Physics 56400 Assignment #5 – Due November 21st

1. The general expressions for c_V^f and c_A^f are

$$c_V^f = I_3^f - 2Q_f \sin^2\theta_W$$
$$c_A^f = I_3^f$$
$$c_L^f = c_V^f + c_A^f$$
$$c_R^f = c_V^f - c_A^f$$

where I_3^f is the third component of the weak isospin and Q_f is the charge of the fermion in units where $Q_e = -1$. Complete the entries, both symbolically and numerically, in the following table of couplings for quarks and leptons in the Standard Model using $\sin^2\theta_W = 0.231$:

f	Q_f	I_3^f	c_V^f	c_A^f	c_L^f	c_R^f
ν_e, ν_μ, ν_τ						
e^-, μ^-, τ^-						
u, c, t						
d, s, b						

2. Show that when $\sqrt{s} = M_Z$, the total cross section for $e^+e^- \to f\bar{f}$, where $f \neq e$, can be written

$$\sigma_{f\bar{f}} = \frac{12\pi\Gamma_e\Gamma_f}{M_Z^2\Gamma_Z^2}$$

where

$$\Gamma_f = N_c \frac{G_F M_Z^3}{6\pi\sqrt{2}}\left(\left(c_V^f\right)^2 + \left(c_A^f\right)^2 \right)$$

and calculate the peak cross section for $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to b\bar{b}$, and $e^+e^- \to c\bar{c}$. Also, explain why this expression is not expected to be valid for $f = e$.

3. Using the expressions

$$\frac{d\sigma}{d\Omega}\left(e_L^- e_R^+ \to f_L \bar{f}_R\right) = \frac{N_c \alpha^2}{4s}(1+\cos\theta)^2 \left|1 + r c_L^e c_L^f\right|^2$$

$$\frac{d\sigma}{d\Omega}\left(e_L^- e_R^+ \to f_R \bar{f}_L\right) = \frac{N_c \alpha^2}{4s}(1-\cos\theta)^2 \left|1 + r c_L^e c_R^f\right|^2$$

$$\frac{d\sigma}{d\Omega}\left(e_R^- e_L^+ \to f_L \bar{f}_R\right) = \frac{N_c \alpha^2}{4s}(1-\cos\theta)^2 \left|1 + r c_R^e c_L^f\right|^2$$

$$\frac{d\sigma}{d\Omega}\left(e_R^- e_L^+ \to f_R \bar{f}_L\right) = \frac{N_c \alpha^2}{4s}(1+\cos\theta)^2 \left|1 + r c_R^e c_R^f\right|^2$$

$$r = \frac{\sqrt{2} M_Z^2 G_F}{s - M_Z^2 + i\Gamma_Z M_Z}\left(\frac{s}{e^2}\right)$$

find an expression for A_{FB} for fermions $f \neq e$. Prepare three graphs showing A_{FB} as a function of $\sqrt{s}$ for $50 < \sqrt{s} < 200$ GeV for the following processes:

$$e^+e^- \to \mu^+\mu^-$$
$$e^+e^- \to b\bar{b}$$
$$e^+e^- \to c\bar{c}$$

assuming that $\sin^2\theta_W = 0.231$.