Physics 56400 Assignment #2 – Due September 10th

1. The elements of the Lorentz transformation matrix for a boost in the +x direction can be written:

$$L^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

   (a) Show that $L^{\rho}{}_{\mu} g_{\rho\sigma} L^{\sigma}{}_{\nu} = g_{\mu\nu}$.

   (b) Show that $a' \cdot b' = a \cdot b$ where $a'^{\mu} = L^{\mu}{}_{\rho} a^{\rho}$ and $b'^{\nu} = L^{\nu}{}_{\sigma} b^{\sigma}$.

   (c) Use the results from (a) and (b) to argue that expressions of the form $a \cdot b$ are Lorentz invariant.

2. Suppose an unstable particle A is moving in the +z direction and decays into two other particles, $A \rightarrow a + b$.

   (a) If particle a is emitted with an polar angle θ^* with respect to the +z axis in the rest frame of particle A, calculate the value of θ^* that will result in the largest angle, θ, with which particle a makes with the +z axis in the lab frame?

   (b) What is the minimum velocity of particle A for which particle a must always be travelling in the +z direction in the lab frame?