $D^+\pi^0$ or $D^0\pi^+$ final states. Calculate the expected branching fractions to these final states and compare with experiment.

4. The quark model wavefunctions for the π^0, ρ, ω and ϕ mesons can be written:

$$\begin{aligned}
|\pi^0\rangle &= \frac{1}{2}\Big(|d\uparrow\overline{d}\downarrow\rangle - |u\uparrow\overline{u}\downarrow\rangle - |d\downarrow\overline{d}\uparrow\rangle + |u\downarrow\overline{u}\uparrow\rangle\Big)\\
|\rho\rangle &= \frac{1}{\sqrt{2}}\Big(|d\uparrow\overline{d}\uparrow\rangle - |u\uparrow\overline{u}\uparrow\rangle\Big)\\
|\omega\rangle &= \frac{1}{\sqrt{2}}\Big(|d\uparrow\overline{d}\uparrow\rangle + |u\uparrow\overline{u}\uparrow\rangle\Big)\\
|\phi\rangle &= |s\uparrow\overline{s}\uparrow\rangle
\end{aligned}$$

The Hamiltonian that describes the coupling of a photon to a meson via the electromagnetic interaction contains a term of the form

$$H_{EM} = -\sqrt{2}\sum_{i=1,2}\frac{q_i}{2m_i}(\sigma_-)_i$$

where $\sigma_-|\uparrow\rangle = |\downarrow\rangle$ and $\sigma_-|\downarrow\rangle = 0$.

(a) Calculate the ratios of the decay rates (partial widths) for $\phi\to\pi^0\gamma$, $\rho\to\pi^0\gamma$ and $\omega\to\pi^0\gamma$.

(b) The ρ and ω mesons have total widths of 150 MeV and 8.44 MeV and measured branching fractions to $\pi^0\gamma$ of (4.5×10^{-4}) and 8.5%, respectively. Compare the predicted ratios of partial widths with the experimental measurements.

5. Using the quark model wavefunctions for the vector mesons:

$$\begin{aligned}
|\rho\rangle &= \frac{1}{\sqrt{2}}\Big(|d\overline{d}\rangle - |u\overline{u}\rangle\Big)\\
|\omega\rangle &= \frac{1}{\sqrt{2}}\Big(|d\overline{d}\rangle + |u\overline{u}\rangle\Big)\\
|\phi\rangle &= |s\overline{s}\rangle\\
|\psi\rangle &= |c\overline{c}\rangle\\
|\Upsilon\rangle &= |b\overline{b}\rangle
\end{aligned}$$

use the Van Royen-Weisskopf formula to calculate the ratios of their partial widths for the electromagnetic decays $V\to e^+e^-$. Compare these with the ratios of the measured partial widths.