Phys 56400 Assignment #4

1. The proper time in the rest frame of the kaon is

$$t = \frac{d}{\gamma\beta} = \frac{dm}{p}$$

The fraction that has not decayed after travelling this distance is

$$f = e^{-t/\tau} = \exp\left(-\frac{dm}{p\tau}\right)$$

where τ is the kaon lifetime.

Hence, $d = -\frac{p\tau}{m} \log f$.

For $p = 120$ GeV/c
$m = 497.6$ MeV/c^2
$\tau = 8.954 \times 10^{-11}$ s
$f = 10^{-4}$,

$$d = -\frac{(120 \text{ GeV/c})(8.954 \times 10^{-11} \text{ s})}{(0.4976 \text{ GeV/c}^2)} \log 10^{-4}$$

$$= (0.199 \times 10^{-6} \text{ s})(2.998 \times 10^{8} \text{ m/s})$$

$$= 59.6 \text{ m}.$$

For $f = 10^{-6}$,

$$d = -\frac{(120 \text{ GeV/c})(8.954 \times 10^{-11} \text{ s})}{(0.4976 \text{ GeV/c}^2)} \log 10^{-6}$$

$$= 89.4 \text{ m}.$$

2. Consider a thin slab of material with cross sectional area A and thickness Δx.

The number of target nuclei is

$$n = N_A \cdot \frac{\rho}{m} A \Delta x$$

and the probability of interacting is

$$\frac{\Delta N}{N} = -\frac{\sigma}{A} \cdot n = -\sigma N_A \frac{\rho}{m} \Delta x .$$

Hence, $N(x) = N_0 e^{-\Gamma x}$

where $\Gamma = \sigma N_A \frac{\rho}{m}$.

For $\bar{K}^0$ with cross section $\bar{\sigma}$ we have

$\bar{N}(x) = \bar{N}_0 e^{-\bar{\Gamma} x}$

where $\bar{\Gamma} = \bar{\sigma} N_A \frac{\rho}{m}$.

3. A pure K_L^0 beam can be expressed in terms of its K^0 and $\bar{K}^0$ components:

$$K_L^0 = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right)$$

The probability that a K^0 emerges from the regenerator is

$$P_{K^0} = e^{-\Gamma x}$$

so the K^0 amplitude as a function of x is

$$K^0(x) = e^{-\Gamma x/2} K^0$$

Hence, the state emerging from the regenerator is

$$|K\rangle = \frac{1}{\sqrt{2}}\left(e^{-\Gamma x/2}|K^0\rangle + e^{-\bar{\Gamma} x/2}|\bar{K}^0\rangle\right)$$

The K_S^0 state is written $|K_S^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$

so the probability that a K_S^0 is in the beam is

$$P = |\langle K_S^0 | K(x)\rangle|^2$$
$$= \frac{1}{2}\left(e^{-\Gamma x/2} - e^{-\bar{\Gamma} x/2}\right)^2$$

If we write $\Gamma = \Gamma_0 + \Delta\Gamma$
$\bar{\Gamma} = \Gamma_0 - \Delta\Gamma$

then $P = e^{-\Gamma_0 x}\left(\frac{e^{-\Delta\Gamma x/2} - e^{\Delta\Gamma x/2}}{2}\right)^2$

Therefore, $P = e^{-\Gamma_0 x} \sinh^2\left(\frac{\Delta\Gamma x}{2}\right)$

where $\Delta\Gamma = \frac{1}{2}(\Gamma - \bar{\Gamma})$

4. If the probability of finding a K^0_s at $z = 0$, immediately after the regenerator is given by the expression above, then the probability as a function of z is

$$P(z) = \frac{p\tau}{m} e^{-zm/p\tau} e^{-\Gamma_0 x} \sinh^2\left(\frac{\Delta\Gamma x}{2}\right)$$

This is the probability of observing a K^0_s decay between z and $z + dz$.